

Mapping motif tunability and robustness in the design of synthetic signaling circuits

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SM1. Supplementary Table I

<i>Linear motif core structure</i>	
$I + L \xrightleftharpoons[k_1^r]{k_1^f} IL \xrightarrow{K_2} I^* + L$	LM Rx 1
$I^* \xrightarrow{K_3} I$	LM Rx 2
$A + I^* \xrightleftharpoons[k_4^r]{k_4^f} AI \xrightarrow{K_5} A^* + I^*$	LM Rx 3
$A^* \xrightarrow{K_6} A$	LM Rx 4
$B + I^* \xrightleftharpoons[k_7^r]{k_7^f} BI \xrightarrow{K_8} B^* + I^*$	LM Rx 5
$B^* \xrightarrow{K_9} B$	LM Rx 6
$X + A^* \xrightleftharpoons[k_{10}^r]{k_{10}^f} XA \xrightarrow{K_{11}} X^* + A^*$	LM Rx 7
$X^* \xrightarrow{K_{12}} X$	LM Rx 8
$Y + B^* \xrightleftharpoons[k_{13}^r]{k_{13}^f} YB \xrightarrow{K_{14}} Y^* + B^*$	LM Rx 9
$Y^* \xrightarrow{K_{15}} Y$	LM Rx 10
<i>Negative feedback additional reactions</i>	
$I^* + X^* \xrightleftharpoons[k_{16}^r]{k_{16}^f} IX \xrightarrow{K_{17}} I + X^*$	NFB Rx 11
$I^* + Y^* \xrightleftharpoons[k_{18}^r]{k_{18}^f} IY \xrightarrow{K_{19}} I + Y^*$	NFB Rx 12
<i>Positive feedback additional reactions</i>	
$I + X^* \xrightleftharpoons[k_{16}^r]{k_{16}^f} IX \xrightarrow{K_{17}} I^* + X^*$	PFB Rx 11
$I + Y^* \xrightleftharpoons[k_{18}^r]{k_{18}^f} IY \xrightarrow{K_{19}} I^* + Y^*$	PFB Rx 12
<i>Positive-negative feedback additional reactions</i>	
$I + X^* \xrightleftharpoons[k_{16}^r]{k_{16}^f} IX \xrightarrow{K_{17}} I^* + X^*$	PNFB Rx 11
$I^* + Y^* \xrightleftharpoons[k_{18}^r]{k_{18}^f} IY \xrightarrow{K_{19}} I + Y^*$	PNFB Rx 12
<i>Negative feedforward additional reactions</i>	
$X^* + I^* \xrightleftharpoons[k_{16}^r]{k_{16}^f} XI \xrightarrow{K_{17}} X + I^*$	NFF Rx 11
$Y^* + I^* \xrightleftharpoons[k_{18}^r]{k_{18}^f} YI \xrightarrow{K_{19}} Y + I^*$	NFF Rx 12
<i>Positive feedforward additional reactions</i>	
$X + I^* \xrightleftharpoons[k_{16}^r]{k_{16}^f} XI \xrightarrow{K_{17}} X^* + I^*$	PFF Rx 11



Positive-negative feedforward additional reactions



Coherent bifan additional reactions



Incoherent bifan additional reactions



Partially coherent bifan additional reactions



Isolated negative feedback additional reactions



Isolated positive feedback additional reactions



Supplementary Table I: Reaction networks describing the dynamics of the signaling motif. The ligand L activates the input source from the inactive (I) to the active (I^*) state. The active input source I^* can then activate intermediate molecules (A^* , B^*), which can further activate downstream components (X^* , Y^*). IL, AI, BI, XA, YB, IX, IY, XI, YI, YA, XB, AX, and BY are

complex signaling molecules that are formed from association of the various motif components.

All active molecules are assumed to be spontaneously deactivated.

