

# Appendix S1

## Derivation of $k$

Here, we derive the coefficient  $k$  in Eq. (3) in the main body of the paper by the condition  $|R'(t)|_{max} = N = |R(t)|_{max}$ . According to Eq. (3),  $R'(t)$  is defined as

$$R'(t) = k \cdot \sum_{i=1}^M \left[ \gamma_i \sum_{j=0}^{i-1} R(t-j) \right],$$

so we have

$$\begin{aligned} R'(t) &= k \cdot \{ \gamma_1 \cdot R(t) + \gamma_2 \cdot [R(t) + R(t-1)] + \cdots + \\ &\quad \gamma_M \cdot [R(t) + R(t-1) + \cdots + R(t-(M-1))] \} \\ &= k \cdot [(\gamma_1 + \gamma_2 + \cdots + \gamma_M) \cdot R(t) + (\gamma_2 + \gamma_3 + \cdots + \gamma_M) \cdot R(t-1) + \cdots + \\ &\quad \gamma_M \cdot R(t-(M-1))]. \end{aligned}$$

Thus, the maximum of  $R'(t)$  is

$$\begin{aligned} |R'(t)|_{max} &= k \cdot [(\gamma_1 + \gamma_2 + \cdots + \gamma_M) \cdot |R(t)|_{max} + \\ &\quad (\gamma_2 + \gamma_3 + \cdots + \gamma_M) \cdot |R(t-1)|_{max} + \cdots + \gamma_M \cdot |R(t-(M-1))|_{max}] \end{aligned}$$

Since  $|R(t)|_{max} = n$  for each  $t$ , we have

$$\begin{aligned} |R'(t)|_{max} &= k \cdot [(\gamma_1 + \gamma_2 + \cdots + \gamma_M) \cdot n + (\gamma_2 + \gamma_3 + \cdots + \gamma_M) \cdot n + \cdots + \gamma_M \cdot n] \\ &= k \cdot \left( \sum_{j=1}^M \gamma_j + \sum_{j=2}^M \gamma_j + \cdots + \sum_{j=M}^M \gamma_j \right) \cdot n \\ &= k \cdot \left( \sum_{i=1}^M \sum_{j=i}^M \gamma_j \right) \cdot n. \end{aligned}$$

We require  $|R'(t)|_{max} = n$ , thus  $k$  is

$$k = 1 / \left( \sum_{i=1}^M \sum_{j=i}^M \gamma_j \right).$$