Supporting Information for "Limitations of inclusive fitness" Calculation of the regression method

Benjamin Allen, Martin A. Nowak, Edward O. Wilson

August 12, 2013

In this Supplementary Information we provide a step-by-step calculation of the values of B, C, and R for the example in Figure 1 of the main text, in order to clearly illustrate the regression method. The given values for this example are shown in Table 1.

Calculating B and C

To perform the regression method, we fit the given values to a linear model of the form

$$w_i = W_0 - Cg_i + Bg_i' + \epsilon_i,$$

Table 1: Values for regression

Fitness w	Own genotype g	Partner's genotype g'
4	1	1
0	1	1
0	1	0
2	0	1
0	0	0
0	0	0
2	0	0

by finding values of W_0, C and B that minimize the sum of squares $E = \sum_{i=1}^{N} \epsilon_i^2$. This quantity is minimized when the partial derivatives $\frac{\partial E}{\partial W_0}$, $\frac{\partial E}{\partial C}$, and $\frac{\partial E}{\partial B}$ all vanish. In the case of W_0 this yields the equation

$$\begin{aligned} 0 &= \frac{\partial E}{\partial W_0} \\ &= 2 \sum_{i=1}^{N} \epsilon_i \frac{\partial \epsilon_i}{\partial W_0} \\ &= 2 \sum_{i=1}^{N} (w_i - W_0 + C g_i - B g_i') \, (-1) \\ &= -2(8 - 8W_0 + 3C - 3B). \end{aligned}$$

For C we obtain

$$\begin{aligned} 0 &= \frac{\partial E}{\partial C} \\ &= 2 \sum_{i=1}^{N} \epsilon_i \frac{\partial \epsilon_i}{\partial C} \\ &= 2 \sum_{i=1}^{N} (w_i - W_0 + C g_i - B g_i') \, g_i \\ &= 2(4 - 3W_0 + 3C - 2B). \end{aligned}$$

For B we obtain

$$\begin{aligned} 0 &= \frac{\partial E}{\partial B} \\ &= 2 \sum_{i=1}^{N} \epsilon_i \frac{\partial \epsilon_i}{\partial B} \\ &= 2 \sum_{i=1}^{N} (w_i - W_0 + C g_i - B g_i') \, (-g_i') \\ &= -2(6 - 3W_0 + 2C - 3B). \end{aligned}$$

Combining the above results, we arrive at the system of equations

$$\begin{aligned} 8W_0 - 3C + 3B &= 8 \\ 3W_0 - 3C + 2B &= 4 \\ 3W_0 - 2C + 3B &= 6, \end{aligned}$$

the solution to which is $W_0 = 5/11$, $C = 3/11$, $B = 19/11$.

Calculating R

R can be calculated directly as

$$R = \frac{\frac{1}{N}\sum_{i=1}^{N} g_i g_i' - \left(\frac{1}{N}\sum_{i=1}^{N} g_i\right)\left(\frac{1}{N}\sum_{i=1}^{N} g_i'\right)}{\frac{1}{N}\sum_{i=1}^{N} g_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} g_i\right)^2} = \frac{7}{15}.$$