Supporting Information

Jüschke et al. 10.1073/pnas.1314984111

SI Materials and Methods

Determination of the Best-Fitting Plane from a Set of Points Using Orthogonal Distance Regression. The orthogonal distance D_i from the point (x_i, y_i, z_i) to the plane $ax + by + cz + d = 0$ is given by

$$D_i = \frac{ax_i + by_i + cz_i + d}{\sqrt{a^2 + b^2 + c^2}}.$$

For a given set of n points we want to find a plane such that we minimize the square of the orthogonal distances. Hence, the function to be minimized is

$$f(a,b,c,d) = \sum_{i=1}^{n} D_i = \sum_{i=1}^{n} \frac{(ax_i + by_i + cz_i + d)^2}{a^2 + b^2 + c^2}. \quad \textbf{[S1]}$$

Setting the partial derivative $\frac{\partial f}{\partial d} = 0$, we can solve for d:

$$\frac{\partial f}{\partial d} = 2\sum_{i=1}^{n} \frac{ax_i + by_i + cz_i + d}{a^2 + b^2 + c^2} = 0$$

$$\sum_{i=1}^{n} ax_i + by_i + cz_i + d = 0$$

$$d = -(a\bar{x} + a\bar{y} + a\bar{z}),$$

where $(\bar{x},\bar{y},\bar{z})$ is the centroid of the data. Substituting d into Eq. **S1**, we get

$$f(a,b,c) = \sum_{i=1}^{n} \frac{\left(a(x_i - \bar{x}) + b(y_i - \bar{y}) + c(z_i - \bar{z})\right)^2}{a^2 + b^2 + c^2}. \quad \textbf{[S2]}$$

By defining **v** and **M**

$$\mathbf{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} x_1 - \bar{x} & y_1 - \bar{y} & z_1 - \bar{z} \\ x_2 - \bar{x} & y_2 - \bar{y} & z_2 - \bar{z} \\ \vdots & \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} & z_n - \bar{z} \end{pmatrix}$$

we can express Eq. **S2** as

$$f(\mathbf{v}) = \frac{(\mathbf{v}^T\mathbf{M}^T)(\mathbf{M}\mathbf{v})}{\mathbf{v}^T\mathbf{v}} = \frac{\mathbf{v}^T(\mathbf{M}^T\mathbf{M})\mathbf{v}}{\mathbf{v}^T\mathbf{v}}.$$

With $\mathbf{A} = \mathbf{M}^T\mathbf{M}$, $f(\mathbf{v})$ has the form of a Rayleigh quotient. It is minimized by the eigenvector of **A** that corresponds to its smallest eigenvalue. The eigenvalues of **A** are the squares of the singular values of **M**, and the eigenvectors of **A** are the singular vectors of **M**. In summary, the best-fitting plane passes through the centroid of the data $(\bar{x},\bar{y},\bar{z})$, and its normal vector **n** is the singular vector of **M** corresponding to its smallest singular value. We use the function svd() implemented in the R programming language (1) to compute the singular-value decomposition of **M** and determine the normal vector **n** of the plane.

Determination of the Surface Area on a Cube. We consider a cube with its center at the origin and an edge length of 2 (Fig. 3*A*). The equatorial distance (i.e., inclination angle $\alpha = 0°$) from the center of the cube to a point on its surface is $d = \sqrt{1 + x^2}$ with $|x| \le 1$. For $|\alpha| \le \arctan\frac{1}{\sqrt{2}}$, the angle α is intersecting the cube surface at $h = d \cdot \tan\alpha$. Hence, the surface area A_ψ between the angles $-\psi$ and ψ (with $0° \le \psi \le \arctan\frac{1}{\sqrt{2}}$) is

$$A_\psi = 16\tan\psi \cdot \int_0^1 \sqrt{1 + x^2}\,dx \quad = 16\tan\psi \cdot \tfrac{1}{2}\left[x\sqrt{1+x^2} + \operatorname{arcsinh} x\right]_0^1$$
$$= 8\left(\sqrt{2} + \operatorname{arcsinh} 1\right)\tan\psi.$$

Considering angles $\psi \ge \arctan\frac{1}{\sqrt{2}}$ as well, the surface area A_ψ is

$$A_\psi = \begin{cases} 8\left(\sqrt{2} + \operatorname{arcsinh} 1\right)\tan\psi, & \text{if } \psi \le \arctan\frac{1}{\sqrt{2}} \approx 35° \\ g(\psi), & \text{if } \arctan\frac{1}{\sqrt{2}} \le \psi \le 45°. \\ 24 - 2\pi\cot^2\psi, & \text{if } \psi \ge 45° \end{cases}$$

We do not determine $g(\psi)$ because we can calculate the probabilities for angles in the range $35° < \psi < 45°$ using the probabilities outside this range. The fraction f_ψ of the area within $-\psi$ and ψ (A_ψ) to the total surface area of the cube $(A_{90°} = 24)$ is

$$f_\psi = \begin{cases} \frac{1}{3}\left(\sqrt{2} + \operatorname{arcsinh} 1\right)\tan\psi, & \text{if } \psi \le \arctan\frac{1}{\sqrt{2}} \approx 35° \\ \frac{1}{24}g(\psi), & \text{if } \arctan\frac{1}{\sqrt{2}} \le \psi \le 45°. \\ 1 - \frac{1}{12}\pi\cot^2\psi, & \text{if } \psi \ge 45° \end{cases}$$

1. R Core Team (2012) *R: A Language and Environment for Statistical Computing* (R Foundation for Statistical Computing, Vienna).