SM2. Supplementary Table II

<i>Linear motif core structure ODE model</i>	
$d[L]/dt = -k_1^f [I][L] + (k_1^r + K_2)[IL]$	LM Eq1
$d[I]/dt = -k_1^f [I][L] + k_1^r [IL] + K_3[I^*]$	LM Eq2
$d[I^*]/dt = K_2[IL] - K_3[I^*] - k_4^f [A][I^*] + (k_4^r + K_5)[AI] - k_7^f [B][I^*] + (k_7^r + K_8)[BI]$	LM Eq3
$d[A]/dt = -k_4^f [A][I^*] + k_4^r [AI] + K_6[A^*]$	LM Eq4
$d[A^*]/dt = K_5[AI] - K_6[A^*] - k_{10}^f [X][A^*] + (k_{10}^r + K_{11})[XA]$	LM Eq5
$d[B]/dt = -k_7^f [B][I^*] + k_7^r [BI] + K_9[B^*]$	LM Eq6
$d[B^*]/dt = K_8[BI] - K_9[B^*] - k_{13}^f [Y][B^*] + (k_{13}^r + K_{14})[YB]$	LM Eq7
$d[X]/dt = -k_{10}^f [X][A^*] + k_{10}^r [XA] + K_{12}[X^*]$	LM Eq8
$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*]$	LM Eq9
$d[Y]/dt = -k_{13}^f [Y][B^*] + k_{13}^r [YB] + K_{15}[Y^*]$	LM Eq10
$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*]$	LM Eq11
$d[IL]/dt = k_1^f [I][L] - (k_1^r + K_2)[IL]$	LM Eq12
$d[AI]/dt = k_4^f [A][I^*] - (k_4^r + K_5)[AI]$	LM Eq13
$d[BI]/dt = k_7^f [B][I^*] - (k_7^r + K_8)[BI]$	LM Eq14
$d[XA]/dt = k_{10}^f [X][A^*] - (k_{10}^r + K_{11})[XA]$	LM Eq15
$d[YB]/dt = k_{13}^f [Y][B^*] - (k_{13}^r + K_{14})[YB]$	LM Eq16
<i>Negative feedback modified and additional ODEs</i>	
$d[I]/dt = -k_1^f [I][L] + k_1^r [IL] + K_3[I^*] + K_{17}[IX] + K_{19}[IY]$	NFB Eq2
$d[I^*]/dt = K_2[IL] - K_3[I^*] - k_4^f [A][I^*] + (k_4^r + K_5)[AI] - k_7^f [B][I^*] + (k_7^r + K_8)[BI] - k_{16}[I^*][X^*] + k_{16}[IX] - k_{18}[I^*][Y^*] + k_{18}[IY]$	NFB Eq3
$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] - k_{16}[I^*][X^*] + (k_{16}^r + K_{17})[IX]$	NFB Eq9
$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] - k_{18}[I^*][Y^*] + (k_{18}^r + K_{19})[IY]$	NFB Eq11
$d[IX]/dt = k_{16}[I^*][X^*] - (k_{16}^r + K_{17})[IX]$	NFB Eq17
$d[IY]/dt = k_{18}[I^*][Y^*] - (k_{18}^r + K_{19})[IY]$	NFB Eq18
<i>Positive feedback modified and additional ODEs</i>	
$d[I]/dt = -k_1^f [I][L] + k_1^r [IL] + K_3[I^*] - k_{16}^f [I][X^*] + k_{16}^r [IX] - k_{18}^f [I][Y^*] + k_{18}^r [IY]$	PFB Eq2

$d[I^*]/dt = K_2[IL] - K_3[I^*] - k_4^f[A][I^*] + (k_4^r + K_5)[AI] - k_7^f[B][I^*] + (k_7^r + K_8)[BI] + K_{17}[IX] + K_{19}[IY]$	<i>PFB Eq3</i>
$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] - k_{16}^f[I][X^*] + (k_{16}^r + K_{17})[IX]$	<i>PFB Eq9</i>
$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] - k_{18}^f[I][Y^*] + (k_{18}^r + K_{19})[IY]$	<i>PFB Eq11</i>
$d[IX]/dt = k_{16}^f[I][X^*] - (k_{16}^r + K_{17})[IX]$	<i>PFB Eq17</i>
$d[IY]/dt = k_{18}^f[I][Y^*] - (k_{18}^r + K_{19})[IY]$	<i>PFB Eq18</i>

Positive-negative feedback modified and additional ODEs

$d[I]/dt = -k_1^f[I][L] + k_1^r[IL] + K_3[I^*] - k_{16}^f[I][X^*] + k_{16}^r[IX] + K_{19}[IY]$	<i>PNFB Eq2</i>
$d[I^*]/dt = K_2[IL] - K_3[I^*] - k_4^f[A][I^*] + (k_4^r + K_5)[AI] - k_7^f[B][I^*] + (k_7^r + K_8)[BI] + K_{17}[IX] - k_{18}^f[I^*][Y^*] + k_{18}^r[IY]$	<i>PNFB Eq3</i>
$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] - k_{16}^f[I][X^*] + (k_{16}^r + K_{17})[IX]$	<i>PNFB Eq9</i>
$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] - k_{18}^f[I^*][Y^*] + (k_{18}^r + K_{19})[IY]$	<i>PNFB Eq11</i>
$d[IX]/dt = k_{16}^f[I][X^*] - (k_{16}^r + K_{17})[IX]$	<i>PNFB Eq17</i>
$d[IY]/dt = k_{18}^f[I^*][Y^*] - (k_{18}^r + K_{19})[IY]$	<i>PNFB Eq18</i>

