Supplementary Material.

Part A: Additional Simulation Results

A1: Method Comparison

Here are more detailed results for the comparison between BINCO and stability selection. The simulation setting is the same as described in Section 3.1. Tables S-1 to S-6 are the results for stability selection and Table S-7 is the result for BINCO.

Table S-1 Power and FDR of Stability Selection, $l = 1$, $\lambda max = 100$, Strong Signal.

			Targeted $FDR = 0.05$		Targeted $FDR = 0.10$				
	FDR.		Power		FDR.		Power		
'min	Mean	SD.	Mean	SD.	Mean	SD.	Mean	SD.	
≤ 40			FDR control not achievable				FDR control not achievable		
50	0.061	0.008	0.818	0.013	0.077	0.009	0.836	0.012	
60	0.060 ¹	0.007	0.783	0.011	0.063	0.008	0.790	0.011	
70	0.054 ¹	0.007	0.725	0.010	0.055	0.007	0.729	0.012	
80	0.050	0.007	0.663	0.010	0.051	0.006	0.666	0.011	
90	0.050	0.006	0.567	0.014	0.050	0.006	0.572	0.014	
100	0.062 ¹	0.008	0.418	0.006	0.061	0.007	0.427	0.016	

 $\overline{1}$ FDR control failed.

TABLE S-2 Power and FDR of Stability Selection, $l = 0.8$, $\lambda_{max} = 100$, Strong Signal.

			Targeted $FDR = 0.05$		Targeted $FDR = 0.10$				
	FDR.		Power		FDR.		Power		
min	Mean	SD.	Mean	SD.	Mean	SD.	Mean	SD	
40			FDR control not achievable		0.099	0.012	0.823	0.011	
50	0.077 ¹	0.009	0.785	0.011	0.091	0.012	0.797	0.011	
60	0.056^{-1}	0.009	0.734	0.011	0.059	0.009	0.739	0.012	
70	0.033	0.008	0.668	0.010	0.036	0.009	0.675	0.009	
80	0.017	0.006	0.568	0.012	0.018	0.007	0.574	0.013	
90	0.010	0.007	0.400	0.015	0.010	0.008	0.408	0.015	
100	0.005	0.007	0.207	0.001	0.007	0.008	0.214	0.010	

 $^{\rm 1}$ FDR control failed.

	Power and FDR of Stability Selection, $l = 0.5$, $\lambda max = 100$, Strong Signal.											
			Targeted $FDR = 0.05$		Targeted $FDR = 0.10$							
	FDR.		Power		FDR.		Power					
λ_{\min}	Mean	SD.	Mean	SD.	Mean	SD	Mean	SD				
40	0.007	0.005	0.706	0.011	0.021	0.006	0.747	0.006				
50	0.007	0.004	0.662	0.009	0.010	0.005	0.676	0.005				
60	0.001	0.002	0.526	0.017	0.002	0.003	0.540	0.003				
70	0	0	0.271	0.012	0	$\overline{0}$	0.287	0.013				
> 80	Omitted for too small power.											

TABLE S-3 Power and FDR of Stability Selection, $l = 0.5$, $\lambda_{max} = 100$, Strong Signal.

TABLE S-4 Power and FDR of Stability Selection, $l = 1$, $\lambda_{max} = 100$, Weak Signal.

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			Targeted $FDR = 0.05$		Targeted $FDR = 0.10$						
		FDR.	Power		FDR.		Power				
γ min	Mean	SD.	Mean	SD.	Mean	SD.	Mean	SD.			
≤ 40			FDR control not achievable		FDR control not achievable						
50	FDR control not achievable			0.006	0.006	0.490	0.020				
60	0.003	0.004	0.434	0.017	0.004	0.004	0.458	0.013			
70	0	0.002	0.288	0.017	< 0.001	0.001	0.298	0.018			
80	Ω	Ω	0.126	0.015	0	0	0.131	0.014			
> 90		Omitted for too small power.									

TABLE S-5

Power and FDR of Stability Selection, $l = 0.8$, $\lambda max = 100$, Weak Signal.

