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Web-based Supplementary Materials for Modeling Seroadaptation and Sexual Behavior among HIV$^+$ Participants with a Simultaneously Multilevel and Multivariate Longitudinal Count Model

Yuda Zhu* and Robert Weiss**

Department of Biostatistics, University of California Los Angeles, Los Angeles, CA

**email:* yudazhu@gmail.com

***email:* robweiss@ucla.edu

The Health Living Project (HLP) study was analyzed using a longitudinal multivariate multilevel count model. In the following sections, a variety of topics are further explored. A simulation study to show the estimation benefits of correctly modeling separate sources of variation is presented in Web Appendix A. A summary table of the participants' demographics at baseline is shown in Web Appendix B. The posterior computation algorithm is given in detail in Web Appendix C. The effects of treatment, site, risk group, and race found from our model are provided in Web Appendix D and posterior summaries of the covariance between outcomes are presented in Web Appendix E.

1. Web Appendix A

Our multilevel model introduces separate sources of variation for modeling the number of protected and unprotected acts. Not only do subjects behave differently but the same subject varies their behavior with different partners bringing another level of heterogeneity to observed outcomes. In the Poisson model, when the variation comes from multiple levels, estimation are gained by correctly modeling these sources of variation. We examine the effects of failing to account for the different levels of heterogeneity through a simulation of univariate random variables with 2 levels of heterogeneity. This is similar to sex acts in our study measured at a single time point but repeatedly observed over multiple partners.

Simulation data is generated assuming $Y_{ik} \sim \text{Po}(\lambda_{ik})$ for participant i=1, …, 12, and partner k=1, … K_i where K_i is itself a zero-truncated Poisson distributed variable with parameter $\lambda_{K_i} = 10$. Mean parameters λ_{ij} are distributed log normal

$$\log \lambda_{ik} = \mu + \beta_i + \delta_{ik} \tag{1}$$

with Gaussian distributed subject latent effects $\beta_i \sim \text{N}(0, \sigma^2)$ and partner latent effects $\delta_{ik} \sim \text{N}(0, d^2)$. Values of μ, σ^2, d^2 are set to 1, 1.5, and 2 respectively with the resulting expected value $\text{E}(Y_{ik}) = \exp\{\mu + 0.5(\sigma^2 + d^2)\} = 15.64$.

We consider 3 separate analyses for making inference on the expected value of Y_{ik}. Analysis 1 uses the full disaggregated data and the correct 2 level heterogeneity model as presented in (1). This is analogous to the ideal situation where all partner level data in the HLP study is observed. Analysis 2 also uses the correct model but assumes we only observe Y_{ik} for $k \leqslant 5$ and the aggregated totals $Y_{iT}=\sum_{k=1}^{K_i} Y_{ik}$. Unobserved disaggregated values $\mathbf{Y}_{i,miss} = (Y_{i6}, \dots Y_{iK_i})$ for $K_i \geqslant 6$ are imputed from observed info as

$$\mathbf{Y}_{i,miss}|Y_{iT}, Y_{i1}, \dots, Y_{i5} \sim \text{Multinomial}(N_i, \mathbf{p}_i)$$

where $N_i = Y_{iT} - \sum_{k=1}^{5} Y_{ik}$, $\mathbf{p}_i = (\lambda_{i6}/\lambda_{iT}, \dots, \lambda_{iK_i}/\lambda_{iT})$, and $\lambda_{iT} = \sum_{k=6}^{K_i} \lambda_{ik}$. This analysis mimics the actual analysis of the HLP study. Analysis 3 uses only aggregated totals, $Y_{iT} \sim \text{Po}(\lambda_i)$, with mean parameters

$$\log \lambda_i = \mu + \beta_i$$

using a single level of Gaussian distributed latent effect $\beta_i \sim \text{N}(0, \sigma^2)$. This mimics the traditional analysis of total sex acts in studies like HLP. Inference in analysis 3 is made from $\text{E}(Y_{iT}) = \text{E}_{K_i}\text{E}(Y_{iT}|K_i) = \text{E}_{K_i}\text{E}(\sum_{k=1}^{K_i} Y_{ik}|K_i) = \text{E}(K_i)\text{E}(Y_{ik})$. Non-influential priors with large variances are chosen in all cases with prior means set at the true value. Inverse-gamma priors are chosen for parameters σ^2 and d^2 while a Gaussian prior is used for μ. Table 1 shows summary results from 600 simulated datasets. Use of aggregated totals in analysis 3 results in longer intervals, an average width of 7.8 as compared to 7.3 from analysis 2 and 7.2 from analysis 1. A bigger mean square error was also found in analysis 3, an average of 8.4 as compared to 6.8 from analysis 2 and 6.5 from analysis 1. This shows that failing to account for subject and partner level variation correctly results in posterior estimates with larger credible intervals.

[Table 1 about here.]

2. Web Appendix B

Table 2 presents a summary of participants' demographics information at baseline.

[Table 2 about here.]

3. Web Appendix C

Posterior sampling of model parameters ($\boldsymbol{\alpha}$, $\boldsymbol{\beta}_{ij}$, $\boldsymbol{\delta}_{ijk}$, $\boldsymbol{\Sigma}$, $\mathbf{D}$, $\mathbf{A}$) for $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant J_i$, $1 \leqslant k \leqslant V_{ij}$ uses Markov Chain Monte Carlo (MCMC) methods (Metropolis et al., 1953; Hastings, 1970; Gelfand and Smith, 1990; Casella and George, 1992). We also simultaneously sample from unobserved partner level outcomes (P_{ijk}, U_{ijk}) for $k \in S_{ij}$ where S_{ij} denotes the set of partners for subject i at time t_{ij} for which partner specific act information was not observed. Detailed sampling algorithms are given here.