Negative feedforward modified and additional ODEs

$d[I^*]/dt = K_2[IL] - K_3[I^*] - k_4^f[A][I^*] + (k_4^r + K_5)[AI] - k_7^f[B][I^*] + (k_7^r + K_8)[BI] - k_{16}^f[X^*][I^*] + (k_{16}^r + K_{17})[IX] - k_{18}^f[Y^*][I^*] + (k_{18}^r + K_{19})[IY]$	<i>NFF Eq3</i>
$d[X]/dt = -k_{10}^f[X][A^*] + k_{10}^r[XA] + K_{12}[X^*] + K_{17}[IX]$	<i>NFF Eq8</i>
$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] - k_{16}^f[X^*][I^*] + k_{16}^r[IX]$	<i>NFF Eq9</i>
$d[Y]/dt = -k_{13}^f[Y][B^*] + k_{13}^r[YB] + K_{15}[Y^*] + K_{19}[IY]$	<i>NFF Eq10</i>
$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] - k_{18}^f[Y^*][I^*] + k_{18}^r[IY]$	<i>NFF Eq11</i>
$d[XI]/dt = k_{16}^f[X^*][I^*] - (k_{16}^r + K_{17})[XI]$	<i>NFF Eq17</i>

Positive feedforward modified and additional ODEs

$d[I^*]/dt = K_2[IL] - K_3[I^*] - k_4^f[A][I^*] + (k_4^r + K_5)[AI] - k_7^f[B][I^*] + (k_7^r + K_8)[BI] - k_{16}^f[X][I^*] + (k_{16}^r + K_{17})[IX] - k_{18}^f[Y][I^*] + (k_{18}^r + K_{19})[IY]$	<i>PFF Eq3</i>
$d[X]/dt = -k_{10}^f[X][A^*] + k_{10}^r[XA] + K_{12}[X^*] - k_{16}^f[X][I^*] + k_{16}^r[IX]$	<i>PFF Eq8</i>
$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] + K_{17}[IX]$	<i>PFF Eq9</i>
$d[Y]/dt = -k_{13}^f[Y][B^*] + k_{13}^r[YB] + K_{15}[Y^*] - k_{18}^f[Y][I^*] + k_{18}^r[IY]$	<i>PFF Eq10</i>
$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] + K_{19}[IY]$	<i>PFF Eq11</i>

$$d[XI]/dt = k_{16}^f [X][I^*] - (k_{16}^r + K_{17})[XI] \quad PFF Eq17$$

Positive-negative feedforward modified and additional ODEs

$$\begin{aligned} d[I^*]/dt &= K_2[IL] - K_3[I^*] - k_4^f[A][I^*] + (k_4^r + K_5)[AI] - k_7^f[B][I^*] + \\ &\quad + (k_7^r + K_8)[BI] - k_{16}^f[X][I^*] + (k_{16}^r + K_{17})[IX] - k_{18}^f[Y^*][I^*] + (k_{18}^r + K_{19})[IY] \end{aligned} \quad PNFF Eq3$$

$$d[X]/dt = -k_{10}^f[X][A^*] + k_{10}^r[XA] + K_{12}[X^*] - k_{16}^f[X][I^*] + k_{16}^r[IX] \quad PNFF Eq8$$

$$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] + K_{17}[IX] \quad PNFF Eq9$$

$$d[Y]/dt = -k_{13}^f[Y][B^*] + k_{13}^r[YB] + K_{15}[Y^*] + K_{19}[IY] \quad PNFF Eq10$$

$$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] - k_{18}^f[Y^*][I^*] + k_{18}^r[IY] \quad PNFF Eq11$$

$$d[XI]/dt = k_{16}^f[X][I^*] - (k_{16}^r + K_{17})[XI] \quad PNFF Eq17$$

Coherent bifan modified and additional ODEs

$$\begin{aligned} d[A^*]/dt &= K_5[AI] - K_6[A^*] - k_{10}^f[X][A^*] + (k_{10}^r + K_{11})[XA] - k_{16}^f[Y][A^*] + (k_{16}^r + K_{17})[YA] \end{aligned} \quad CB Eq5$$

$$d[B^*]/dt = K_8[BI] - K_9[B^*] - k_{13}^f[Y][B^*] + (k_{13}^r + K_{14})[YB] - k_{18}^f[X][B^*] + (k_{18}^r + K_{19})[XB] \quad CB Eq7$$

$$d[X]/dt = -k_{10}^f[X][A^*] + k_{10}^r[XA] + K_{12}[X^*] - k_{18}^f[X][B^*] + k_{18}^r[XB] \quad CB Eq8$$

$$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] + K_{19}[XB] \quad CB Eq9$$

$$d[Y]/dt = -k_{13}^f[Y][B^*] + k_{13}^r[YB] + K_{15}[Y^*] - k_{16}^f[Y][A^*]k_{16}^r[YA] \quad CB Eq10$$

$$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] + K_{17}[YA] \quad CB Eq11$$