			Targeted $FDR = 0.05$		Targeted $FDR = 0.10$					
		FDR.	Power		FDR.		Power			
λ_{\min}	Mean	SD.	Mean	SD.	Mean	SD.	Mean	SD.		
40			FDR control not achievable		FDR control not achievable					
50	0.002	0.003	0.407	0.026	0.006	0.006	0.485	0.016		
60	0.001	0.003	0.328	0.016	0.001	0.003	0.342	0.014		
70	0	0	0.143	0.016	0	0	0.149	0.015		
80	Omitted for too small power.									

TABLE $\operatorname{S-6}$ Power and FDR of Stability Selection, $l = 0.5$, $\lambda_{max} = 100$, Weak Signal.

			Targeted $FDR = 0.05$		Targeted $FDR = 0.10$			
Signal	FDR.		Power		FDR.		Power	
Strength	Mean	SD	Mean	SD.	Mean	SD.	Mean	SD.
Strong	0.026	0.016	0.801	0.023	0.056	0.025	0.835	0.016
Weak	0.034	0.011	0.569	0.032	0.059	0.017	0.610	0.028

TABLE S-7 Power and FDR of BINCO

A2: U-shape Diagnostics for Empty and Power-law Networks.

Fig S-1: Diagnostic on the empirical distribution of selection frequencies from empty network: no "U-shape" characteristic is observed for λ in a wide range.

Fig S-2: Diagnostic on the empirical distribution of selection frequencies from power-law network: "U-shape" characteristic is observed for λ in a wide range. Note the "U-shape" characteristic is also observed for the empirical selection frequency distributions from empirical and hub networks. Those diagnostic plots are very similar to this one and hence omitted.

A3: Additional Simulation Results for BINCO.

Here we investigate the impact of the number of components in the networks on BINCO's performance. We consider the power-law network with sample size $n = 200$ and the number of nodes $p = 500$. The signal strength is fixed at the strong level as in Section 3.1. Networks are generated by varying the number of components $C = 5, 2$ to 1.

Since components are independent, the model dimensionality is the size of each component which is smaller for larger C. Thus, as the number of components decreases, it might be more challenging to detect the network due to the increasing dimensionality for each component. Nevertheless, for all three networks with different numbers of components, BINCO provides proper (and slightly conservative) control for FDR and decent power (Table S-8).

TABLE S-8 Investigation of the Impact of Different Number of components in Power-law Networks on BINCO Performance

			Targeted $FDR = 0.05$		Targeted $FDR = 0.10$				
Number of		FDR.		Power		FDR.		Power	
components	Mean	SD.	Mean	SD.	Mean	SD.	Mean	SD.	
5	0.046	0.009	0.810	0.013	0.096	0.013	0.845	0.013	
$\overline{2}$	0.048	0.012	0.783	0.011	0.096	0.016	0.814	0.011	
	0.039	0.015	0.804	0.011	0.095	0.020	0.836	0.013	

Part B: Details of the Hub Genes Detected by BINCO on the Breast Cancer Data

¹ The rank of the number of connected edges (from the largest to the smallest) for each gene based on the estimated network by BINCO.

Fig S-3: A scatter-plot of the SD of degree ranks (small to large) versus the mean of degree ranks (large to small) of each gene/probe. The solid circles are the top 10 genes.

Part C: Examples of p_{ij} and \tilde{p}_{ij} being Close

EXAMPLE 1. Subsampling.

Since subsampling $Y'_{(m)}$ of size m from a random sample $Y_{(n)}$ is equivalent to directly sampling a random sample $Y_{(m)}$ of size m, the asymptotic behavior of p_{ij} and \tilde{p}_{ij} should be the same. In particular, if $p_{ij}^{(n)}$ has a limit, then $\tilde p_{ij}^{(m)}$ converges to the same limit and hence $p_{ij}^{(n)}-\tilde p_{ij}^{(m)}\to 0$ as $m, n \to \infty$.

Example 2. Lasso with selection consistency under linear regression settings.

