Electronic Supplementary Information for Stabilization of the Inverse Laplace Transform of Multiexponential Decay through Introduction of a Second Dimension

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1. Cramer-Rao Lower Bound Analysis

The advantage of the 2D approach as demonstrated in the paper is dependent upon the number of points, m, sampled in the indirect dimension, as well as the ratio R_{T1} of the T_1 relaxation time constants. We calculated the Cramer-Rao lower bound (CRLB) [1, 2, 3] as a function of m, using, as an example, a signal comprised of two with components (T_1, T_2, weight) = (25 ms, 100 ms, 60%) and (35 ms, 300 ms, 40%) at SNR = 100. Fig 1 shows that precision improved rapidly up to $\sim m = 6$, after which this improvement was less marked. We therefore selected m = 6 in our simulations and experiments as representing a reasonable tradeoff between experimental speed and accuracy. Similarly, we found the 2D approach to provide no benefit in the case of equal component T_1's, and to become increasingly advantageous with increasing difference between T_1 relaxation time constants (Fig. 2). Substantial improvements were seen for $R_{T1} > \sim 1.5$, with the degree of improvement tapering off for $R_{T1} > 2$. It is clear that these results for m and R_{T1} must be regarded as representative; they will clearly be dependent upon component SNR, T_2s and weights.

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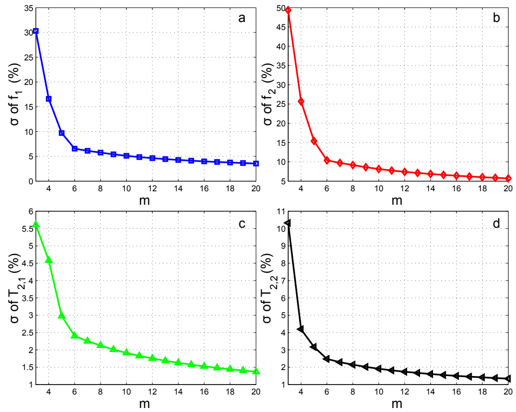

Figure 1: Cramer-Rao lower bound as a function of m for a simulated signal with two components with (T_1, T_2, weight) = (25 ms, 100 ms, 60%) and (35 ms, 300 ms, 40%) at SNR = 100.

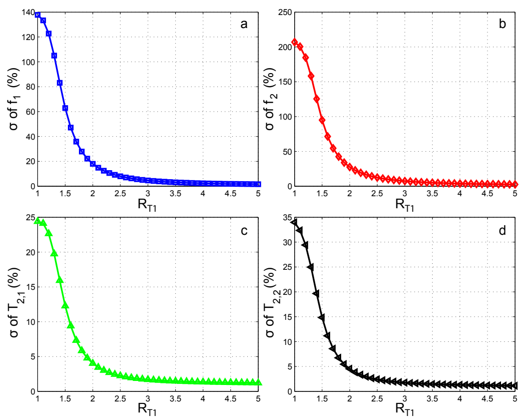

Figure 2: Cramer-Rao lower bound as a function of the ratio R_{T1} of the T_1 relaxation time constants for a simulated signal with two components with (T_1, T_2, weight) = (25 ms, 100 ms, 60%) and (35 ms, $R_{T1}\times$100 ms, 40%) at SNR = 100.

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statistical parameters, Bull. Calcutta Math. Soc. (37) (1945) 81–91.