Supporting Information

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Flat-Plate and Delta-Wing Approaches to Estimating Tail-Generated Lift

The delta-wing model of avian tail function assumes that the tail generates lift in such a way that the amount of lift produced is proportional to the square of the maximum chord width (MCW) of the tail but is independent of the area of the tail. Thomas (1) explicitly argued that parallel-sided or tapering sections of the tail distal to the point at which MCW occurs would not contribute to lift generation.

An alternative approach to estimating the lift produced by an avian tail, although one whose applicability has not been defended by detailed arguments comparable with those supplied by Thomas (1) for the delta-wing model, is based on generic equations for a thin, flat airfoil. For such an airfoil, the lift coefficient (C_L) is zero if the angle of attack (α) is zero. For small angles of attack, the amount of increase in C_L per radian of α is given by the lift slope, a (2):

$$a = \frac{a_0}{1 + \frac{a_0}{\pi R_a}}. \quad \textbf{[S1]}$$

In this equation, R_a is the aspect ratio of the airfoil (defined as b^2/S, where b is the span of the airfoil and S is the area) and a_0 is the lift slope for an airfoil of infinite aspect ratio. Because for a thin, flat airfoil a_0 is conventionally taken to equal 2π/rad (3), the lift slope can be computed as:

$$a = \frac{2\pi}{1 + \frac{2S}{b^2}}. \quad \textbf{[S2]}$$

And the lift coefficient becomes:

$$C_L = \frac{2\pi\alpha}{1 + \frac{2S}{b^2}}. \quad \textbf{[S3]}$$

Because $L = 0.5 C_L \rho U^2 S$, where L is lift, ρ is air density, U is air speed, and S is airfoil area (2), the final theoretical expression for lift generation by a finite flat-plate airfoil is:

$$L_{FP} = \frac{\pi\alpha\rho U^2 S}{1 + \frac{2S}{b^2}}. \quad \textbf{[S4]}$$

For comparison, the delta-wing equation for lift generation developed by Thomas (1) and subsequently applied to *Archaeopteryx* by Gatesy and Dial (4) is:

$$L_{DW} = \frac{\pi\rho\alpha U^2 b^2}{4}. \quad \textbf{[S5]}$$

It should be noted that *b* is equivalent to MCW. Furthermore, Eq. **S4** becomes identical to Eq. **S5** for the special case of a triangular tail whose anteroposterior length is equal to its MCW.

We use the standard value of 1.23 kg/m^3 for air density and follow Gatesy and Dial (4) in considering an air speed of 5 m/s and an angle of attack for the tail of 15° = 0.26 rad. We treat the frond and fan of *Jeholornis* as separate lift-generating surfaces. Span and area estimates for the tails of *Archaeopteryx* and *Jeholornis*, in addition to flat-plate and delta-wing lift estimates for these tails obtained using Eqs. **S4** and **S5** respectively, are given in Table S1.

1. Thomas ALR (1993) On the aerodynamics of birds' tails. *Philos Trans R Soc Lond B Biol Sci* 340:361–380.
2. Pennycuick CJ (2008) *Modelling the Flying Bird* (Academic, New York), Vol 4.
3. O'Connor JK, et al. (2009) Phylogenetic support for a specialized clade of Cretaceous enantiornithine birds with information from a new species. *J Vertebr Paleontol* 29:188–204.
4. Gatesy SM, Dial KP (1996) From frond to fan: *Archaeopteryx* and the evolution of short-tailed birds. *Evolution* 50:2037–2048.

Fig. S1. Full-slab photograph of STM2-18 preserving remiges and proximal tail fan; distal end of the tail is not preserved. Proximal tail fan (area in red box) is enlarged in the *Inset*. (Scale bar: 1 cm.)

Fig. S2. Full-slab photograph of STM3-3 proximal tail fan; forelimbs and distal end of the tail are not preserved.

Fig. S3. Full-slab photograph of STM2-11 preserving body coverts on the neck. Coverts on the neck (area in red box) are enlarged in the *Inset* to show detail.