Consider a linear regression model

$$
Y = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}
$$

where, for sample size n, Y is an $n \times 1$ response, $\mathbf{X} = (X_1, \ldots, X_p)$ is the $n \times p$ design matrix and ϵ is the random error with mean **0** and covariance **I**. β is the coefficient vector that needs to estimate.

Denote the covariance matrix of **X** by $C = E(\mathbf{X}'\mathbf{X})$ and write C as

$$
C = \left(\begin{array}{cc} C_{11} & C_{12} \\ C_{21} & C_{22} \end{array} \right),
$$

where C_{11} is the covariance matrix of relevant variables in **X**, C_{22} is the covariance matrix of irrelevant variables in **X** and $C_{12} = C_{21}$ is the matrix of covariance between relevant and irrelevant variables in X . When p is fixed, the selection consistency of the Lasso procedure is equivalent to the irrepresentable condition (Zhao and Yu, 2006)

(S-1)
$$
|C_{12}(C_{11})^{-1}sign(\beta(1))| < 1 - \eta
$$

where $\beta(1)$ is the non-zero coefficients for the relevant variables in the linear model, η is a positive constant vector and sign(\cdot) maps positive entry to 1, negative entry to -1 and zero to zero. Denote the Lasso estimator of β by $\hat{\beta}$ and the one based on bootstrap data by $\tilde{\hat{\beta}}$. Also define the event

$$
S \equiv \{ sign(\hat{\beta}) = sign(\beta) \}
$$

and

$$
\tilde{S} \equiv \{ sign(\tilde{\hat{\beta}}) = sign(\beta) \}.
$$

Note that $\tilde{\cdot}$ is used to represent the counterpart in the bootstrap sample space to that in the sample space. Below we will show that $P(S) \to 1$ implies $P(\tilde{S}) \to 1$, i.e., the Lasso procedure is also consistent on bootstrap resample data. Thus, denote the selection probability of the ith feature w.r.t. the sample space by p_i and that w.r.t. the bootstrap resample space by \tilde{p}_i , then p_i and \tilde{p}_i converge to the same limit (1 or 0, depending on whether the ith feature is a true or irrelevant one) for all $1 \leq i < p$.

We use the notation consistent with Zhao and Yu (2006). First we see that, under the finitemoment assumption of **X**, both the sample covariance C^n and bootstrap resample covariance \tilde{C}^n converge to the same limit C (Arenal-Gutierrez et al. 1996), which means Proposition 1 in Zhao and Yu (2006) can be applied to the bootstrap resample data. Then,

$$
1 - P(\tilde{S}) \le \sum_{i=1}^{q} P\left(|\tilde{z}_i^n| \ge \sqrt{n}(|\beta_i| - \frac{\lambda_n}{2n} \tilde{b}_i^n) \right) + \sum_{i=1}^{p-q} P\left(|\tilde{C}_i^n| \ge \frac{\lambda_n}{2\sqrt{n}} \tilde{\eta}_i \right)
$$

where $(\tilde{z_1}^n, \ldots, \tilde{z_q}^n, \tilde{\zeta_1}^n, \ldots, \tilde{\zeta_{p-q}})' = \tilde{D}^n \tilde{W}^n$ with

$$
\tilde{D}^n = \left(\begin{array}{cc} (\tilde{C}_{11}^{\tilde{n}})^{-1} & \mathbf{0} \\ \tilde{C}_{21}^{\tilde{n}} (\tilde{C}_{11}^{\tilde{n}})^{-1} & -\mathbf{1} \end{array} \right),
$$

 $\tilde{W}^n = \tilde{\mathbf{X}}'\tilde{\epsilon}/\sqrt{n}, \text{ and } \tilde{b} = (\tilde{b_1}^n, \dots, \tilde{b_q}^n) = (\tilde{C_{11}}^n)^{-1} sign\beta(1).$

