Evolutionary Dynamics of Nitrogen Fixation in the Legume–Rhizobia Symbiosis

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Supporting Information

Text S2. Increasing cost function ($C'(x) > 0$)

Because the benefit function $B(x) \geq 0$ is an increasing function of x, we have

$$B(\bar{x}) < B(x) \quad \text{for} \quad y < \bar{x} < x \quad \text{and} \quad B(\bar{x}) > B(x) \quad \text{for} \quad y > \bar{x} > x, \tag{S2.1}$$

where $\bar{x} = ((n-1)x + y)/n$. If the cost function $C(x) \geq 0$ is a similarly increasing function of x, then

$$C(y) < C(x) \text{ for } y < x \text{ and } C(y) > C(x) \text{ for } y > x. \tag{S2.2}$$

From Eqs. (S2.1) and (S2.2), the following conditions must be met:

$$B(\bar{x})C(y) < B(x)C(x) \quad \text{for } y < x \text{ and } \quad B(\bar{x})C(y) > B(x)C(x) \quad \text{for } y > x. \tag{S2.3}$$

Because the invasibility of mutant y is $w(x, y) = B(\bar{x})C(y) - B(x)C(x)$, we obtain

$$w(x,y) < 0 \text{ for } y < x \text{ and } w(x,y) > 0 \text{ for } y > x. \tag{S2.4}$$

This invasibility condition is identical to that of case (ii) “Maximum evolution” in section 3.1 (Figure 2C). That is, a resident population can be invaded and replaced by mutants adopting higher strategies, but not by those adopting lower strategies. This result shows that the maximum activity of nitrogen fixation ($x = 1$) will evolve and stably persist if both $B(x)$ and $C(x)$ are increasing functions of x.