Supporting Material: Shape-induced asymmetric diffusion in dendritic spines allows efficient synaptic AMPA receptor trapping

Supporting Material I: Geometrical model

In this SI we provide more information concerning the geometrical model which we have invoked in our paper to model the surface of a dendritic spine. The parameterization of the rotational symmetric body has been chosen to model the most notable classes of spine shapes of a dendritic spine, while retaining a surface that is computationally manageable. The shape can be varied from thin, stubby to mushroom shaped spines by varying only one parameter A. The parameterization which represents this surface is given by:

$$
\begin{aligned}
x(u,v) &= R\sin u\cos v \\
y(u,v) &= R\sin u\sin v \\
z(u,v) &= B - \frac{R\cos u}{A\,u}
\end{aligned}
\tag{1}
$$

where R is the maximal radius of the shape and B the height of the spine as shown in Fig S1 ($u_c < u < \pi$, $0 < v < 2\pi$). The absorbing boundary, representing the base of the neck, is placed at the plane $z = 0$. The cut-off value for u_c can be calculated by solving the implicit equation: $B - \frac{R\cos u_c}{Au_c} = 0$. Similarly, it is possible to include a boundary at the top of the spine, resembling the Post Synaptic Density.

To determine the Laplace-Beltrami operator of this surface, one has to calculate the metric of this surface, which has the following two non-zero elements:

$$
g_{u\,v} = \begin{pmatrix} R^2\left(cos^2 u + \frac{(\cos u + u\sin u)^2}{A^2u^4}\right) & 0 \\ 0 & R^2\sin^2 u \end{pmatrix}, \tag{2}
$$

from which we can calculate the Christoffel symbols $\Gamma^k_{i\,j}$:

$$
\begin{aligned}
\Gamma^u_{vv} &= \frac{1}{\tan u} \\
\Gamma^u_{vv} &= \frac{-2\sin u\cos u + \frac{2\cos u\,(\cos u + u\sin u)}{A^2u^3} + \frac{4(\cos u + u\sin u)^2}{A^2u^5}}{2\left(cos^2 u + \frac{(\cos u + u\sin u)^2}{A^2u^4}\right)} \\
\Gamma^v_{uv} = \Gamma^v_{uv} &= \frac{\cos u\sin u}{\left(cos^2 u + \frac{(\cos u + u\sin u)^2}{A^2u^4}\right)}
\end{aligned}
\tag{3}
$$

Using this, we can calculate the Lapace-Beltrami operator:

$$
\nabla^2 = g^{uu}\frac{\partial^2}{\partial u^2} + g^{vv}\frac{\partial^2}{\partial v^2} - g^{vv}\Gamma^u_{vv}\frac{\partial}{\partial u} - g^{uu}\Gamma^u_{vv}\frac{\partial}{\partial u} \tag{4}
$$

Where $\Gamma^k_{i\,j}$ are the Christoffel symbols and g^{ik} the inverse metric.

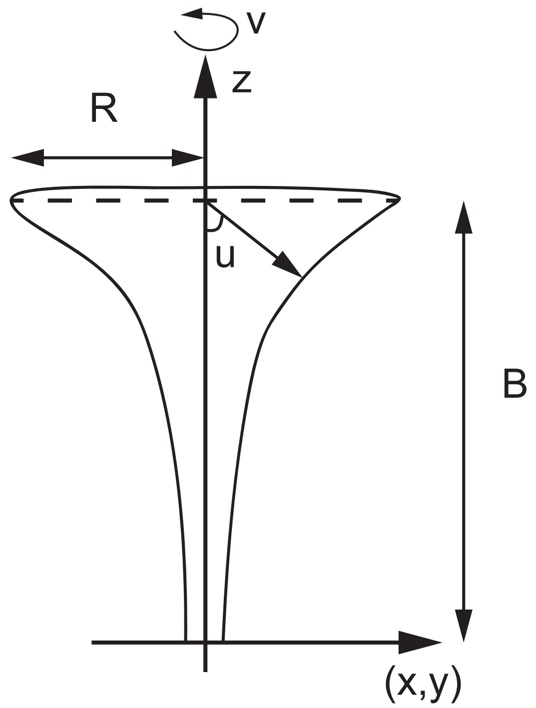

Figure S1: Representation of the parametrization of the dendritic spine as, given by Eq. 1

Supporting Material II: Mean Square Displacement

To show the Random Walk simulation we have used indeed locally results in Brownian diffusion we have plotted the low time behaviour of the MSD for 100 particles released on the surface of a spine. The Mean squared Displacement is calculated via the following equation:

$$MSD = g_{uu}\Delta u^2 + g_{vv}\Delta v^2$$

Indeed, as we see in Fig. S2 the MSD is linear with time, with a slope of 4D, where D is the imposed diffusion coefficient.

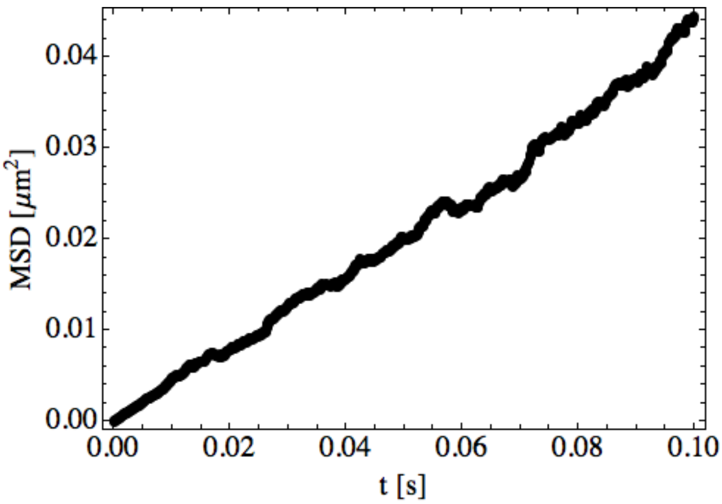

Figure S2: Mean Square Displacement (MSD) as function of time (t) for the release of 100 Receptors on the surface of the spine, given a diffusion coefficient of 0.1 $\mu m^2/s$. The slope of the line is equal to 4D.