Supporting Information for DiME: A scalable disease module identification algorithm with application to glioma progression

Yunpeng Liu¹, Daniel A. Tennant², Zexuan Zhu⁴, John K. Heath³, Xin Yao¹, Shan He^{1,3*}

1 School of Computer Science, University of Birmingham, Birmingham, UK

2 School of Cancer Sciences, University of Birmingham, Birmingham, UK

3 Centre for Systems Biology, School of Biological Sciences, University of Birmingham, Birmingham, UK

4 College of Computer Science and Software Engineering, Shenzhen University, China

∗ E-mail: s.he@cs.bham.ac.uk

The B-score Algorithm Pseudo-code

Algorithm 1 The B-score Algorithm

```
1: function Bscore(modele C)2: t \leftarrow 0, C_0 \leftarrow C, B_0 \leftarrow \emptyset, n_C \leftarrow |C|3: while t < n_C - 1 do
 4: for each i \in C_t do
 5: Calculate p_i as p_i = \sum_{q=k_i^{init}}^{k_i} f(q|C)6: end for
 7: w_{t+1} \leftarrow vertex in C_t with highest p_i8: w_{t+2} \leftarrow vertex in C_t with second highest p_i9: B_{t+1} \leftarrow B_t \bigcup \{w_{t+1}\}, C_{t+1} \leftarrow C_t \setminus \{w_{t+1}\}10: Recalculate p values for all nodes currently in B_{t+1}11: p_l \leftarrow lowest p value of vertices in B_{t+1}12: if p_{w_{t+2}} > p_l then
13: \text{swap}(p_{w_{t+2}}, p_l)14: end if
15: Compute Pr(< S_{t+1} | C_{t+1}, B_{t+1}, p_{w_{t+2}}), where S_{t+1} = \sum_{i \in B_{t+1}} p_i16: t \leftarrow t + 117: end while
18: Return min<sub>t</sub> Pr(< S_t | C_t, B_t, p_{w_{t+1}})19: end function
```
The B-score measure assumes a null model where edges within the module (community) of interest is held unchanged while the remaining connections in the network are randomly shuffled. A probabilistic measure based on hypergeometric distribution is then calculated for each module member to evaluate the likelihood that the observed number of within-module edges would arise from the null model. Such a probability is then summed over the number of possible within-module connections (from the observed value to the maximum possible value - the total degree of the node) to give a cumulative probability p_i of observing an intra-module degree equal to or larger than the observed value under the null model.

The above p_i is calculated for all nodes in the module and sorted to identify the "worst" node in the module - the node with the highest p_i . The B-score algorithm assumes that, for a truly non-random module, the probability of observing such a worst p_i value as the *minimum* among all nodes currently not belonging to the module is expected to be very low under the null model for its calculation. The original B-score algorithm also incorporated a stochastic element into the calculation of p_i , took into consideration a list of k worst nodes in the module and utilized the probability that the sum of the scores of these worst nodes in a module obtained from a random background model is smaller than the observed value as the final statistical significance measure (the B-score) for the module. The B-score is calculated over multiple runs and the average is used for evaluation of module statistical significance. Such an additional step has been shown to act as a resampling step and guard against possible significant community structure from random graphs.

Calculation of the conservation score

We first define a reference network which can be seen as the ground true network without noise. We extract the modules from the network as reference modules: R_i $(i = 1, \ldots, n_r)$. We also define a set of m noisy co-expression networks by introducing m different levels of edge noise to the reference network.

We extract the modules N_{i_j} $(i = 1, \ldots, n_n)$ from the jth noisy network and repeat this for all m noisy networks. Then we perform the following steps to calculate the conservation score.

1. Find the best matching module N_{k_j} in the jth noisy network for the kth reference modules R_k , where k_j is obtained by:

$$
k_j = \arg\max_i \frac{|R_k \bigcap N_{i_j}|}{\min(|R_k|, |N_{i_j}|)}
$$

- 2. Repeat step 1 to find the best matching modules in all m noisy networks: N_{k_j} , $j = 1, \ldots, m$.
- 3. Calculate the conservation score for the reference module R_k using the following formula:

$$
ConservationScore(R_k) = \frac{|R_k \bigcap_{j=1}^{m} N_{k_j}|}{|R_k|}
$$

4. Repeat steps 1-3 to calculate conservation scores of all reference modules $ConservationScore(R_i)$, $i=1,\ldots,n_r$.

Derivation of $\Delta \tilde{W}$

Let x_i be the boolean variable indicating whether the *i*th node is selected as a community member. Denote the entire set of nodes as V , and N as the total number of nodes in V . Let A be the adjacency matrix of the entire network. Following the denotations used by Zhao et al. (2011), we have

$$
\tilde{W}_S = O_S \cdot \frac{|S_c|}{|S|} + O_S - (O_S + B_S) = \sum_{i,j \in S} A_{ij} x_i x_j \left(\frac{|S_c|}{|S|} + 1 \right) - \sum_{i \in S, j \in V} A_{ij} x_i \tag{1}
$$

If node k is in S, the only move that will change \tilde{W} is to move it from S to S_c . The new \tilde{W} after moving will be:

$$
\tilde{W}_{S'} = \left(\sum_{i,j \in S} A_{ij} x_i x_j - 2 \sum_{j \in S} A_{kj} x_j\right) \left(\frac{|S_c| + 1}{|S| - 1} + 1\right) - \left(\sum_{i \in S, j \in V} A_{ij} x_i - \sum_{j \in V} A_{kj}\right);
$$
\n(2)

and if node k is in S_c , the only move that will change \tilde{W} is to move it from S_c to S. The new \tilde{W} after moving will be:

$$
\tilde{W}_{S'} = \left(\sum_{i,j \in S} A_{ij} x_i x_j + 2 \sum_{j \in S} A_{kj} x_j\right) \left(\frac{|S_c| - 1}{|S| + 1} + 1\right) - \left(\sum_{i \in S, j \in V} A_{ij} x_i + \sum_{j \in V} A_{kj}\right) \tag{3}
$$

