Supporting information for: Controllable emission of a dipolar source coupled with a magneto-dielectric resonant subwavelength scatterer

Brice Rolly[1], Jean-Michel Geffrin[1], Redha Abdeddaim[1], Brian Stout[1], Nicolas Bonod[1*]

[1] CNRS, Aix-Marseille Université, Ecole Centrale Marseille, Institut Fresnel, Campus de St Jérôme, 13397 Marseille, France

[*] Corresponding author: nicolas.bonod@fresnel.fr

1 Derivation of the quadrupolar expression of the emission pattern

In this supporting information, we derive the analytic expression for the electric field radiated by the emitter-particle system that includes electric and magnetic contributions up to quadrupole order. This expression is derived from a Multipole T-matrix theory which can be systematically extended to any multipole order, and the expressions are rendered more compact by introducing the following abbreviations:

$$e^{i\varphi} \equiv \exp(-ik\hat{\mathbf{r}}\cdot\mathbf{r_1}) = \exp(-ikd\sin\theta\cos\phi). \tag{1a}$$

$$K_\mathrm{c} \equiv \frac{1}{4\pi\epsilon_0\epsilon_h}k^2 \ , \quad K_\mathrm{r} \equiv \frac{e^{ikr}}{r} \ , \quad K \equiv K_\mathrm{r}K_\mathrm{c} = \frac{e^{ikr}k^2}{4\pi\epsilon_0\epsilon_h r} \tag{1b}$$

where $\hat{\mathbf{r}}$ indicates the far-field observation direction, and ϵ_h the relative dielectric constant of the host material. The vector $\mathbf{r_1} = d\hat{\mathbf{x}}$ indicates the position vector of the center of the particle with respect to the emitter, while $e^{i\varphi}$ is the far-field phase shift associated with this separation. The constant K_c is associated with unit conversions, while K_r describes far-field spatial dependence.

The far-fields produced by the different dipoles (electric dipolar emitter, induced electric and magnetic dipoles excited in the particle) can be respectively cast:

$$\mathbf{E}_0^{\mathrm{ff}}(r,\theta,\phi) = p_0 K\sin\theta\hat{\mathbf{e}_\theta} \tag{2a}$$

$$\mathbf{E}_{\mathrm{d,e}}^{\mathrm{ff}}(r,\theta,\phi) = p_1 K e^{i\varphi}\sin\theta\hat{\mathbf{e}_\theta} \tag{2b}$$

$$\mathbf{E}_{\mathrm{d,m}}^{\mathrm{ff}}(r,\theta,\phi) = m_1 K e^{i\varphi}\frac{n_0}{c}(\cos\phi\hat{\mathbf{e}}_\theta - \cos\theta\sin\phi\hat{\mathbf{e}}_\phi) \tag{2c}$$

where p_0 is the emitter dipole moment while p_1 and m_1 are the induced electric and magnetic moments of the spherical particle.

The expressions generated by T-matrix theory can be rendered more physically transparent by introducing dimensionless "polarizabilities", $\tilde{\alpha}_n^{e(m)}$ defined in terms of their respective T-matrix coefficients, $t_n^{e(m)}$ (cf. ref.[1]):

$$\widetilde{\alpha}_n^e \equiv \frac{\alpha_n^e}{4\pi a^{n+2}} \equiv \frac{3}{2i(ka)^{n+2}} t_n^e \quad , \qquad \widetilde{\alpha}_n^m \equiv \frac{\alpha_n^m}{4\pi a^{n+2}} \equiv \frac{3}{2i(ka)^{n+2}} t_n^m \tag{3}$$

The ordinary (dimensioned) "polarizabilities", $\alpha_n^{e(m)}$, are defined here such that electric dipole polarizability, α_1^e, is in accord with its conventional definition of $\mathbf{p}_1 = \epsilon_0 \varepsilon_h \alpha_1^e \mathbf{E}$. This choice provides a clear definition of multipole polarizability, but since there is no universally accepted convention for higher orders, care must be taken when comparing our multipole polarizability with those found in other works.