$$d[YA]/dt = k_{16}^f[Y][A^*] - (k_{16}^r + K_{17})[YA] \quad CB Eq17$$

Incoherent bifan modified and additional ODEs

$$\begin{aligned} d[A^*]/dt &= K_5[AI] - K_6[A^*] - k_{10}^f[X][A^*] + (k_{10}^r + K_{11})[XA] - k_{16}^f[Y^*][A^*] + (k_{16}^r + K_{17})[YA] \end{aligned} \quad IB Eq5$$

$$d[B^*]/dt = K_8[BI] - K_9[B^*] - k_{13}^f[Y][B^*] + (k_{13}^r + K_{14})[YB] - k_{18}^f[X^*][B^*] + (k_{18}^r + K_{19})[XB] \quad IB Eq7$$

$$d[X]/dt = -k_{10}^f[X][A^*] + k_{10}^r[XA] + K_{12}[X^*] + K_{19}[XB] \quad IB Eq8$$

$$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] - k_{18}^f[X^*][B^*] + k_{18}^r[XB] \quad IB Eq9$$

$$d[Y]/dt = -k_{13}^f[Y][B^*] + k_{13}^r[YB] + K_{15}[Y^*] + K_{17}[YA] \quad IB Eq10$$

$$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] - k_{16}^f[Y^*][A^*]k_{16}^r[YA] \quad IB Eq11$$

$$d[YA]/dt = k_{16}^f[Y^*][A^*] - (k_{16}^r + K_{17})[YA] \quad IB Eq17$$

Partially coherent bifan modified and additional ODEs

$$\begin{aligned} d[A^*]/dt &= K_5[AI] - K_6[A^*] - k_{10}^f[X][A^*] + (k_{10}^r + K_{11})[XA] - k_{16}^f[Y][A^*] + (k_{16}^r + K_{17})[YA] \end{aligned} \quad PCB Eq5$$

$$d[B^*]/dt = K_8[BI] - K_9[B^*] - k_{13}^f[Y][B^*] + (k_{13}^r + K_{14})[YB] - k_{18}^f[X^*][B^*] + (k_{18}^r + K_{19})[XB] \quad PCB Eq7$$

$$d[X]/dt = -k_{10}^f[X][A^*] + k_{10}^r[XA] + K_{12}[X^*] + K_{19}[XB] \quad PCB Eq8$$

$$d[X^*]/dt = K_{11}[XA] - K_{12}[X^*] - k_{18}^f[X^*][B^*] + k_{18}^r[XB] \quad PCB Eq9$$

$$d[Y]/dt = -k_{13}^f[Y][B^*] + k_{13}^r[YB] + K_{15}[Y^*] - k_{16}^f[Y][A^*]k_{16}^r[YA] \quad PCB Eq10$$

$$d[Y^*]/dt = K_{14}[YB] - K_{15}[Y^*] + K_{17}[YA] \quad PCB Eq11$$

$$d[YA]/dt = k_{16}^f[Y][A^*] - (k_{16}^r + K_{17})[YA] \quad PCB Eq17$$

Isolated negative feedback modified and additional ODEs

$$d[A]/dt = -k_4^f[A][I^*] + k_4^r[AI] + K_6[A^*] + K_{17}[AX] \quad iNFB Eq4$$

$$d[A^*]/dt = K_5[AI] - K_6[A^*] - k_{10}^f[X][A^*](k_{10}^r + K_{11})[XA] - k_{16}^f[A^*][X^*] + k_{16}^r[AX] \quad iNFB Eq5$$

$$d[B]/dt = -k_7^f[B][I^*] + k_7^r[BI] + K_9[B^*] + K_{19}[BY] \quad iNFB Eq6$$

$$d[B^*]/dt = K_8[BI] - K_9[B^*] - k_{13}^f[Y][B^*] + (k_{13}^r + K_{14})[YB] - k_{18}^f[B^*][Y^*] + k_{18}^r[BY] \quad iNFB Eq7$$

$$d[X^*]/dt = -k_{16}^f[A^*][X^*] + (k_{16}^r + K_{17})[AX] \quad iNFB Eq9$$

$$d[Y^*]/dt = -k_{18}^f[B^*][Y^*] + (k_{18}^r + K_{19})[BY] \quad iNFB Eq11$$

$$d[AX]/dt = k_{16}^f[A^*][X^*] - (k_{16}^r + K_{17})[AX] \quad iNFB Eq17$$

$$d[BY]/dt = k_{18}^f[B^*][Y^*] - (k_{18}^r + K_{19})[BY] \quad iNFB Eq18$$

Isolated positive feedback modified and additional ODEs

$$d[A]/dt = -k_4^f[A][I^*] + k_4^r[AI] + K_6[A^*] - k_{16}^f[A][X^*] + k_{16}^r[AX] \quad iPFB Eq 4$$