In this section, we use the notation $\widehat{r}$ to denote the current iteration of parameter r. A proposal function $q(r_{prop}|\widehat{r}) \sim Q(h(\widehat{r}))$, where Q is a specified distribution, denotes that $q(r_{prop}|\widehat{r})$ is the density of the distribution Q with parameters defined by $h(\widehat{r})$ evaluated at r_{prop}.

(1) The conditional posterior distributions of $\boldsymbol{\alpha}_v^+$ and $\boldsymbol{\alpha}_v^-$ have the same form. We only present the sampling density $f(\boldsymbol{\alpha}_v^+|.)$ here. Sample $\boldsymbol{\alpha}_v^+$ from

$$f(\boldsymbol{\alpha}_v^+|.) \propto \exp\left[\sum_{i=1}^{n}\sum_{j=1}^{J_i}\left\{V_{ij}^+\mathbf{x}_{ij}'\boldsymbol{\alpha}_v^+ - \lambda_{v,ij}^+\right\} - G(\boldsymbol{\alpha}_v^+)\right]\{1-\exp(-\lambda_{v,i1}^+ - \lambda_{v,i1}^-)\}^{-1} \quad (2)$$

where $\lambda_{v,ij}^+ = \exp(\mathbf{x}_{ij}'\boldsymbol{\alpha}_v^+ + \beta_{v,ij}^+)$, $\lambda_{v,ij}^- = \exp(\mathbf{x}_{ij}'\boldsymbol{\alpha}_v^- + \beta_{v,ij}^-)$ and $G(\boldsymbol{\alpha}_v^+) = \frac{1}{2}(\boldsymbol{\alpha}_v^+ - \boldsymbol{\mu}_{\alpha_v^+})'\boldsymbol{\Sigma}_{\alpha_v^+}(\boldsymbol{\alpha}_v^+ - \boldsymbol{\mu}_{\alpha_v^+})$ comes from the prior of $\boldsymbol{\alpha}_v^+$. Prior parameters $\boldsymbol{\mu}_{\alpha_v^+}$ and $\boldsymbol{\Sigma}_{\alpha_v^+}$ are correspondingly a vector with each element set to $\mu_\alpha = 0$ and a diagonal matrix with each diagonal element set to $\sigma_\alpha^2 = 10$. We use a second-order Taylor approximation of equation (2) as the adaptive proposal function. When $\mu_\alpha = 0$ and $\boldsymbol{\Sigma}_{\alpha_v^+}$ is diagonal, we use a multivariate normal proposal function $q(\boldsymbol{\alpha}^+_{v,prop}|\widehat{\boldsymbol{\alpha}}^+_v) \sim \text{MVN}(\mathbf{T}^{-1}_{v^+}\mathbf{M}'_{v^+}, \mathbf{T}^{-1}_{v^+})$ with

$$\mathbf{T}_{v^+} = \Sigma^{-1}_{\alpha^+_v} + \sum_{i=1}^{n}\sum_{j=1}^{J_i}\widehat{\lambda}^+_{v,ij}\mathbf{x}_{ij}\mathbf{x}'_{ij} - \sum_{i=1}^{n} R_i\mathbf{x}_{i1}\mathbf{x}'_{i1},$$

$$\mathbf{M}_{v^+} = \sum_{i=1}^{n}\sum_{j=1}^{J_i}(\widehat{\lambda}^+_{v,ij}\widehat{\boldsymbol{\alpha}}^{+\prime}_v\mathbf{x}_{ij} - \widehat{\lambda}^+_{v,ij} + V^+_{ij})\mathbf{x}'_{ij} - \sum_{i=1}^{n}(R_i\widehat{\boldsymbol{\alpha}}^{+\prime}_v\mathbf{x}_{i1} - H_i)\mathbf{x}'_{i1},$$

where $\widehat{\lambda}^+_{v,ij} = \exp(\mathbf{x}'_{ij}\widehat{\boldsymbol{\alpha}}^+_v + \beta^+_{v,ij})$, $H_i = -\widehat{\lambda}^+_{v,i1}\{1-\exp(-\widehat{\lambda}^+_{v,i1} - \lambda^-_{v,i1})\}^{-1}\exp(-\widehat{\lambda}^+_{v,i1} - \lambda^-_{v,i1})$, and $R_i = H_i^2 - H_i\widehat{\lambda}^+_{v,i1} + H_i$.

(2) Sampling from the conditional posterior distributions of $\boldsymbol{\alpha}^+_u, \boldsymbol{\alpha}^-_u, \boldsymbol{\alpha}^+_p$, and $\boldsymbol{\alpha}^-_p$ is similar in form to sampling $\boldsymbol{\alpha}^+_v$ but uses the partner level observations. We present here only the posterior sampling algorithm for $\boldsymbol{\alpha}^+_u$. To sample from posterior distributions of $\boldsymbol{\alpha}^+_u$, define set Φ_i such that $k \in \Phi_i$ denotes all baseline partners for subject i who are HIV$^-$ or HIV$^+$ and not categorized as a primary partner. We also reorder partner observations for subject i at each time point j such that partners $1, \ldots, V^+_{ij}$ are HIV$^+$ and the rest are HIV$^-$ for notational convenience. We sample $\boldsymbol{\alpha}^+_u$ from

$$f(\boldsymbol{\alpha}^+_u|.) \propto \exp\left[\sum_{i=1}^{n}\sum_{j=1}^{J_i}\sum_{k=1}^{V^+_{ij}}\left\{U_{ijk}\mathbf{x}'_{ijk}\boldsymbol{\alpha}^+_u - \lambda_{u,ijk}\right\} - G(\boldsymbol{\alpha}^+_u)\right]\{1-\exp(-\sum_{k\in\Phi_i}\lambda_{u,i1k})\}^{-1} \tag{3}$$