Denote the counterpart of \tilde{W}^n and \tilde{D}^n w.r.t. the sample space by W^n and D^n . Note that $W^n \rightarrow_d$ $N(\mathbf{0}, C)$ and by Theorem 2.2 of Bickel and Freedman (1981) $\tilde{W}^n - W^n \rightarrow_d N(\mathbf{0}, C)$. Also note that $\tilde{D}^n - D^n \to 0$ and $D^n \to D$ where

$$
D = \left(\begin{array}{cc} (C_{11})^{-1} & \mathbf{0} \\ C_{21}(C_{11})^{-1} & -\mathbf{1} \end{array} \right).
$$

By the Slutsky's Theorem,

$$
D^n W^n \to_d D \cdot N(0, C)
$$

and

$$
\tilde{D}^n \tilde{W}^n - D^n W^n = \tilde{D}^n (\tilde{W}^n - W^n) + (\tilde{D}^n - D^n) W^n \to_d D \cdot N(0, C),
$$

which implies that $\tilde{z}_i^n - z_i^n$ and z_i^n have the same limiting distribution. Thus, for λ_n such that

$$
\lambda_n/n \to 0, \ \lambda_n/n^{\frac{1+c}{2}} \to \infty \ \ with \ 0 \le c < 1,
$$
\n
$$
\sum_{i=1}^q P\left(|\tilde{z_i^n}| \ge \sqrt{n}(|\beta_i| - \frac{\lambda_n}{2n}\tilde{b_i^n})\right)
$$
\n
$$
= \sum_{i=1}^q P\left(|\tilde{z_i^n} - z_i^n + z_i^n| \ge \sqrt{n}(|\beta_i| - \frac{\lambda_n}{2n}\tilde{b_i^n})\right)
$$
\n
$$
\le \sum_{i=1}^q P\left(|\tilde{z_i^n} - z_i^n| + |z_i^n| \ge \sqrt{n}(|\beta_i| - \frac{\lambda_n}{2n}\tilde{b_i^n})\right)
$$
\n(S-2)\n
$$
\le \sum_{i=1}^q \left[P\left(|\tilde{z_i^n} - z_i^n| \ge \frac{1}{2}\sqrt{n}(|\beta_i| - \frac{\lambda_n}{2n}\tilde{b_i^n})\right) + P\left(|z_i^n| \ge \frac{1}{2}\sqrt{n}(|\beta_i| - \frac{\lambda_n}{2n}\tilde{b_i^n})\right) \right]
$$
\n(S-3)\n
$$
= o(e^{-n^c}),
$$

where $(S-3)$ uses the result from the proof of Theorem 1 in Zhao and Yu (2006) while for $(S-2)$ it is because $P(Z_1 + Z_2 \ge t) \le P(max(Z_1, Z_2) + max(Z_1, Z_2) \ge t) = P(Z_1 \ge t/2 \text{ or } Z_2 \ge t/2) \le t$ $P(Z_1 \ge t/2) + P(Z_2 \ge t/2)$. Similarly, it can be shown that

$$
\sum_{i=1}^{p-q} P\left(|\tilde{\zeta_i^n}| \ge \frac{\lambda_n}{2\sqrt{n}} \tilde{\eta_i}\right) = o(e^{-n^c}).
$$

Therefore, $P(\tilde{S}) \rightarrow 1$.

Analogues to the above, it can be shown that lasso is consistent w.r.t. both the sample and the bootstrap resample space under (S-1) and additional regularity conditions when p is allowed to grow as n grows.

Example 3. Space procedure (Peng et al., 2009) with selection consistency for network construction.

Similar to the irrepresentable condition for the lasso regression case, the selection consistency for the space procedure is implied by a condition imposed on the second derivative of the objective loss function which converges to the same limit for both sample data and bootstrap resample data. Under this condition and additional regularity conditions, the probability of space procedure being inconsistent based on bootstrap resamples can be bounded above by a small number in a similar way as the bound based on samples, which implies that the space procedure is also consistent w.r.t. the bootstrap resmaple space and hence $p_{ij} - \tilde{p}_{ij} \to 0$ for all $(i, j) \in \Omega$.