$$d[A^*]/dt = K_5[AI] - K_6[A^*] - k_{10}^f[X][A^*] + (k_{10}^r + K_{11})[XA] + K_{17}[AX] \quad iPFB Eq 5$$

$$d[B]/dt = -k_7^f[B][I^*] + k_7^r[BI] + K_9[B^*] - k_{18}^f[B][Y^*] + k_{18}^r[BY] \quad iPFB Eq 6$$

$$d[B^*]/dt = K_8[BI] - K_9[B^*] - k_{13}^f[Y][B^*] + (k_{13}^r + K_{14})[YB] + K_{19}[BY] \quad iPFB Eq 7$$

$$d[X^*]/dt = -k_{16}^f[A][X^*] + (k_{16}^r + K_{17})[AX] \quad iPFB Eq 9$$

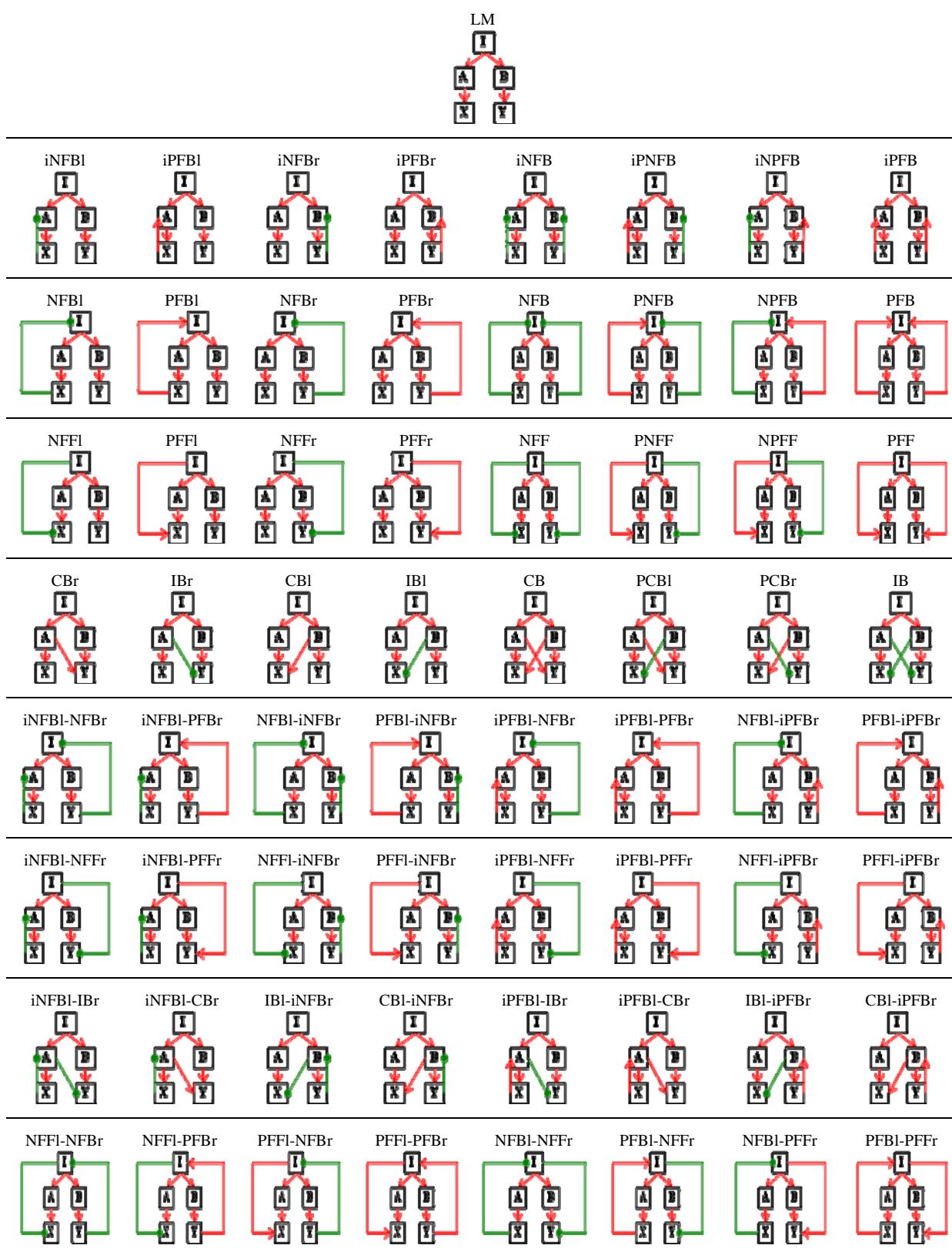
$$d[Y^*]/dt = -k_{18}^f[B][Y^*] + (k_{18}^r + K_{19})[BY] \quad iPFB Eq 11$$

