File S3

Proof of Result 3

The asymmetric equilibrium $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ is determined by the solutions of the quadratic equation

$$T(x;\mu_B) = (1-m)s^2x^2 - sx\big[s(1-m) - \mu_B(s+2)(1-2m)\big] - \mu_B(2m+s) = 0. \quad (S3.1)$$

When $\mu_B = 0$, equation (S3.1) reduces to

$$T(x;0) = -(1-m)s^2x(1-x) = 0, \qquad (S3.2)$$

giving the two solutions $\hat{x} = 0$ and $\hat{x} = 1$. As $T(0;\mu_B) < 0$, when $\mu_B > 0$ the solution $x = 0$ shifts to a negative solution of (S3.1). Hence, when μ_B is positive and small, the positive root $\hat{x}(\mu_B)$ of $T(x;\mu_B) = 0$ is close to $x = 1$. That is, when μ_B is small the corresponding asymmetric equilibrium is close to the fixation of AB where $\hat{x} = \hat{y} = 1$. Moreover, by continuity, if μ_B is small, their stability is the same. Near fixation of AB, $w = 1-x$ and $z = 1-y$ are small, and up to non-linear terms, when $\mu_B = 0$, we have

$$\begin{aligned} w' &= \frac{1-m}{1+s}w + m(s+1)z \\ z' &= \frac{m}{1+s}w + (1-m)(s+1)z. \end{aligned} \qquad (S3.3)$$

The characteristic polynomial $P(\lambda)$ of (S3.3) is

$$P(\lambda) = \lambda^2 - (1-m)\left[(1+s) + \frac{1}{1+s}\right]\lambda + (1-2m) \qquad (S3.4)$$

and

$$P(1) = 1 - (1-m)\frac{(1+s)^2+1}{1+s} + 1 - 2m. \qquad (S3.5)$$

In fact, it can be easily seen that

$$(1+s)P(1) = -s^2(1-m). \qquad (S3.6)$$

As $P(+\infty) > 0$ and $P(1) < 0$, since $s > 0$, $0 < m < 1$, $P(\lambda)$ has a root larger than 1. Thus, when μ_B is small, fixation in AB is internally locally unstable and so is the asymmetric equilibrium when μ_B is small.