Part D: Other Simulation Results

D1: An Example where BINCO's FDR Cannot be Controlled at Stringent Levels when the Valley Point Value is too Large.

We simulate data from the power-law network as in Fig $5(a)$, but the signal strength is set at the "very weak" level (see details in the simulation where we investigate the effect of signal strength on BINCO in Section 3.2). The selection frequencies are generated by applying space on bootstrap resamples from the simulated data. In this example, the empirical selection frequency distribution is not U-shaped (Fig S-4) with a valley point value (0.96) greater than the threshold (0.8) set in Step 2.3 in the U-shape Detection Procedure. The smallest FDR of an aggregation-based selection $S_c^{\lambda} = \{(i, j) : X_{ij}^{\lambda} \geq c\}$ is 0.07, achieved at $c = 1$, which means the FDR of BINCO's slelction can not be less than 0.07.

Fig S-4: Non-U-shaped empirical distribution of selection frequencies where the FDR of BINCO's selection can not be less than 0.07. The selection frequencies are generated under $\lambda = 30$

D2: An Example where Stability Selection Fails to Control False Positives.

In this example we simulate selection frequencies from a setting where the exchangeability assumption needed by *stability selection* is violated. Specifically, we generate 20 independent samples of selection frequencies under the following setting:

(a) Consider 100,000 candidate variables, of which 99,500 are null variables and 500 are true variables.

(b) The selection probabilities for first 90,000 null variables are set to be $p_0 = 0.01$. The distribution of selection probabilities (denoted by $F(p)$) of the other 9,500 null variables is $F(p)$ = $Pr(p_{ij} \leq p) = \sqrt{p}$, such that, e.g., 10% of 9,500 null variables (less than 1% of total null edges) have selection probabilities greater than 0.81 but less than 1.

- (c) The selection probabilities for all 500 true variables are set to be $p_1 = 0.99$.
- (d) We repeat this for $B = 100$.

The empirical mixture densities are all U-shaped (Fig S-5), although the valley point value is large. We observe (Fig S-6) that BINCO provides reasonable estimates for the null distribution and hence controls the FDR well, but the *stability selection* fails to control the false positives (Fig S-7).

Fig S-5: A typical empirical mixture distribution of selection frequencies.

Fig S-6: The null distribution (in dots) is well estimated by BINCO (solid line).

Number of False Positives for Stability Selection

Fig S-7: The actual number of false positives (around 1300, dots) is significantly larger than the theoretical upper bound (below 500, solid line) suggested by the stability selection method, for all 20 samples.

There are correlations between edges in both simulated and real data (see Fig S-8), and the correlation distributions for both are similar.

Fig S-8: Distributions of correlation between edges for a simulated data (left panel, which is the same data set used in Figs 1-3 of the main text) and for the real data (right panel, the BC data analyzed in Section 4 of the main text). The mean and MAD for the simulated data are 0.005 and 0.111, respectively, and the mean and MAD for the real data are 0.002 and 0.111, respectively.

D4: U-Shaped Empirical Distributions of Selection Frequencies Generated from Non-normal Data.

We simulate two non-normal data sets as follows. First we apply Cholesky decomposition on the correlation matrix (Σ) , which we used to generate the normal data for the power-law network in Section 3, i.e., we decompose Σ as $\Sigma = LL^*$ where L is an lower triangular matrix with strictly positive diagonal entries and L^* is the conjugate transpose of L . Then we simulate uncorrelated vector u following some non-normal marginal distribution (we use the t-distribution $(df=5)$ for one data set and the uniform distribution on [-1,1] for the other) and apply L to this u to obtain the jointly non-normal data Lu. We apply *space* on the two simulated non-normal data sets and both yield U-shaped empirical distributions of selection frequencies (Figs S-9, S-10). Applying BINCO on these U-shaped distributions yields edge selection results with well-controlled FDR and decent power (details omitted).

Fig S-9: Selection frequency distribution for data simulated from a t-distribution (df=5).

Fig S-10: Selection frequency distribution for data simulated based on uniform distribution.

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