# Appendix S5

## How $N$ affects the model parameters and simulation results

Since $\alpha$ is determined from $\alpha + \beta = 2$ and $\alpha/\beta = V_+/V_-$, and $\Delta r$ from $\Delta r = \frac{1}{2}(d_{bear} - d_{bull})$, both $\alpha$ and $\Delta r$ are not dependent on the number of agents (denoted by $N$). On the other hand, $\Delta R$ is calculated from the linear relation between $\Delta R$ and $\Delta r$, which is obtained through the simulation of the model. As shown below in Table 1, the slope of the linear relation increases as $N$ grows, and it leads to a larger magnitude of $\Delta R$ for larger $N$. In the main body of the paper, for simplicity, $\Delta R$ is approximated to an integer in the simulation for each index. To investigate how $N$ affects the simulation results, however, $\Delta R$ should be more accurate, and it is now corrected to one decimal place. The influence of $N$ on $L(t)$ is displayed in Fig. 1. The amplitude of $L(t)$ increases with $N$, but gradually converges for larger $N$. For $A(t)$ and $P(|r(t)| > x)$, the cases are similar.

Table 1: **The values of $\Delta R$ and the relations between $\Delta R$ and $\Delta r$ for different $N$ for the S&P 500 and Shanghai indices.**

| $N$ | relation of $\Delta R$ and $\Delta r$ | S&P 500 Index | | Shanghai Index | |
|---|---|---|---|---|---|
| | | $\Delta r$ | $\Delta R$ | $\Delta r$ | $\Delta R$ |
| 1250 | $\triangle R = 12.4\Delta r$ | 0.067 | 0.8 | -0.043 | -0.5 |
| 2500 | $\triangle R = 17.6\Delta r$ | 0.067 | 1.2 | -0.043 | -0.8 |
| 5000 | $\triangle R = 25.4\Delta r$ | 0.067 | 1.7 | -0.043 | -1.1 |
| 10000 | $\triangle R = 38.2\Delta r$ | 0.067 | 2.6 | -0.043 | -1.6 |
| 20000 | $\triangle R = 61.6\Delta r$ | 0.067 | 4.1 | -0.043 | -2.6 |

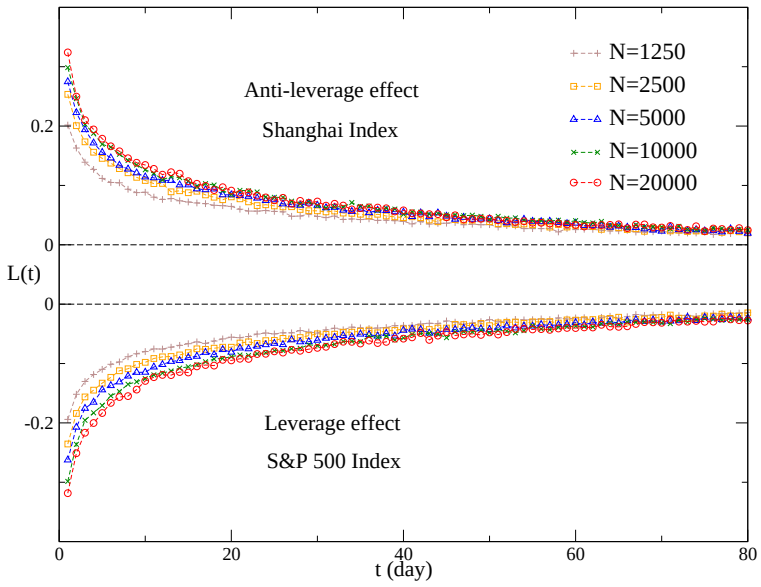


Figure 1: $L(t)$ **from the simulations with different $N$ for the S&P 500 and Shanghai indices.** The S&P 500 Index exhibits the leverage effect, and the Shanghai Index shows the anti-leverage effect. For each $N$, the simulation is performed 100 times for average.