where $\lambda_{u,ijk} = \exp(\mathbf{x}'_{ijk}\boldsymbol{\alpha}^+_u + \beta^+_{u,ij} + \delta_{u,ijk})$ for $k \leqslant V^+_{ij}$, $\lambda_{u,ijk} = \exp(\mathbf{x}'_{ijk}\boldsymbol{\alpha}^-_u + \beta^-_{u,ij} + \delta_{u,ijk})$ for $k > V^+_{ij}$, and $G(\boldsymbol{\alpha}^+_u) = \frac{1}{2}(\boldsymbol{\alpha}^+_u - \boldsymbol{\mu}_{\alpha^+_u})'\boldsymbol{\Sigma}_{\alpha^+_u}(\boldsymbol{\alpha}^+_u - \boldsymbol{\mu}_{\alpha^+_u})$ comes from the prior of $\boldsymbol{\alpha}^+_u$. Prior parameters $\boldsymbol{\mu}_{\alpha^+_u}$ and $\boldsymbol{\Sigma}_{\alpha^+_u}$ are correspondingly a vector with each element set to $\mu_\alpha = 0$ and a diagonal matrix with each diagonal element set to $\sigma^2_\alpha = 10$. Similar to the proposal function $q(\boldsymbol{\alpha}^+_{v,prop}|\widehat{\boldsymbol{\alpha}}^+_v)$, we use a second-order Taylor approximation of equation (3) as the adaptive proposal function $q(\boldsymbol{\alpha}^+_{u,prop}|\widehat{\boldsymbol{\alpha}}^+_u)$. When $\mu_\alpha = 0$ and $\boldsymbol{\Sigma}_{\alpha^+_u}$ is diagonal,

$q(\boldsymbol{\alpha}^{+}_{u,prop}|\widehat{\boldsymbol{\alpha}}^{+}_{u}) \sim \text{MVN}(\mathbf{T}^{-1}_{u^+}\mathbf{M}'_{u^+}, \mathbf{T}^{-1}_{u^+})$ with

$$\mathbf{T}_{u^+} =\Sigma^{-1}_{\alpha^+_v} + \sum_{i=1}^{n}\sum_{j=1}^{J_i}\sum_{k=1}^{V^+_{ij}} \widehat{\lambda}_{u,ijk}\mathbf{x}_{ijk}\mathbf{x}'_{ijk} - \sum_{i=1}^{n}\sum_{k\in\Phi_i} R_{i,u^+}\mathbf{x}_{i1k}\mathbf{x}'_{i1k},$$

$$\mathbf{M}_{u^+} = \sum_{i=1}^{n}\sum_{j=1}^{J_i}\sum_{k=1}^{V^+_{ij}} (\widehat{\lambda}_{u,ijk}\widehat{\boldsymbol{\alpha}}^{+\prime}_{u}\mathbf{x}_{ijk} - \widehat{\lambda}^{+}_{u,ijk} + U_{ijk})\mathbf{x}'_{ijk} - \sum_{i=1}^{n}\sum_{k\in\Phi_i} (R_{i,u^+}\widehat{\boldsymbol{\alpha}}^{+\prime}_{u}\mathbf{x}_{i1k} - H_{i,u^+})\mathbf{x}'_{i1k},$$

where

$$\widehat{\lambda}_{u,ijk} = \exp(\mathbf{x}'_{ijk}\widehat{\boldsymbol{\alpha}}^{+}_{u} + \beta^{+}_{u,ij} + \delta_{u,ijk}) \text{ for } k \leqslant V^{+}_{ij},$$

$$H_{i,u^+} = -\,\widehat{\lambda}^{+}_{u,i1T}\{1-\exp(-\widehat{\lambda}^{+}_{u,i1T} - \lambda^{-}_{u,i1T})\}^{-1}\exp(-\widehat{\lambda}^{+}_{u,i1T} - \lambda^{-}_{u,i1T}),$$

$$\widehat{\lambda}^{+}_{u,i1T} = \sum_{k}\widehat{\lambda}_{u,i1k} \text{ for } (k\in\Phi_i)\cap(k\leqslant V^{+}_{ij}),$$

$$\lambda^{-}_{u,i1T} = \sum_{k}\lambda_{u,i1k} \text{ for } (k\in\Phi_i)\cap(k > V^{+}_{ij}),$$

$$R_{i,u^+} = H^2_{i,u^+} - H_{i,u^+}\widehat{\lambda}^{+}_{u,i1T} + H_{i,u^+}.$$

(3) Sample $\boldsymbol{\beta}_{i1},\ldots,\boldsymbol{\beta}_{iJ}$ for $1\leqslant i\leqslant n$ from

$$f(\boldsymbol{\beta}_{i1}|.) \propto f(\mathbf{Y}_{i1}|\boldsymbol{\beta}_{i1})f(\boldsymbol{\beta}_{i1}|\mathbf{L})f(\boldsymbol{\beta}_{i2}|\mathbf{A},\boldsymbol{\Sigma},\boldsymbol{\beta}_{i1}),$$

$$f(\boldsymbol{\beta}_{ij}|.) \propto f(\mathbf{Y}_{ij}|\boldsymbol{\beta}_{ij})f(\boldsymbol{\beta}_{ij}|\mathbf{A},\boldsymbol{\Sigma},\boldsymbol{\beta}_{i(j-1)})f(\boldsymbol{\beta}_{i(j+1)}|\mathbf{A},\boldsymbol{\Sigma},\boldsymbol{\beta}_{ij}), \qquad \text{for } 2\leqslant j< J,$$