With our definitions, the induced dipole moments can be compactly expressed:

$$p_1 = p_0 \gamma_1^e \tilde{\alpha}_1^e \ , \qquad \gamma_1^e \equiv e^{ikd}(k^2d^2 + ikd - 1)(a/d)^3 \tag{4a}$$

$$m_1 = p_0 \gamma_1^m \tilde{\alpha}_1^m c/n_0 \ , \qquad \gamma_1^m \equiv e^{ikd}(k^2d^2 + ikd)(a/d)^3 \tag{4b}$$

where $\gamma_1^{\mathrm{e,m}}$ are dimensionless coupling coefficients between the dipole emitter and the electric/magnetic induced dipole.

The fields produced by both induced quadrupoles can be derived following the method detailed in [1]. One finds therein that the field coefficient of a electric dipole emitter along the z axis is expressed $\mathbf{f}$, is $f_{\mathrm{n=1,m=0}} = -2i\sqrt{2\pi} p_0 K_{\mathrm{c}}/\sqrt{3}$. The coupling between an electric dipole emitter and the quadrupoles of the scatterer is then determined by the nonzero coefficients of the irregular translation-addition coefficients between the scatterer and the source [2]:

$$\begin{aligned} A_{2,-1,1,0}(kd, \pi/2, \pi) &= -A_{2,1,1,0}(kd, \pi/2, \pi) \\ &= -\sqrt{15}\frac{\exp(ikd)(k^3d^3 + 3ik^2d^2 - 6kd - 6i)}{2\sqrt{2}(kd)^4} \end{aligned} \tag{5a}$$

$$\begin{aligned} B_{2,-2,1,0}(kd, \pi/2, \pi) &= -B_{2,2,1,0}(kd, \pi/2, \pi) \\ &= \sqrt{15}\frac{\exp(ikd)(k^2d^2 + 3ikd - 3)}{2\sqrt{2}(kd)^3} \end{aligned} \tag{5b}$$

(the other $A_{2,\mathrm{m},1,0}(kd, \pi/2, \pi)$ and $B_{2,\mathrm{m},1,0}(kd, \pi/2, \pi)$ being null).

The induced electric and magnetic quadrupole fields in this notation are then:

$$\mathbf{E}^{\mathrm{ff}}_{\mathrm{Q,e,0}} \equiv \lim_{r\to\infty} \left[A_{2,-1,1,0}(kd, \pi/2, \pi)\mathbf{N}_{2,-1}(r, \theta, \phi) + A_{2,1,1,0}(kd, \pi/2, \pi)\mathbf{N}_{2,1}(r, \theta, \phi) \right] t_2^e f_{1,0} \tag{6a}$$

$$\mathbf{E}^{\mathrm{ff}}_{\mathrm{Q,m,0}} \equiv \lim_{r\to\infty} \left[B_{2,-2,1,0}(kd, \pi/2, \pi)\mathbf{M}_{2,-2}(r, \theta, \phi) + B_{2,2,1,0}(kd, \pi/2, \pi)\mathbf{M}_{2,2}(r, \theta, \phi) \right] t_2^m f_{1,0} \tag{6b}$$

where the vector partial waves $\mathbf{N}_{\mathrm{n,m}}$ and $\mathbf{M}_{\mathrm{n,m}}$ are also found in Ref.[1]. Invoking the analytical expressions for the A and B coefficients[2], and the dimensionless polarizabilities of Eq.(3), these far fields are simply expressed:

$$\mathbf{E}^{\mathrm{ff}}_{\mathrm{Q,e,0}} = KQ^e \bigg(\cos 2\theta \cos\phi \hat{\mathbf{e}}_\theta - \cos\theta \sin\phi \hat{\mathbf{e}}_\phi \bigg) \tag{7a}$$

