File S2

Proof of Result 2

If an asymmetric polymorphism exists, then (11) holds, namely, (with $\mu = \mu_B$),

$$1 + sy = \frac{(1-2\mu)(1+s)}{1+sx}. \qquad (S2.1)$$

That is,

$$y = \frac{s(1-x) - 2\mu(1+s)}{s(1+sx)}, \qquad 1 - y = \frac{s(1+s)x + 2\mu(1+s)}{s(1+sx)}. \qquad (S2.2)$$

Substituting these relations into the equilibrium equation for x from (8), we find, after some simplification, that

$$x = \frac{1-m}{1+sx}\big[(1-\mu)(1+s)x + \mu(1-x)\big] + \frac{m}{s}(sx + 2\mu + \mu s). \qquad (S2.3)$$

Equation (S2.3) is equivalent to the quadratic equation

$$T(x) = (1-m)s^2x^2 - sx\big[s(1-m) - \mu(s+2)(1-2m)\big] - \mu(2m+s) = 0. \qquad (S2.4)$$

As μ, m, s are positive and $m < 1$, we have $T(0) < 0$ and $T(\pm\infty) > 0$, implying that $T(x)$ has two real roots, one positive and one negative. Now

$$\begin{aligned} T(1) &= (1-m)s^2 - s\big[s(1-m) - \mu(s+2)(1-2\mu)\big] - \mu(2m+s) \\ &= \mu\big[s(s+2)(1-2m) - (2m+s)\big]. \end{aligned} \qquad (S2.5)$$

$T(1;m)$ is a linear function of m and

$$\begin{aligned} &T(1;0) = \mu s(s+1) > 0 \\ &T(1;\tfrac{1}{2}) = -\mu(2m+s) < 0 \\ &T(1;m_0) = 0. \end{aligned} \qquad (S2.6)$$

Hence if $0 < m < m_0$, $T(1;m) > 0$ and a unique $0 < \hat{x} < 1$ exists such that $T(\hat{x}) = 0$. In order for $\hat{x}$ to be an equilibrium, its corresponding $\hat{y}$ should satisfy $0 < \hat{y} < 1$, where

$$1 - \hat{y} = \frac{1+s}{1+s\hat{x}}\frac{s\hat{x}+2\mu}{s} \qquad (S2.7)$$

and $0 < \hat{y} < 1$ if and only if

$$(1+s)(s\hat{x}+2\mu) < s(1+s\hat{x}) \qquad (S2.8)$$

or

$$\hat{x} < \frac{s - 2\mu(1+s)}{s}. \tag{S2.9}$$

So $0 < \hat{x} < 1$ if $0 < \mu < \mu_0 = \frac{1}{2}\frac{s}{s+1}$, and $\big[s - 2\mu(1+s)\big] > 0$. We compute $T\big(\frac{s-2\mu(1+s)}{s}\big)$, which equals

$$(1-m)\big[s-2\mu(1+s)\big]^2 - \big[s-2\mu(1+s)\big]\big[s(1-m)-\mu(s+2)(1-2m)\big] - \mu(2m+s). \tag{S2.10}$$

So

$$T\big(\tfrac{s-2\mu(1+s)}{s}\big) = 2\mu^2(1+s)(s+2m) + s\mu(s+2)(1-2m) - \mu(2m+s). \tag{S2.11}$$

But when $0 < m < m_0$,

$$T(1) = s\mu(s+2)(1-2m) - \mu(2m+s) > 0, \tag{S2.12}$$

therefore $T\big(\frac{s-2\mu(1+s)}{s}\big) > 0$, and (S2.9) holds.