$$f(\boldsymbol{\beta}_{iJ}|.) \propto f(\mathbf{Y}_{iJ}|\boldsymbol{\beta}_{iJ})f(\boldsymbol{\beta}_{iJ}|\mathbf{A},\boldsymbol{\Sigma},\boldsymbol{\beta}_{i(J-1)}),$$

where $f(Y_{ij}|\boldsymbol{\beta}_{ij}) = f(V^{+}_{ij}|\lambda^{+}_{v,ij})f(V^{-}_{ij}|\lambda^{-}_{v,ij})\prod_{k=1}^{V_{ij}} f(U_{ijk}|\lambda_{u,ijk})f(P_{ijk}|\lambda_{p,ijk})$, $\lambda_{p,ijk} = \exp(\mathbf{x}'_{ijk}\boldsymbol{\alpha}^{+}_{p} + \beta^{+}_{p,ij} + \delta_{p,ijk})$ for $k\leqslant V^{+}_{ij}$, $\lambda_{p,ijk} = \exp(\mathbf{x}'_{ijk}\boldsymbol{\alpha}^{-}_{p} + \beta^{-}_{p,ij} + \delta_{p,ijk})$ for $k > V^{+}_{ij}$. We use a random walk Gaussian proposal function $q(\boldsymbol{\beta}_{ij,prop}|\widehat{\boldsymbol{\beta}}_{ij}) \sim \text{MVN}(\widehat{\boldsymbol{\beta}}_{ij},\boldsymbol{\Sigma}_{\beta,q})$ where $\boldsymbol{\Sigma}_{\beta,q}$ is a diagonal matrix with diagonal elements chosen to obtain an acceptable rate of acceptance for $\boldsymbol{\beta}_{ij,prop}$.

(4) Sample $\boldsymbol{\delta}_{ijk} = (\delta_{p,ijk},\delta_{u,ijk})^T$ for $1\leqslant i\leqslant n$, $1\leqslant j\leqslant J_i$, $1\leqslant k\leqslant V_{ij}$ from

$$f(\boldsymbol{\delta}_{ijk}|.) \propto f(P_{ijk}|\lambda_{p,ijk})f(U_{ijk}|\lambda_{u,ijk})f(\boldsymbol{\delta}_{ijk}|\mathbf{D})$$

using the M-H algorithm with a random walk Gaussian proposal function $q(\boldsymbol{\delta}_{ijk,prop}|\widehat{\boldsymbol{\delta}}_{ijk}) \sim$

MVN($\widehat{\boldsymbol{\delta}}_{ijk}, \boldsymbol{\Sigma}_{\delta,q}$) where $\boldsymbol{\Sigma}_{\delta,q}$ is a diagonal matrix with diagonal elements chosen to obtain an acceptable rate of acceptance for $\boldsymbol{\delta}_{ijk,prop}$.

(5) Sample $\boldsymbol{\Sigma}$ from

$$f(\boldsymbol{\Sigma}|.) \propto \prod_{i=1}^{n}\prod_{j=1}^{J_i} f(\boldsymbol{\beta}_{ij}|\boldsymbol{\Sigma})\pi(\boldsymbol{\Sigma})$$

where the prior $\pi(\boldsymbol{\Sigma})$ is defined in the Prior Specification Section of our paper. A proposal function, $q(\boldsymbol{\Sigma}_{prop}|\widehat{\boldsymbol{\Sigma}}) \sim \text{IW}(\boldsymbol{\Psi}_{\Sigma,q}, m_{\Sigma,q})$ is used to approximate $f(\boldsymbol{\Sigma}|.)$ where $\boldsymbol{\Psi}_{\Sigma,prop} = \sum_{i=1}^{n}\sum_{j=2}^{J_i}(\boldsymbol{\beta}_{ij} - \mathbf{A}\boldsymbol{\beta}_{i(j-1)})(\boldsymbol{\beta}_{ij} - \mathbf{A}\boldsymbol{\beta}_{i(j-1)})' + \boldsymbol{\Psi}_{\Sigma}$ and $m_{\Sigma,prop} = m_{\Sigma} + \sum_{i=1}^{n}\sum_{j=2}^{J_i} 1$. The proposal function $q(\boldsymbol{\Sigma}_{prop}|\boldsymbol{\Sigma}_{current})$ is modestly overdispersed compared to $f(\boldsymbol{\Sigma}|.)$ due to the normal prior terms for the diagonal elements.

(6) Sample $\mathbf{D}$ from

$$f(\mathbf{D}|.) \propto \prod_{i=1}^{n}\prod_{j=1}^{J_i}\prod_{k=1}^{V_{ij}} f(\boldsymbol{\delta}_{ijk}|\mathbf{D})\pi(\mathbf{D})$$

where the prior $\pi(\mathbf{D})$ is defined in the Prior Specification Section of our paper. A proposal function, $q(\mathbf{D}_{prop}|\widehat{\mathbf{D}}) \sim \text{IW}(\boldsymbol{\Psi}_{D,q}, m_{D,q})$ is used to approximate $f(\mathbf{D}|.)$ where $\boldsymbol{\Psi}_{D,q} = \sum_{i=1}^{n}\sum_{j=1}^{J_i}\sum_{k=1}^{V_{ij}} \boldsymbol{\delta}_{ijk}\boldsymbol{\delta}'_{ijk} + \Psi_D$ and $m_{D,q} = m_D + \sum_{i=1}^{n}\sum_{j=1}^{J_i}\sum_{k=1}^{V_{ij}} 1$.

