Weber's law - mathematical and computational supplement

## **1** Supplemental Figures



Figure 1: Tuning curves appropriate for Weber's law. a. Monotonically increasing functions. b. Monotonically decreasing functions. Plots are shown for different values of  $\theta_0$ . In the monotonically increasing case they are defined only for  $\theta \geq \theta_0$  and in the decreasing case only for  $\theta < \theta_0$ . The plots assume Poisson statistics with  $\tau = 0.5$  sec,  $\alpha = 0.25$ ,  $\beta = 1$  and  $\rho = 0.5$ . Note that changes to  $\alpha, \beta$  and  $\tau$  do not change the shape of the curves, which ensure linear scaling, but do effect their magnitudes (which changes the Weber fraction  $\alpha$ ).



Figure 2: Weber's law with a diverse population of neurons. **a.** The summed firing rate (dashed red line) from a population of 15 simulated neurons with sigmoidal tuning curves (thin colored lines at the bottom) is nearly identical to a log-power tuning curve (solid black line, behind dashed line). Each of these neurons is assumed to code for the input magnitude and is independently parameterized. **b.** The mean magnitude (red + symbols) estimated from 50 independent trials (individual trial estimates indicated by green dots) using this population of 15 neurons accurately reflects the true input magnitude. **c.** Standard deviation ( $\sigma_{est}$ , blue + symbols) of the population magnitude estimates scales linearly with input magnitude in accord with Weber's law.



Figure 3: Examples of neural data fit by log-power curves. These data represent contrast tuning curves extracted from the two cited papers from A.F Dean in 1981. We have fit them to a log-power curve,  $y = k \cdot ln(\theta/\theta_0)^n$ , and found a range of parameters. a. From figure 1 in Dean 1981, Exp. Brain Res. The value n = 2.21 corresponds to a noise exponent of  $\rho = 0.547$  very close to the exponent measured in the same paper  $\rho = 0.58$ . b. From Dean 1981, J. Physiology, Figure 2. Here, n = 2.33 corresponding to  $\rho = 0.57$ . c. From Dean 1981, J. Physiology, Figure 3. Here n = 1.27 corresponding to  $\rho = 0.21$  which is much lower than the experimentally observes  $\rho$ . d. Here there is one cell from Dean 1981, J. Physiology. The two curves are for two different spatial frequencies. Fits are strongly dependent on other parameters (such as spatial frequency). For the lower spatial frequency which drives this cell better n = 2.74, corresponding to  $\rho = 0.63$ . For higher spatial frequency n = 4.56 corresponding to  $\rho = 0.78$  which is much larger than the value of the measured noise exponent. A range of exponents and threshold values are estimated for the different cells.