Therefore, we can calculate the change in the value of \tilde{W} when node k is in S:

$$
\Delta \tilde{W}_k = \tilde{W}_{S'} - \tilde{W}
$$

= $\left(O_S - 2 \sum_{j \in S} A_{k_j} x_j \right) \left(\frac{|S_c| + 1}{|S| - 1} \right) - O_S \cdot \frac{|S_c|}{|S|} - \sum_{j \in S} A_{k_j} x_j + \sum_{j \in S} A_{k_j}$ (4)

$$
= O_S \cdot \frac{N}{|S|(|S|-1)} - 2\frac{N}{|S|-1} \sum_{j \in S} A_{k_j} x_j + \sum_{j \in S} A_{k_j}, \tag{5}
$$

Similarly, we can obtain $\Delta \tilde{W}$ when node k is in S_c

$$
\Delta \tilde{W}_k = \tilde{W}_{S'} - \tilde{W}
$$
\n
$$
= \left(O_S + 2 \sum_{j \in S} A_{k_j} x_j \right) \left(\frac{|S_c| - 1}{|S| + 1} \right) - O_S \cdot \frac{|S_c|}{|S|} + \sum_{j \in S} A_{k_j} x_j - \sum_{j \in S} A_{k_j} \tag{6}
$$

$$
= -O_S \cdot \frac{N}{|S|(|S|+1)} + 2\frac{N}{|S|+1} \sum_{j \in S} A_{k_j} x_j - \sum_{j \in S} A_{k_j}, \tag{7}
$$

Combine the above two equations, we finally derive the equation for $\Delta \tilde{W}$:

$$
\Delta \tilde{W}_k = \begin{cases} O_S \cdot \frac{N}{|S|(|S|-1)} - 2\frac{N}{|S|-1} \sum_{j \in S} A_{k_j} x_j + \sum_{j \in S} A_{k_j} & \text{if } k \in S \\ -O_S \cdot \frac{N}{|S|(|S|+1)} + 2\frac{N}{|S|+1} \sum_{j \in S} A_{k_j} x_j - \sum_{j \in S} A_{k_j} & \text{if } k \in S_c \end{cases}
$$

where $O_S = \sum_{i,j \in S} A_{ij} x_i x_j$.

It is easy to observe from (9) and (10) that the change in \tilde{W} for any flipping of node membership can be calculated in linear time (O_S) itself can be updated in linear time for each flip too).

Two Unique DiME Modules in the Grade II and IV Glioma Coexpression Networks

Figure Legends

Tables

	Technique								
	DiME			MCODE			Modularity		
B-score Cutoff	0.05	0.001	1×10^{-5}	0.05	0.001	1×10^{-5}	0.05	0.001	1×10^{-5}
Rembrandt Data. (GBM)	32.97% (574/ 1741)	50.09% (872) 1741)	54.68% (952/ 1741)	58.09% (452) 778)	81.36% (633/ 778)	83.03% (646) 778)	41.16% (1073) 2607)	49.33% (1286) 2607)	99.04% (2582/ 2607)
TCGA Data (GBM)	30.19% (358/ 1186)	42.50% (504/ 1186)	51.85% (615) 1186)	33.75% (188/ 557)	39.14% (218/ 557)	45.60% (254/ 557)	2.14% (36/ 1681)	45.63% (767) 1681)	90.78% (1526) 1681)
Rembrandt Data (grade II Glioma)	47.27% (1230) 2602)	62.95% (1638) 2602)	68.14\% (1773/ 2602)	61.96% (728/ 1175)	65.79% (773/ 1175)	69.96% (822/ 1175)	94.97% (3546) 3734)	98.23% (3668) 3734)	98.93% (3694) 3734)
GEO Data (grade II Glioma)	42.46% (1106) 2605)	66.64% (1736) 2605)	71.48% (1862) 2605)	56.59% (466) 822)	67.76% (557) 822)	74.70% (614) 822)	94.76% (3255) 3435)	99.33% (3412) 3435)	99.71% (3425/ 3435)

Table S1. Relative loss of genes under different B-score cutoffs

Table S3. Module Members in A Unique DiME Module (Grade IV Glioma) Larger than 10 Genes

Module Name	Gene Symbol Gene Product						
	MGAT1	$(Alpha-1,3-)-Glycoprotein$ $Beta-1, 2-N-Acetyl-$ Mannosyl					
		glucosaminyltransferase					
	RAB32	Ras-Related Protein Rab-32					
	<i>ANKRD13B</i>	Ankyrin Repeat Domain-Containing Protein 13B					
	SBK1	SH3 Domain Binding Kinase 1					
Regulation of	<i>SGMS2</i>	Sphingomyelin Synthase 2					
vesicle-related	PLBD1	Phospholipase B Domain Containing 1					
processes	<i>LRRTM2</i>	Leucine Rich Repeat Transmembrane Neuronal 2					
(grade IV glioma)	FSD1	Fibronectin Type III And SPRY Domain Containing 1					
	WIPI1	WD Repeat Domain, Phosphoinositide Interacting 1					
	<i>BICC1</i>	Bicaudal C Homolog 1					
	SOX8	SRY (Sex Determining Region Y)-Box 8					
	RRAS	Related RAS Viral (R-Ras) Oncogene Homolog					
	LMF1	Lipase Maturation Factor 1					
	<i>GATAD2B</i>	GATA Zinc Finger Domain Containing 2B					