(7) Since parameter $\mathbf{A}$ is diagonal with diagonal elements $A_{l,l}$, we can sample $\mathbf{A}$ from

$$f(\mathbf{A}|.) \propto \prod_{i=1}^{n}\left\{ f(\boldsymbol{\beta}_{i1}|\mathbf{L})\prod_{j=2}^{J_i} f(\boldsymbol{\beta}_{ij}|\mathbf{A}, \boldsymbol{\Sigma}, \boldsymbol{\beta}_{i(j-1)})\right\}\prod_{l=1}^{6}\pi(A_{l,l})$$

where $\pi(A_{l,l}) = 1/2\ \mathbf{1}(-1 \leqslant A_{l,l} \leqslant 1)$ is a uniform distribution from -1 to 1. We propose the l^{th} diagonal element of A, $A_{l,l}$, using a random walk truncated Gaussian proposal function

$$q(A_{(l,l),prop}|\widehat{A}_{l,l}) \sim \text{truncN}(\widehat{A}_{l,l}, \sigma_A^2)$$

where $A_{(l,l),prop}$ is between -1 to 1.

(8) Let $P_{ijT} = \sum_{k=1}^{V_{ij}} P_{ijk}$ and $U_{ijT} = \sum_{k=1}^{V_{ij}} U_{ijk}$ respectively be the total protected and unprotected acts for subject i at time t_{ij}. Also let S_{ij} be the set of partners k with subject i at time t_{ij} for which partner specific protected and unprotected acts is not recorded

and let $\mathbf{P}_{ijS_{ij}}$ and $\mathbf{U}_{ijS_{ij}}$ be the set of protected and unprotected acts corresponding to these partners. We sample $\mathbf{P}_{ijS_{ij}}$ and $\mathbf{U}_{ijS_{ij}}$ for $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant J_i$ from

$$\mathbf{P}_{ijS_{ij}} | \boldsymbol{\lambda}_{ij}, P_{ijT}, (P_{ijk} \text{ for } k \notin S_{ij}) \sim \text{Multinomial}(N_{P,ij}, \boldsymbol{\pi}_{P_ij}),$$

$$\mathbf{U}_{ijS_{ij}} | \boldsymbol{\lambda}_{ij}, U_{ijT}, (P_{ijk} \text{ for } k \notin S_{ij}) \sim \text{Multinomial}(N_{U,ij}, \boldsymbol{\pi}_{U_ij}),$$

where $\boldsymbol{\lambda}_{ij} = (\lambda^{+}_{v,ij}, \lambda^{-}_{v,ij}, \lambda_{p,ij1}, \lambda_{u,ij1}, \ldots, \lambda_{p,ijV_{ij}}, \lambda_{u,ijV_{ij}})^T$, $N_{P,ij} = P_{ijT} - \sum_{k \notin S_{ij}} P_{ijk}$, $N_{U,ij} = U_{ijT} - \sum_{k \notin S_{ij}} U_{ijk}$, $\boldsymbol{\pi}_{P_{ij}} = (\pi_{P_{ijk}}) = (\lambda_{p,ijk}/\lambda_{p,ijT})$ for every $k \in S_{ij}$, $\lambda_{p,ijT} = \sum_{k \in S_{ij}} \lambda_{p,ijk}$, $\boldsymbol{\pi}_{U_{ij}} = (\pi_{U_{ijk}}) = (\lambda_{u,ijk}/\lambda_{u,ijT})$ for every $k \in S_{ij}$, and $\lambda_{u,ijT} = \sum_{k \in S_{ij}} \lambda_{u,ijk}$.

We repeat Steps 1 through 8 until convergence and to collect a sample from the posterior.

4. Web Appendix D

4.1 *Treatment Over Time*

We evaluated treatment efficacy of the HLP trial at each followup comparing the treatment group to the control group looking for evidence of any of the following scenarios:

- decrease in the number of HIV^-/unknown partners
- increase in the number of protected acts per HIV^-/unknown partner
- decrease in the number of unprotected acts per HIV^-/unknown partner.

A comparison of sexual behavior profiles for the treatment and control groups across time are shown in Figure 1. An overall decrease in the average number of both HIV^+ and HIV^-/unknown partners is observed across the entire study population. The average number of unprotected acts per partner also decreases across both treatment and control groups across partners of either serostatus while protected acts per partner stays fairly consistent throughout the study. However, the treatment group does not appear to behave differently from the control group at any time point. We show the difference between the treatment and control groups at each followup time period adjusting for baseline differences in Table 3. Values are reported as the ratio of the estimated outcome between treatment and control at followup divided by the ratio of the estimated outcome between treatment and control at baseline. Values greater than 1 imply higher estimated counts in the treatment group than the control group after adjusting for initial differences in the two groups at baseline while values less than 1 imply lower estimated counts. For example, our estimates suggest the treatment group reported only 0.92 (0.74, 1.12) times as many HIV^-/unknown partners as the control group after the first followup after adjusting for baseline differences.

[Figure 1 about here.]

[Table 3 about here.]

4.2 *Site, Risk Group, and Race*

The HLP study was carried out across multiple geographical sites, contained multiple HIV transmission risk groups, and included multiple ethnicities of participants. Table 4 shows estimates for these covariate effects.

San Francisco appears to be the most risky population for HIV transmission followed by Los Angeles. New York and Milwaukee are less risky but for different reasons. Participants in Milwaukee behaved differently from those in Los Angeles, San Francisco, and New York, reporting significantly fewer HIV^+ partners likely due to the demographics of the population there. Participants at the New York site reported more protected sex with their partners than all other sites. Participants in San Francisco reported greater numbers of partners than the other locations. Most of the difference comes from the number of HIV^-/unknown serostatus partners.