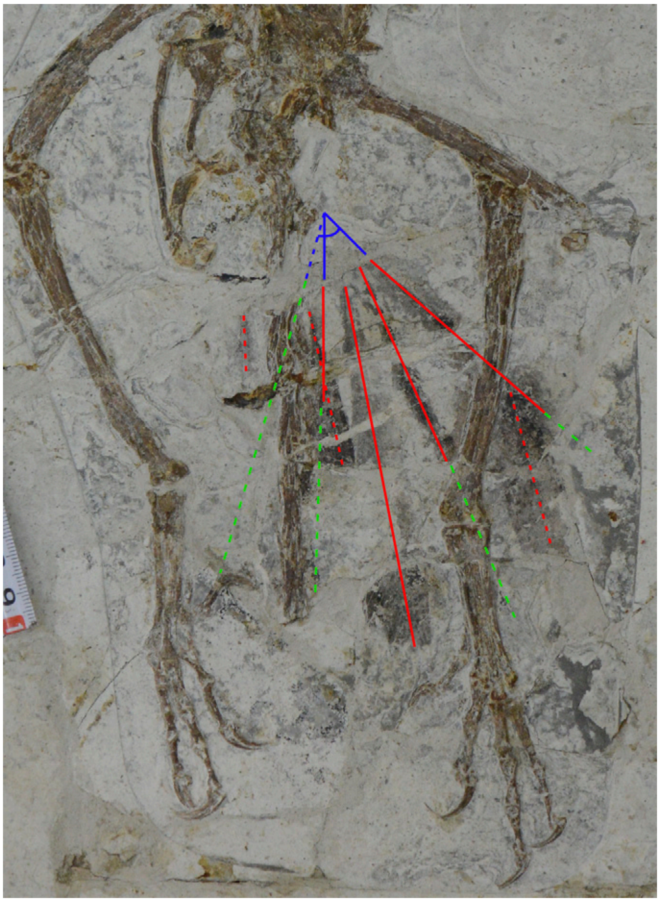

Fig. S4. Close up of the proximal tail fan of STM2-37 showing the estimated margins of the tail fan used for aerodynamic models.

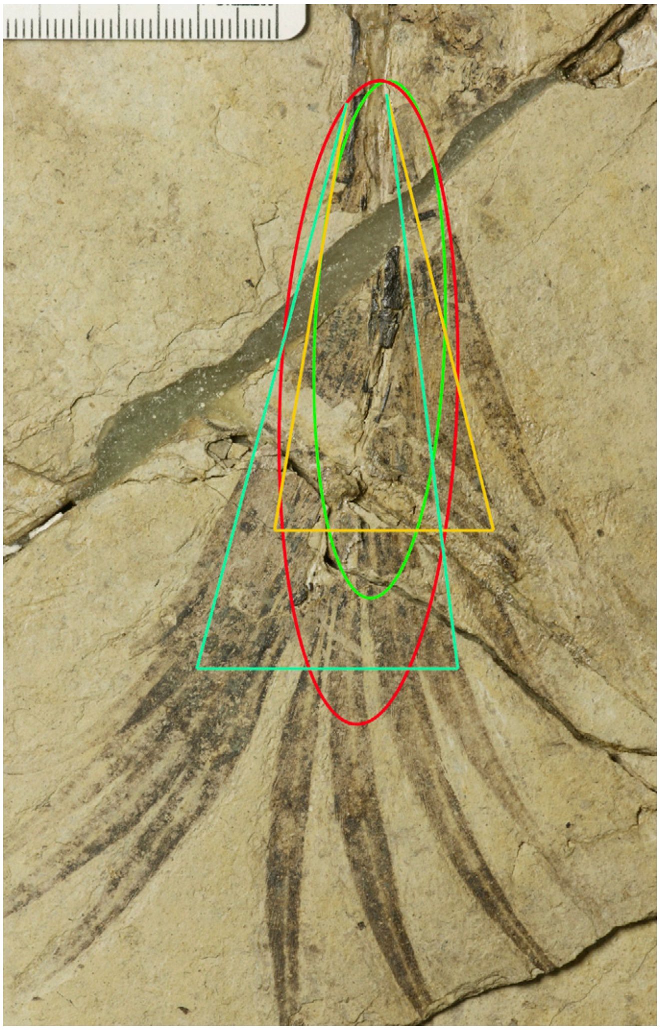

Fig. S5. Close up of the distal tail frond of SDM20090109.1 showing the estimated margins of the tail fan used for aerodynamic models.

Table S1. Tail plumage parameters and lift estimates for *Archaeopteryx* and *Jeholornis*

Tail	Span, m	Area, m^2	Flat-plate lift, N	Delta-wing lift, N
Archaeopteryx (min)	0.090	0.018	0.084	0.051
Archaeopteryx (max)	0.090	0.021	0.086	0.051
Jeholornis (fan, min)	0.080	0.006	0.053	0.040
Jeholornis (fan, max)	0.103	0.007	0.076	0.067
Jeholornis (frond, min)	0.033	0.001	0.009	0.007
Jeholornis (frond, max)	0.040	0.002	0.014	0.010

Minimum and maximum span and area estimates for the frond and fan of *Jeholornis* were obtained graphically from photos of STM2-37 (fan) and SDM 20090109 (frond). Because the former specimen is slightly larger, the span and area estimates for the frond were scaled up based on the discrepancy in femur length between the two specimens. Minimum and maximum area estimates for the tail frond of *Archaeopteryx* were obtained graphically from a photo of the London specimen. Span estimate for the tail frond of *Archaeopteryx* was taken from Gatesy and Dial (4).