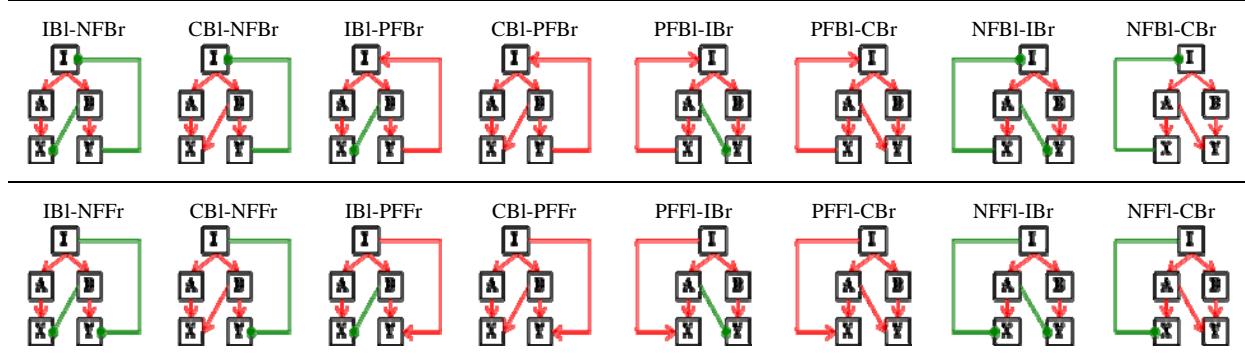
$$d[AX]/dt = k_{16}^f[A][X^*] - (k_{16}^r + K_{17})[AX] \quad iPFB Eq17$$

$$d[BY]/dt = k_{18}^f[B][Y^*] - (k_{18}^r + K_{19})[BY] \quad iPFB Eq18$$

Supplementary Table II: Ordinary differential equation (ODE) modeling describing the dynamics of the signaling motif. The components in square brackets represent the concentrations of the signaling species, and the k's represent the kinetic rate constants.