Among the 4 risk groups, females report the largest numbers of unprotected sex acts with HIV^- partners. While MSMs have reported significantly greater numbers of HIV^-/unknown partners, their transmission risk is mitigated by their increased propensity to use protection with these partners. The IDU group also exhibits unsafe behavior, reporting the lowest number of protected sex acts per partner and the second highest number of unprotected sex acts among the 4 risk groups. However, their HIV transmission risk is partially mitigated because relative to other risk groups, a larger proportion of their partners are HIV$^+$ serostatus partners.

The race of the participant did not have as pronounced an effect on transmission risk as risk group and location. We found no significant differences between whites and others. The Hispanic group reported significantly larger numbers of HIV$^-$/unknown serostatus partners than all other groups but also report fewer unprotected sex acts with them. The African American group reported significantly fewer numbers of HIV$^-$/unknown serostatus partners than whites and Hispanics but also reported slightly higher numbers of unprotected sex acts per partner.

[Table 4 about here.]

5. Web Appendix E

Table 5 shows estimated covariance between log mean parameters of the corresponding outcomes, $(V_{ij}^+, V_{ij}^-, P_{ijk}^+, U_{ijk}^+, P_{ijk}^-, U_{ijk}^-)$, which are a function of parameters $\mathbf{L}$ and $\mathbf{D}$ in the model. Significant positive (negative) covariance estimates represent significant positive (negative) correlation between corresponding outcomes. Estimates and 95% posterior intervals for the autoregressive parameter $\mathbf{A}$ are also shown.

[Table 5 about here.]

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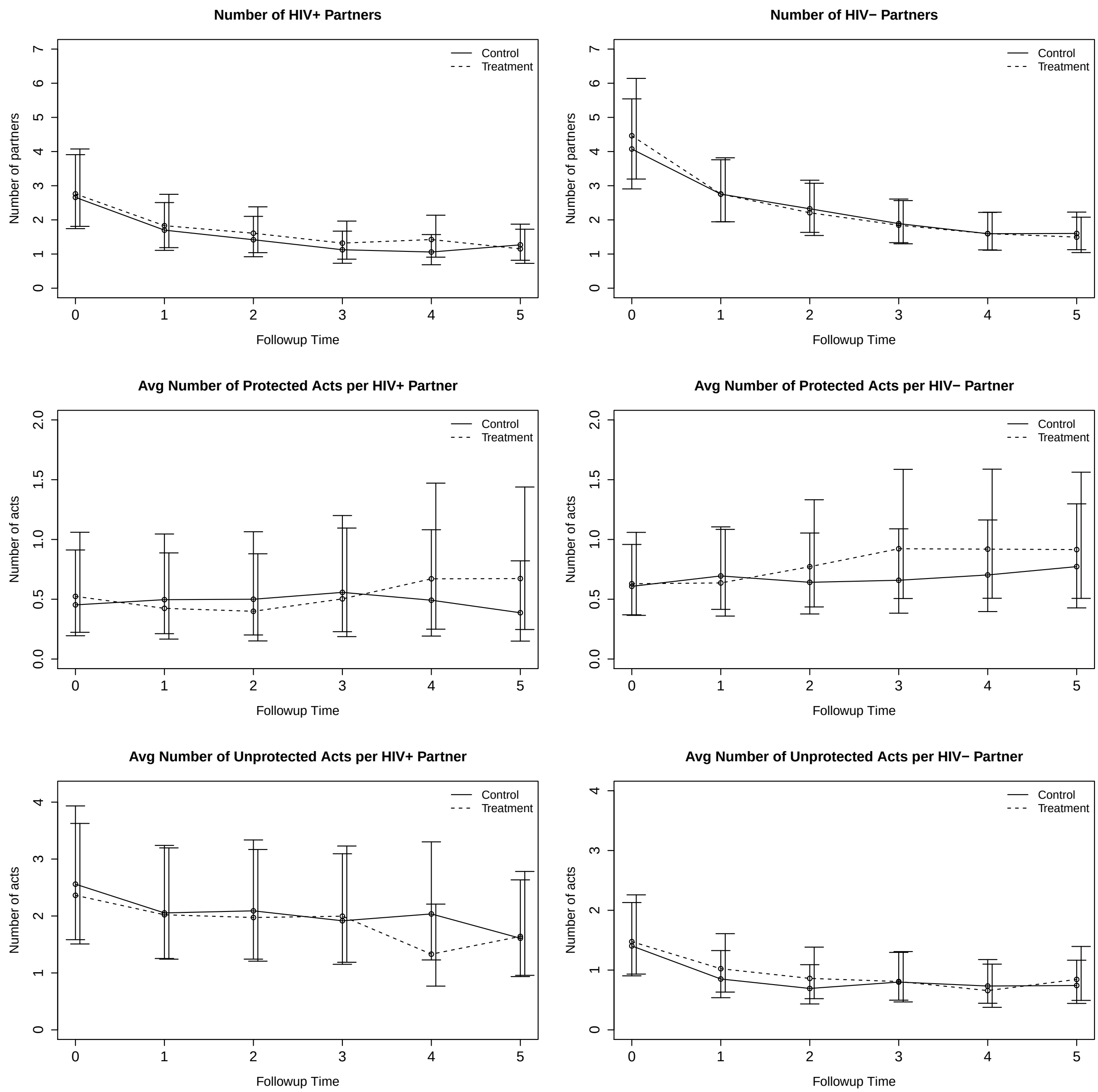

Figure 1. Sexual behavior profiles of treatment and control groups over time. Graphs represent averages for participants who are white, MSM, less than 40 years old, from Los Angeles, did not graduate high school, were out of work, and had no history of hard drug use. Baseline calculations allow for 0 partners.