2. Xie Y, Jüschke C, Esk C, Hirotsune S, Knoblich JA (2013) The phosphatase PP4c controls spindle orientation to maintain proliferative symmetric divisions in the developing neocortex. *Neuron* 79(2):254–265.

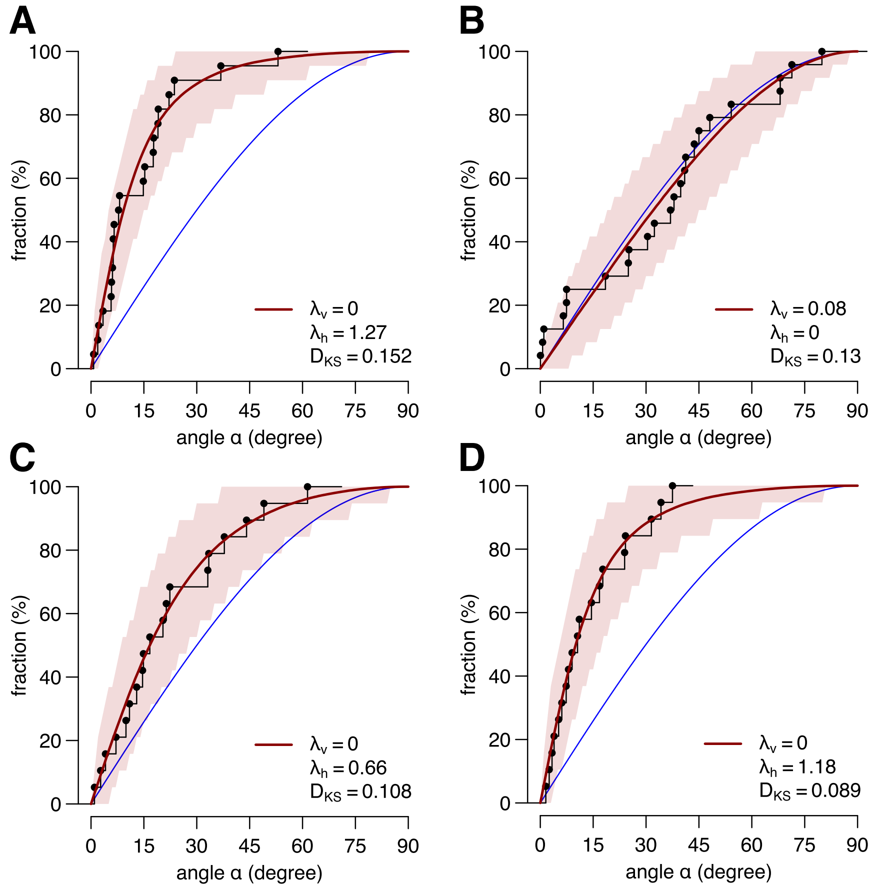

Fig. S1. Protein phosphatase 4 catalytic subunit (PP4c) is required for horizontal spindle orientation. Cumulative distributions of spindle orientation angles determined from mitotic radial glial cells after in utero electroporation with short hairpin (sh) constructs. (*A*) sh-Scrambled control. (*B*) Knock-down of PP4c by sh-PP4c leads to spindle randomization. (*C*) Coexpression of sh-PP4c with a PP4c rescue construct partially restores spindle orientation. (*D*) Coexpression of sh-PP4c with a nonphosphorylatable Ndel1 construct rescues horizontal spindle orientation. The blue line indicates the random distribution; the 95% confidence interval is shaded in pink. Data were obtained from ref. 2.

Table S1. Probabilities for random spindle orientation angles in a 3D cube

Term	Horizontal		Oblique		Vertical	
Range ψ_1–ψ_2	0°–30°		30°–60°		60°–90°	
Probability *P*, %	44.2		47.1		8.7	
Range ψ_1–ψ_2	0°–15°	15°–30°	30°–45°	45°–60°	60°–75°	75°–90°
Probability *P*, %	20.5	23.7	29.6	17.5	6.8	1.9

Table S2. Probabilities for random spindle orientation angles in a 2D circle

Term	Horizontal		Oblique		Vertical	
Range ψ_1–ψ_2	0°–30°		30°–60°		60°–90°	
Probability *P*, %	33.3		33.3		33.3	
Range ψ_1–ψ_2	0°–15°	15°–30°	30°–45°	45°–60°	60°–75°	75°–90°
Probability *P*, %	16.7	16.7	16.7	16.7	16.7	16.7

Other Supporting Information Files

Dataset S1 (CSV)
R Script S2 (PDF)
R Script S3 (PDF)