File S5

Proof of Result 5

At a symmetric equilibrium $y = x$, and also, by (32), $\tilde{y} = \tilde{x}$. Thus (30) and (31) imply that

$$\tilde{x} = \frac{\big[(s+1)(1-m_B) - m_B\big]x + m_B}{sx+1} \tag{S5.1}$$

and

$$x = \frac{\big[(1-m_B) - m_B(s+1)\big]\tilde{x} + m_B(1+s)}{(1+s) - s\tilde{x}}. \tag{S5.2}$$

Substituting (S5.1) into (S5.2) gives the quadratic equation

$$(s+2)m_B\left\{sx^2 + \big[2 - m_B(s+2)\big]x - (1-m_B)\right\} = 0. \tag{S5.3}$$

As $0 < m, \mu_B < 1$, $s > 0$ and $m_B = m(1-\mu_B) + \mu_B(1-m) > 0$, x satisfies the equation $R(x) = 0$ with $R(x)$ given in (36). As $0 < m_B < 1$ we have $R(0) < 0$, and as $R(\pm\infty) > 0$, $R(x) = 0$ has two real roots, one positive and one negative. Observe that

$$R(1) = s + \big[2 - m_B(s+2)\big] - (1-m_B) = (1-m_B)(s+1) > 0 \tag{S5.4}$$

and

$$R\left(\frac{1}{2}\right) = \frac{s}{4} + \frac{1}{2}\cdot\big[2 - m_B(s+2)\big] - (1-m_B) = \frac{s}{4}(1-2m)(1-2\mu_B) \tag{S5.5}$$

as $1 - 2m_B = 1 - 2m - 2\mu_B + 4m\mu_B = (1-2m)(1-2\mu_B)$. Therefore when $0 < m, \mu_B < \frac{1}{2}$ we have $R(\frac{1}{2}) > 0$ and $0 < \bar{x} < \frac{1}{2}$.