File S8

Proof of Result 8

i. In a constant environment, the mean fitness w^* at the symmetric equilibrium $(\bar{\mathbf{x}}^*, \bar{\mathbf{y}}^*)$ is $w^* = 1+sx^*$, and it is a decreasing function of μ_B if $\frac{\partial x^*}{\partial \mu_B}$ is negative, or equivalently if $\frac{\partial x^*}{\partial m_B}$ is negative (since $m_B = m+\mu_B(1-2m)$ and $0 \le m < \frac{1}{2}$). Using the equilibrium equation (14),

$$\frac{\partial x^*}{\partial m_B} = \frac{1-(s+2)x^*}{2sx^* + \big[(s+2)m_B - s\big]}. \qquad (S8.1)$$

As $x^* > \frac{1}{s+2}$, in order for $\frac{\partial x^*}{\partial m_B}$ to be negative, it is sufficient that

$$x^* > \frac{s - m_B(s+2)}{2s}. \qquad (S8.2)$$

This follows easily from the fact that $Q(x)$ of (14) satisfies $Q(0) < 0$, $Q(x^*) = 0$, and $Q\left(\frac{s-m_B(s+2)}{2s}\right) < 0$.

ii. With a fitness cycle of period 2, the mean fitness $\bar{w}$ at the symmetric equilibrium $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ is

$$\bar{w} = (1+s) + sm_B\big[(s+2)\bar{x} - 1\big]. \qquad (S8.3)$$

$\bar{w}$ is an increasing function of μ_B if $\frac{\partial \bar{w}}{\partial \mu_B} > 0$ or equivalently if $\frac{\partial \bar{w}}{\partial m_B} > 0$. Now

$$\frac{\partial \bar{w}}{\partial m_B} = s\big[(s+2)\bar{x} - 1\big] + s(s+2)m_B\frac{\partial \bar{x}}{\partial m_B}. \qquad (S8.4)$$

Thus $\frac{\partial \bar{w}}{\partial m_B} > 0$ provided $\frac{\partial \bar{x}}{\partial m_B} > 0$. Using the equilibrium equation $R(x) = 0$ for $\bar{x}$, we have

$$\frac{\partial \bar{x}}{\partial m_B} = \frac{\bar{x}(s+1) - 1}{2s\bar{x} + \big[2 - m_B(s+2)\big]}. \qquad (S8.5)$$

Since $\big[\bar{x}(s+1) - 1\big] > 0$, we conclude that $\frac{\partial \bar{x}}{\partial m_B} > 0$ if

$$\bar{x} > \frac{m_B(s+2) - 2}{2s}, \qquad (S8.6)$$

which follows from $R(0) < 0$, $R(\bar{x}) = 0$, and $R\left(\frac{m_B(s+2)-2}{2s}\right) < 0$.