Table 1

A summary of the average estimated posterior median ($PMed$), posterior mean (PM), lower limit (L_{CI}) and upper limit (U_{CI}) of the 95% equal-tail credible interval (CI), coverage probability of the CI (CP), length of the CI (G_{CI}), and mean square error MSE from 3 analysis methods across 600 simulated datasets with 2 levels of heterogeneity to examine the effects of disaggregation. Analysis 1 uses the true model with complete disaggregated observations. Analysis 2 uses the true model with partial disaggregated observations. Analysis 3 looks at only aggregated totals and assumes data with only a single level of heterogeneity.

Analysis	$PMed$	PM	L_{CI}	U_{CI}	CP	G_{CI}	MSE
1	16.0	16.1	13.0	20.2	97.3	7.2	6.5
2	16.0	16.2	13.0	20.3	96.9	7.3	6.8
3	16.2	16.3	13.0	20.8	95.4	7.8	8.4

Table 2

General demographics of the $n = 936$ subjects at baseline with stratification by intervention assigment.

Variable	Treatment (n=467)	Control (n=469)	Total (n=936)
Site, n (%)			
Los Angeles	163 (34.9)	170 (36.2)	333 (35.6)
Milwaukee	43 (9.2)	44 (9.4)	87 (9.3)
New York	127 (27.2)	118 (25.2)	245 (26.2)
San Francisco	134 (28.7)	137 (29.2)	271 (29.0)
Risk Group, n (%)			
MSM	256 (54.8)	278 (59.3)	534 (57.1)
IDU	57 (12.2)	50 (10.7)	107 (11.4)
FEM	103 (22.1)	93 (19.8)	196 (20.9)
HTM	51 (10.9)	48 (10.2)	99 (10.6)
Education, n (%)			
Less than HS	88 (18.8)	97 (20.7)	185 (19.8)
HS Grad	126 (27.0)	99 (21.1)	225 (24.0)
Some College	176 (37.7)	183 (39.0)	359 (38.4)
College Grad	77 (16.5)	90 (19.2)	167 (17.8)
Race, n (%)			
White	143 (30.6)	157 (33.5)	300 (32.1)
Black	231 (49.5)	190 (40.5)	421 (45.0)
Hispanic	61 (13.1)	82 (17.5)	143 (15.3)
Other	32 (6.9)	40 (8.5)	72 (7.7)
Gender, n (%)			
Male	364 (77.9)	376 (80.2)	740 (79.1)
Female	103 (22.1)	93 (19.8)	196 (20.9)
Mean Age (sd)			
Age	39.57 (7.15)	40.11 (7.68)	39.84 (7.42)

Table 3

Difference between treatment and control groups at each followup time after adjusting for initial differences at baseline. Values are multiplicative and reported as posterior mean (PM) followed by the 95% equal-tail credible interval (L_{CI}, U_{CI}).

Variable	Followup 1	Followup 2	Followup 3	Followup 4	Followup 5
Number of Partners					
HIV^+	1.04 (0.81, 1.33)	1.10 (0.83, 1.46)	1.14 (0.83, 1.55)	1.31 (0.92, 1.81)	0.89 (0.61, 1.24)
HIV^-/unknown	0.92 (0.74, 1.12)	0.87 (0.68, 1.11)	0.90 (0.68, 1.15)	0.92 (0.69, 1.21)	0.86 (0.63, 1.15)
Protected Acts					
HIV^+ partner	0.78 (0.38, 1.46)	0.74 (0.34, 1.47)	0.84 (0.33, 1.68)	1.29 (0.49, 2.75)	1.64 (0.62, 3.47)
HIV^-/unknown partner	0.90 (0.61, 1.26)	1.19 (0.77, 1.75)	1.39 (0.87, 2.10)	1.30 (0.78, 2.04)	1.19 (0.71, 1.88)
Unprotected Acts					
HIV^+ partner	1.08 (0.73, 1.55)	1.05 (0.68, 1.56)	1.16 (0.71, 1.79)	0.73 (0.43, 1.18)	1.13 (0.68, 1.79)
HIV^-/unknown partner	1.16 (0.81, 1.64)	1.21 (0.79, 1.80)	0.99 (0.59, 1.49)	0.87 (0.53, 1.38)	1.11 (0.65, 1.76)

Table 4

*The effects of covariates on sexual behavior. Multiplicative effects from the comparison group is reported as posterior mean PM followed by the 95% equal-tail credible interval (L_{CI}, U_{CI}). The comparison group are white MSMs assigned to the control group who are less than 40 years old, from Los Angeles, did not graduate high school, out of work, and had no history of hard drug use. Values with * indicate statistically significant evidence of difference from the comparison group.*