SM3. Supplementary Table III

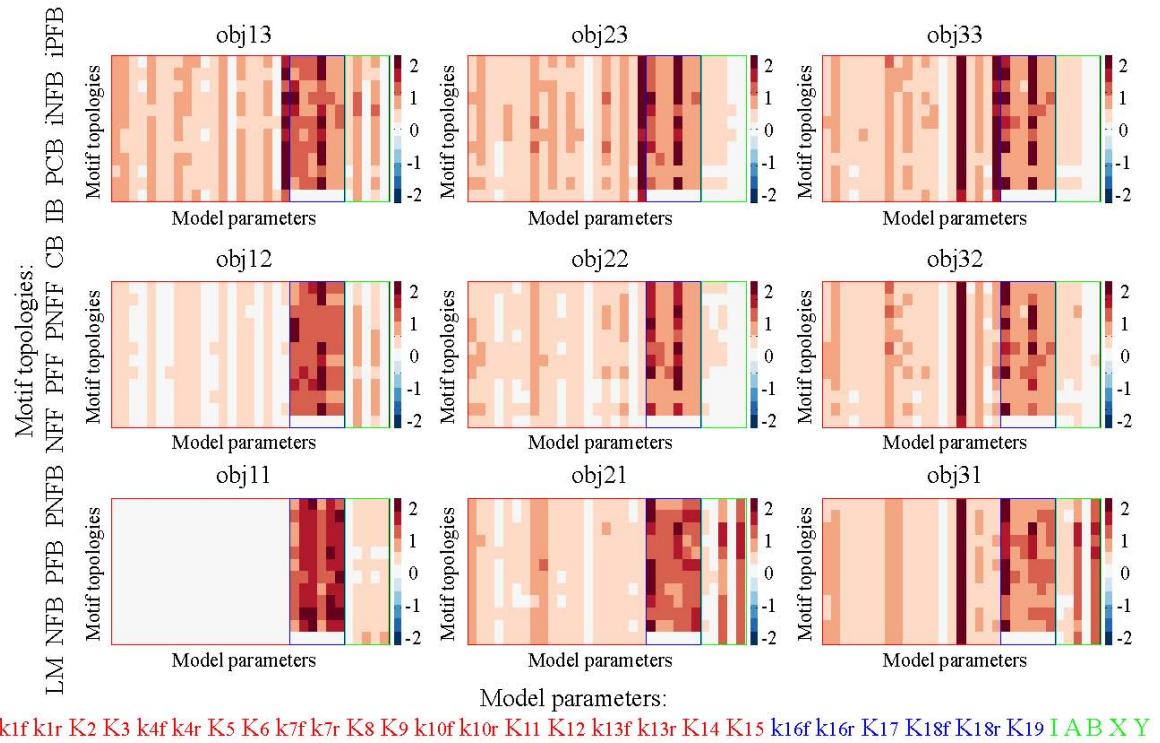




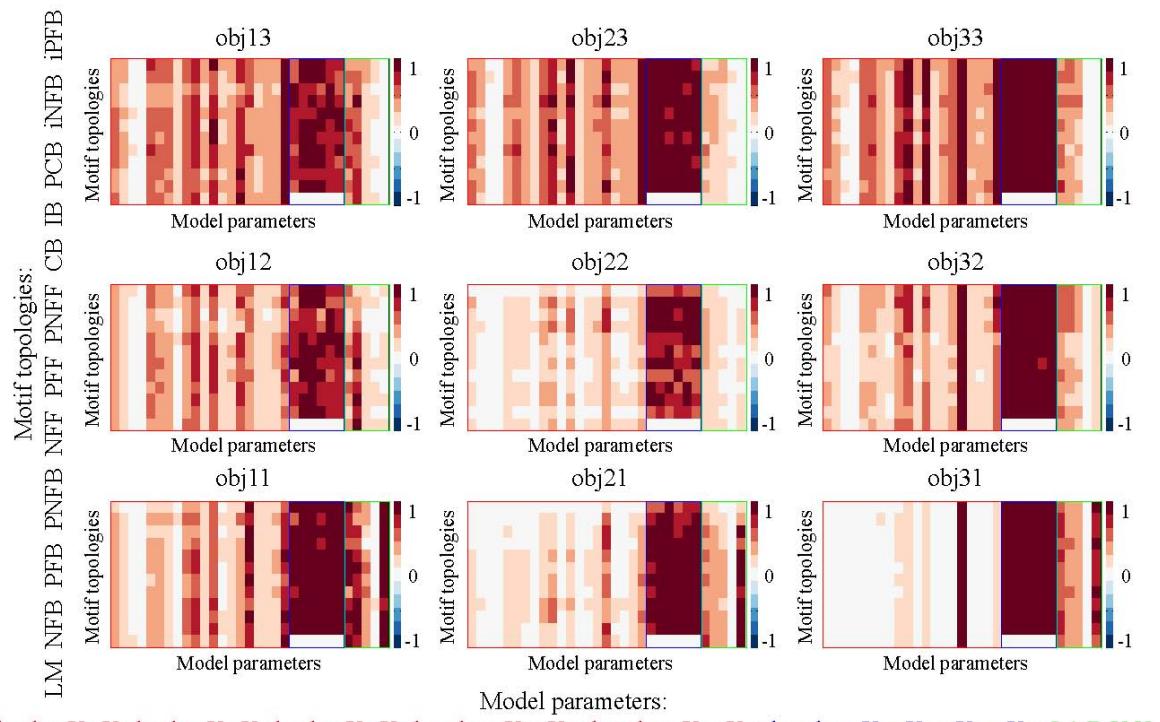
Supplementary Table III: Motif topologies that interconnect the linear, feedback, feed-forward, bifan, and isolated feedback motifs.