$$\mathbf{E}^{\mathrm{ff}}_{\mathrm{Q,m,0}} = KQ^m \bigg(\sin\theta \cos 2\phi \hat{\mathbf{e}}_\theta - \frac{\sin 2\theta \sin 2\phi}{2} \hat{\mathbf{e}}_\phi \bigg), \tag{7b}$$

where we introduced electric and magnetic “quadrupole” moments, Q^e and Q^m, respectively defined:

$$Q^e \equiv p_0 \gamma_2^e \tilde{\alpha}_2^e \ , \qquad \gamma_2^e \equiv -\frac{5}{3}(k^3d^3 + 3ik^2d^2 - 6kd - 6i)\left(\frac{a}{d}\right)^4 e^{ikd} \tag{8a}$$

$$Q^m \equiv p_0 \gamma_2^m \tilde{\alpha}_2^m \ , \quad \gamma_2^m \equiv \frac{5}{3}(k^3d^3 + 3ik^2d^2 - 3kd)\left(\frac{a}{d}\right)^4 e^{ikd} \tag{8b}$$

We caution the reader however that different normalizations of the quadrupole moments exist in the literature and that our choice was motivated for reasons of notational simplicity.

Just like for the dipoles, the quadrupoles are centered at $\mathbf{r_1}$, and must be multiplied by the far-field phase shift:

$$\mathbf{E}^{\text{ff}}_{\text{Q,e}}(r,\theta,\phi) = Ke^{i\varphi}Q^e\left(\cos 2\theta\cos\phi\hat{\mathbf{e}}_\theta - \cos\theta\sin\phi\hat{\mathbf{e}}_\phi\right) \tag{9a}$$

$$\mathbf{E}^{\text{ff}}_{\text{Q,m}}(r,\theta,\phi) = Ke^{i\varphi}Q^m\left(\sin\theta\cos 2\phi\hat{\mathbf{e}}_\theta - \frac{\sin 2\theta\sin 2\phi}{2}\hat{\mathbf{e}}_\phi\right) \tag{9b}$$

The total, normalized far-field irradiance (with respect to the maximum electric dipole emitter far-field irradiance) can then be cast:

$$\begin{aligned} I(\theta,\phi) =& \frac{1}{p_0^2K^2}\left|\mathbf{E}^{\text{ff}}_0 + \mathbf{E}^{\text{ff}}_{\text{d,e}} + \mathbf{E}^{\text{ff}}_{\text{d,m}} + \mathbf{E}^{\text{ff}}_{\text{Q,e}} + \mathbf{E}^{\text{ff}}_{\text{Q,m}}\right|^2 \\ =& \Big| \left(\sin\theta(1 + e^{i\varphi}\gamma_1^e\tilde{\alpha}_1^e) + e^{i\varphi}\left(\gamma_1^m\tilde{\alpha}_1^m\cos\phi + \gamma_2^e\tilde{\alpha}_2^e\cos 2\theta\cos\phi + \gamma_2^m\tilde{\alpha}_2^m\sin\theta\cos 2\phi\right)\right)\hat{\mathbf{e}}_\theta \\ & - \left(\cos\theta\sin\phi(\gamma_1^m\tilde{\alpha}_1^m + \gamma_2^e\tilde{\alpha}_2^e) + \frac{1}{2}\gamma_2^m\tilde{\alpha}_2^m\sin 2\theta\sin 2\phi\right)e^{i\varphi}\hat{\mathbf{e}}_\phi\Big|^2 \end{aligned} \tag{10}$$

which leads to Eq.(1) of the main text.

References

[1] Stout, B., Devilez, A., Rolly, B., and Bonod, N. *J. Opt. Soc. Am. B* **28** (5), 1213–1223 (2011).

[2] Stout, B., Auger, J.-C., and Lafait, J. *J. Mod. Opt.* **49** (13), 2129–2152 (2002).