Variable	Number of Partners	Protected Acts	Unprotected Acts
HIV^+ Partner			
Site			
Milwaukee	0.34 (0.23, 0.49)*	0.68 (0.31, 1.71)	1.29 (0.78, 2.12)
New York	1.10 (0.87, 1.38)	1.34 (0.85, 2.25)	0.94 (0.70, 1.23)
San Francisco	1.15 (0.93, 1.43)	0.78 (0.51, 1.19)	0.85 (0.66, 1.11)
Risk Group			
Female	0.14 (0.10, 0.19)*	2.06 (1.16, 3.76)*	1.98 (1.37, 2.94)*
HTM	0.36 (0.25, 0.51)*	3.35 (1.77, 6.13)*	1.12 (0.75, 1.66)
IDU	1.21 (0.94, 1.54)	0.50 (0.30, 0.85)*	1.65 (1.22, 2.18)*
Education			
HS grad or some college	0.92 (0.70, 1.18)	1.43 (0.94, 2.20)	1.00 (0.76, 1.30)
College graduate	1.34 (0.95, 1.87)	0.75 (0.39, 1.47)	0.98 (0.69, 1.45)
Age			
More than 40 yrs old	0.89 (0.75, 1.08)	1.07 (0.76, 1.52)	0.99 (0.82, 1.22)
Career			
Working	1.22 (1.08, 1.37)*	1.09 (0.82, 1.42)	1.14 (0.97, 1.36)
Race			
Black	0.96 (0.76, 1.22)	2.70 (1.81, 4.08)*	0.83 (0.65, 1.05)
Hispanic	0.83 (0.61, 1.12)	3.37 (1.98, 5.94)*	0.87 (0.64, 1.22)
Other	0.77 (0.53, 1.07)	1.70 (0.95, 3.15)	1.05 (0.72, 1.55)
Drug Use			
Hard drug usage (Lifetime)	1.31 (1.05, 1.66)*	0.94 (0.60, 1.42)	0.92 (0.71, 1.16)
Recent hard drug usage	1.09 (0.76, 1.56)	0.82 (0.43, 1.43)	1.13 (0.76, 1.64)
HIV^- or unknown Partner			
Site			
Milwaukee	1.08 (0.82, 1.41)	1.07 (0.73, 1.58)	1.07 (0.72, 1.56)
New York	0.95 (0.78, 1.16)	1.72 (1.28, 2.34)*	0.95 (0.74, 1.23)
San Francisco	1.50 (1.24, 1.80)*	0.93 (0.72, 1.22)	0.88 (0.67, 1.11)
Risk Group			
Female	0.69 (0.57, 0.84)*	1.70 (1.22, 2.30)*	1.88 (1.41, 2.54)*
HTM	0.58 (0.45, 0.77)*	2.39 (1.57, 3.56)*	1.29 (0.88, 1.87)
IDU	0.60 (0.47, 0.76)*	0.51 (0.35, 0.77)*	1.66 (1.19, 2.32)*
Education			
HS grad or some college	1.09 (0.90, 1.33)	1.05 (0.76, 1.44)	0.88 (0.68, 1.15)
College graduate	1.95 (1.53, 2.53)*	0.76 (0.49, 1.16)	0.84 (0.58, 1.23)
Age			
More than 40 yrs old	0.84 (0.73, 0.98)*	0.96 (0.79, 1.21)	0.79 (0.63, 0.96)*
Career			
Working	1.13 (1.02, 1.26)*	0.99 (0.83, 1.17)	1.15 (0.97, 1.37)
Race			
Black	0.83 (0.68, 0.99)*	1.26 (0.94, 1.73)	1.05 (0.80, 1.34)
Hispanic	1.36 (1.09, 1.69)*	1.90 (1.35, 2.73)*	0.78 (0.57, 1.06)
Other	0.86 (0.65, 1.14)	1.37 (0.89, 2.10)	0.94 (0.63, 1.40)
Drug Use			
Hard drug usage (Lifetime)	0.95 (0.79, 1.14)	0.90 (0.68, 1.17)	1.18 (0.90, 1.53)
Recent hard drug usage	0.98 (0.74, 1.34)	0.95 (0.64, 1.41)	1.41 (1.00, 1.96)*
Partner type			
Main Partner		3.70 (3.41, 4.00)*	

Table 5

Summary of variance and covariance for the log mean parameters of the outcomes and the autoregressive parameter A. Values are reported as posterior mean PM on the first line and the 95% equal-tail credible interval (L_{CI}, U_{CI}) on the second.

Outcomes	V^+_{ij}	V^-_{ij}	P^+_{ijk}	U^+_{ijk}	P^-_{ijk}	U^-_{ijk}
Different Partners						
V^+_{ij}	2.05	0.11	-0.38	-0.47	-0.14	0.06
	(1.84, 2.28)	(0.01, 0.21)	(-0.66, -0.12)	(-0.65, -0.31)	(-0.29, 0.03)	(-0.10, 0.22)
V^-_{ij}	–	1.68	-0.13	-0.22	-0.61	-0.60
	–	(1.55, 1.84)	(-0.34, 0.07)	(-0.35, -0.10)	(-0.78, -0.46)	(-0.74, -0.44)
$P^+_{ijk'}$	–	–	5.09	-1.17	2.08	-0.82
	–	–	(4.34, 5.79)	(-1.45, -0.91)	(1.72, 2.55)	(-1.17, -0.43)
$U^+_{ijk'}$	–	–	–	1.84	-0.35	0.98
	–	–	–	(1.58, 2.10)	(-0.57, -0.13)	(0.74, 1.21)
$P^-_{ijk'}$	–	–	–	–	2.84	-0.46
	–	–	–	–	(2.52, 3.20)	(-0.64, -0.27)
$U^-_{ijk'}$	–	–	–	–	–	2.46
	–	–	–	–	–	(2.18, 2.75)
Same Partner						
P^+_{ijk}	–	–	6.71	-1.44	–	–
	–	–	(5.96, 7.41)	(-1.72, -1.17)	–	–
U^+_{ijk}	–	–	–	3.45	–	–
	–	–	–	(3.19, 3.72)	–	–
P^-_{ijk}	–	–	–	–	4.46	-0.72
	–	–	–	–	(4.11, 4.86)	(-0.91, -0.54)
U^-_{ijk}	–	–	–	–	–	4.07
	–	–	–	–	–	(3.79, 4.36)
Across time correlation parameter						
A	0.77	0.72	0.64	0.67	0.75	0.67
	(0.74, 0.80)	(0.69, 0.75)	(0.59, 0.70)	(0.62, 0.72)	(0.72, 0.79)	(0.63, 0.72)