SM4. Supplementary Figure 1

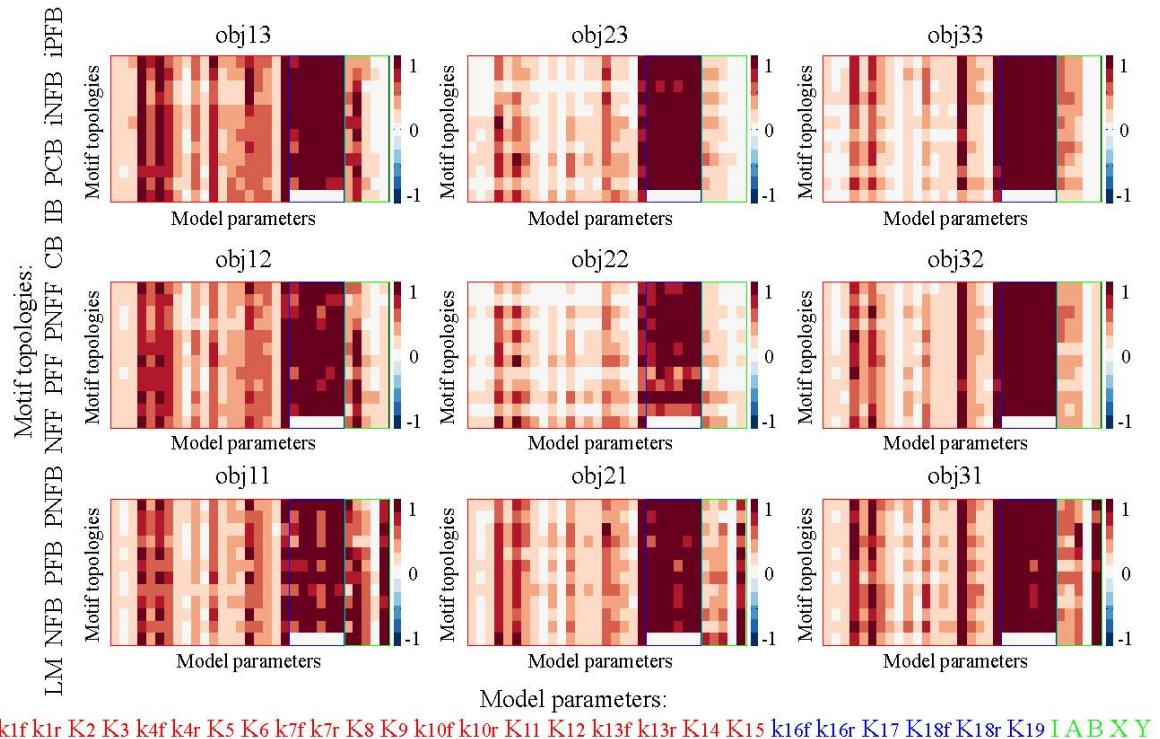
A



B

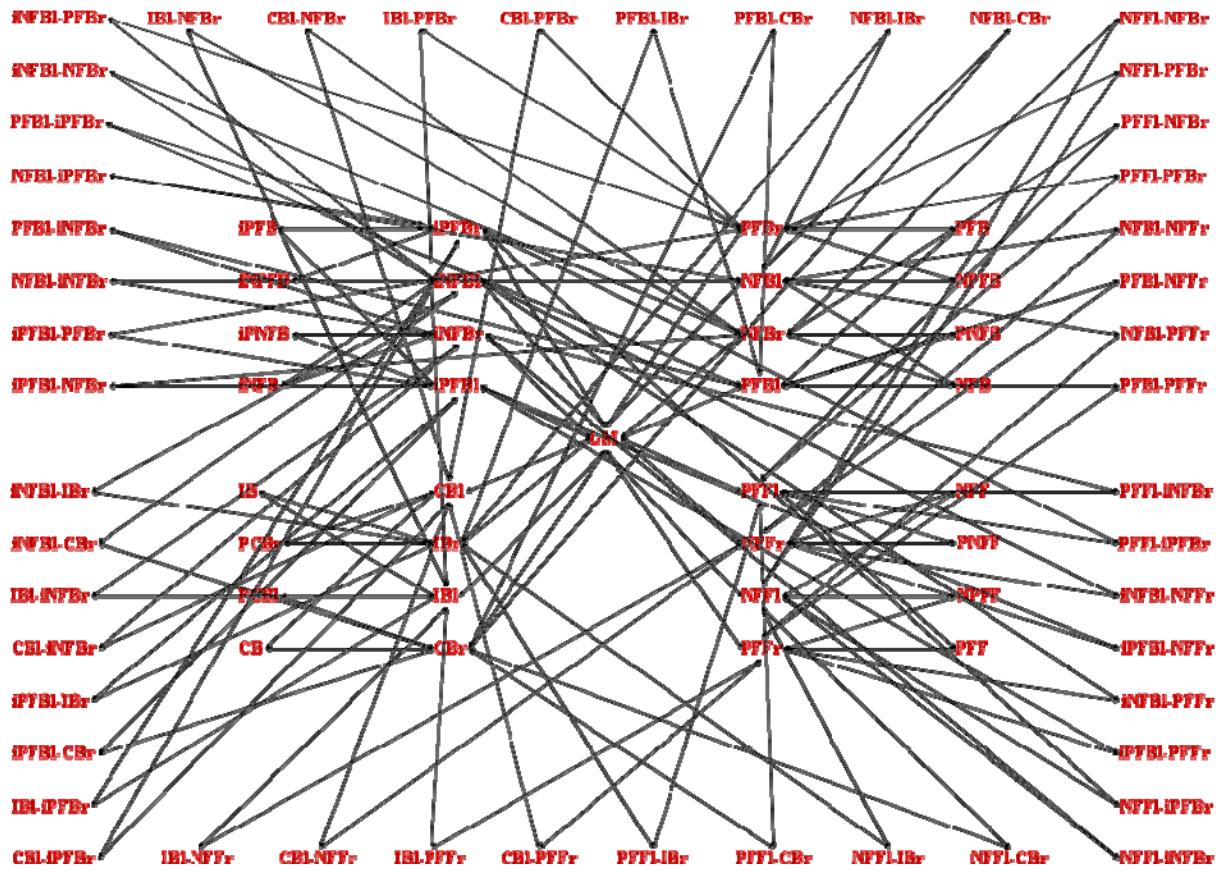


C



Supplementary Figure 1: Motif tunability to static output objectives obtained through PSO sampling of model parameters necessary for ODE implementation. Particle positions were initialized to points A) $xy_{1.5_8.5}$; B) $xy_{8.5_1.5}$; and C) $xy_{8.5_8.5}$.

SM5. Supplementary Figure 2



Supplementary Figure 2: Interaction network of 81 motif topologies that linked the linear, feedback, feed-forward, bifan, and isolated feedback motifs.