Supplementary information for

Optical orbital angular momentum conservation during the transfer process from plasmonic vortex lens to light

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Theoretical analysis on the generating process surface plasmon (SP) vortices

The excitation of whole plasmonic vortex lens (PVLs) can be considered as convolution of a point source as following: [1]

$$E^{sp} \propto \sum_{n=1}^{p} \delta(r_m - r) G[\hat{n} \otimes \hat{n}] E_{\pm}^{in} \qquad (1).$$

The in-plane component of near-field SP electric field $E_{0+,m}$ in the slits of PVLs can be expressed as following:

$$E^{sp}{}_{0+,m}(r_i, z = 0) \propto G \times [E^{in}(r_m, z = 0) \times \hat{n}] \hat{n} \qquad (2),$$

where, $G = \frac{e^{ik_{sp}|r_i - r_m|}}{|r_i - r_m|^{1/2}}$ is the Huygens-Fresenel palsmonic propagator, $\hat{n} = f^{-1}(\frac{d^2 r_m}{ds^2})$ is the local unit normal vector determined from the curvature *f* and the arc length *s* of the nanoslit, and E^{in} is the incident electric field. The resulted SP electric filed is the integral of elementary point sources over the whole PVL as:

$$\begin{aligned} E^{sp} &= C_{in} \bullet E^{in} \\ &\propto \int_0^{2\pi} d\varphi e^{im\varphi - ik_{sp} r_i \cos\varphi} \hat{r} \otimes \hat{r} \bullet E^{in} \end{aligned} \qquad (3).$$

The incident electric field E^{in} is the $LG_{p,l}$ laser mode with linear polarization along X direction $\hat{x}$:

$$\begin{aligned} E^{in}(r, \varphi, z) &\propto \frac{p}{w} \exp[\frac{-ikr^2 z}{2(z_R^2 + z^2)} - \frac{r^2}{w^2}] \exp[-i(2p + l + 1) \arctan(z / z_R)] \\ &\times \exp[-il\phi] (-1)^p (\frac{\sqrt{2} r}{w})^{|l|} L_p^{|l|} (\frac{2r^2}{w^2}) \hat{x} \end{aligned} \qquad (4).$$

Here, *p* and *l* are the radial and azimuthal indexes of the mode respectively, *k* is the wavenumber of light in free space, *w* is the beam waist, z_R is the Rayleigh rang of the mode, and $L_p^{|l|}(x)$ is the Laguerre polynomials.

Since the Bessel function is $J_a(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{i(at - x \sin t)} dt$, with the waveguide model,[1-5] the corresponding z-directional field electric field of the SP vortices is mathematically represented by the *n*-th order Bessel function $J_n(x)$ and Laguerre-Gaussian function with a spiral phase profile given with a scalar equation by:

$$E_{z,n}(r, \varphi, z) \propto J_{|n|}(k_{sp} r) \exp[in(\varphi + \frac{\pi}{2})] L_p^{|l|}(\frac{2r^2}{w^2}) \exp(-\frac{r^2}{w^2}) \quad (5).$$

With the angular momentum operator $\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi}$, it can be found that the $LG_{p,l}$ laser mode possesses the orbital angular momentum of $l\hbar$ per photon, and the generated SP vortices have orbital angular momentum of $n\hbar = (l + m)\hbar$ per photon which represents the twisted wavefront.

Angular momentum conservation with the m=-7 PVL

The achieved experimental patterns and theoretical analysis of SP vortices with l=-1 and -2, and m=-7 are presented in Fig.1. The intensity distribution of cross sections (red line) through the center are presented in Fig.1 (c) and (d). The theoretical analysis shows that the total angular momentum per photon of SP vortices is $n = l + m$ and the total angular momentum is conserved.

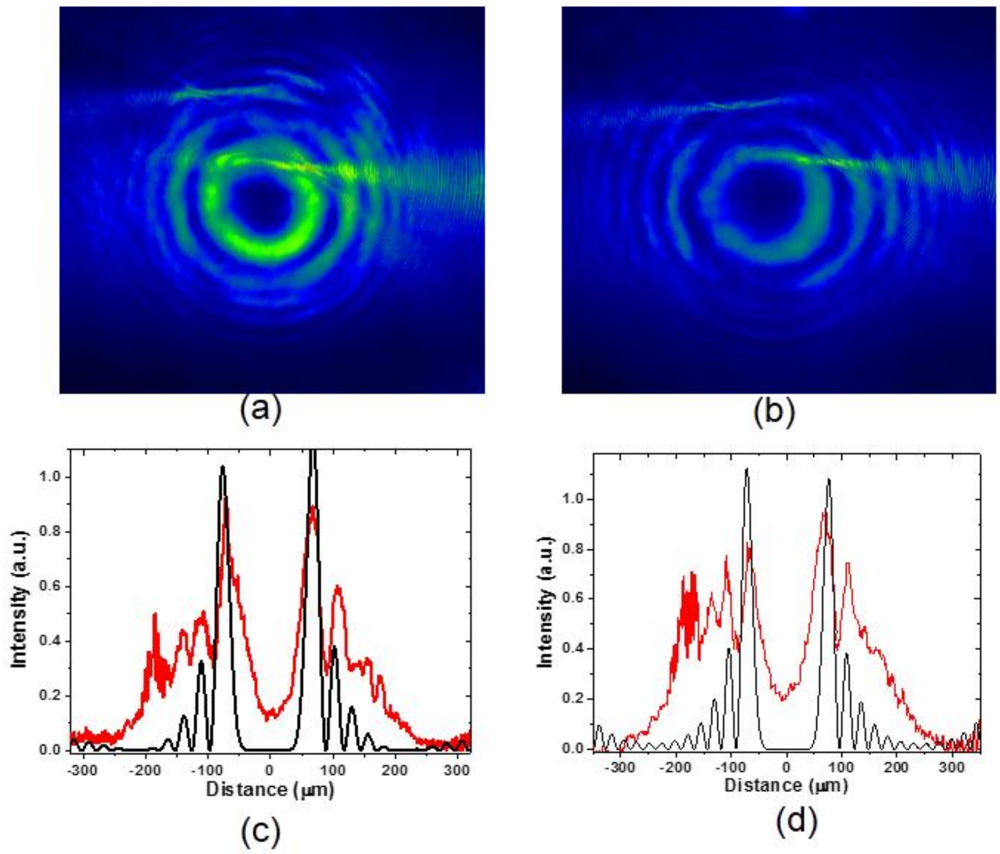

Figure 1 Experimental patterns and theoretical analysis of the SP vortices using a PVL with m=-7. (a) and (b) Experimental patterns using linear polarized $LG_{0,-1}$ and $LG_{0,-2}$ modes and a PVL with m=-7. (c) and (d) Experimental data (red lines) of the intensity distribution through the center of obtained SP vortex patterns corresponding the patterns (a) and (b), respectively, and the theoretical fitting (black lines) of experimental data (red lines) with respective $\left|E_{z,n}\right|^2$ functions shown in eq. (5) with n=-8 and -9.

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