File S1

Proof of Result 1

1. When $y = x$, the mean fitnesses in the two demes E_x and E_y are equal:

$$w_x = 1 + sx = 1 + sy = w_y, \tag{S1.1}$$

and, from (8), the equilibrium equation is, (with $\mu = \mu_B$),

$$(1+sx)x = (1-m)\big[(1-\mu)(1+s)x + \mu(1-x)\big] + m\big[(1-\mu)(1-x) + \mu(1+s)x\big]. \tag{S1.2}$$

Thus

$$(1+sx)x = (1+s)x\big[(1-m)(1-\mu) + m\mu\big] + (1-x)\big[\mu(1-m) + m(1-\mu)\big], \tag{S1.3}$$

or

$$x + sx^2 = (1+s)x\big[1 - m - \mu + 2m\mu\big] + (1-x)\big[m + \mu - 2m\mu\big]. \tag{S1.4}$$

This is equivalent to

$$Q(x) = sx^2 + \big[(s+2)(m+\mu-2m\mu) - s\big]x - (m+\mu-2m\mu) = 0. \tag{S1.5}$$

Now, as $0 < m,\ \mu < 1$, we have

$$(m + \mu - 2m\mu) = m(1-\mu) + \mu(1-m) > 0. \tag{S1.6}$$

Therefore

$$Q(0) = -(m+\mu-2m\mu) < 0 \tag{S1.7}$$

and

$$Q(1) = (s+1)(m+\mu-2m\mu) > 0. \tag{S1.8}$$

As $Q(\pm\infty) > 0$, we conclude that the equation (S1.5) has a unique root x^* with $0 < x^* < 1$. Thus there is a unique symmetric polymorphism $(\mathbf{x}^*, \mathbf{y}^*)$, given by (13).

2. Near the equilibrium $(\mathbf{x}^*, \mathbf{y}^*)$, on the boundary where only B is present, $(z - x)$ is small, and from (10), the internal local stability of $(\mathbf{x}^*, \mathbf{y}^*)$ in the boundary is determined by the factor

$$C^* = \frac{(1-2\mu)(1+s)}{(1+sx^*)(1+sz^*)}. \tag{S1.9}$$

As $x^* = z^*$, $C^* < 1$ if $(1+s) < (1+sx^*)^2$, and as $s > 0$ this is true if $s(x^*)^2 + 2x^* > 1$. From the equilibrium equation (S1.5), as $Q(x^*) = 0$ we have

$$\begin{aligned} s(x^*)^2 + 2x^* &= -\big[(s+2)(m+\mu-2m\mu) - s\big]x^* + (m+\mu-2m\mu) + 2x^* \\ &= -(s+2)(m+\mu-2m\mu-1)x^* + (m+\mu-2m\mu). \end{aligned} \tag{S1.10}$$

Thus $s(x^*)^2 + 2x^* > 1$ if and only if

$$(s+2)(1-m-\mu+2m\mu)x^* > (1-m-\mu+2m\mu). \tag{S1.11}$$

But $(1-m-\mu+2m\mu) = (1-m)(1-\mu) + m\mu > 0$ as $0 < m,\ \mu < 1$, and so $C^* < 1$ provided $\mathbf{x}^* > \frac{1}{s+2}$. As $Q(1) > 0$ and $Q(x^*) = 0$, it is sufficient to show that $Q\big(\frac{1}{s+2}\big) < 0$. Indeed

$$\begin{aligned} Q\left(\frac{1}{s+2}\right) &= \frac{s}{(s+2)^2} + \big[(s+2)(m+\mu-2m\mu) - s\big]\frac{1}{s+2} - (m+\mu-2m\mu) \\ &= \frac{s}{(s+2)^2} - \frac{s}{s+2} = -\frac{s(s+1)}{(s+2)^2} < 0. \end{aligned} \tag{S1.12}$$

3. We compute $Q\big(\frac{1}{2}\big)$ using (14),

$$Q\left(\frac{1}{2}\right) = \frac{s}{4} + \frac{1}{2}\big[(s+2)(m+\mu-2m\mu) - s\big] - (m+\mu-2m\mu). \tag{S1.13}$$

In fact,

$$Q\left(\frac{1}{2}\right) = -\frac{s}{4}\big[1 - 2(m+\mu-2m\mu)\big]. \tag{S1.14}$$

But $1 - 2(m+\mu-2m\mu) = (1-2m)(1-2\mu) > 0$ when $0 < m,\ \mu < \frac{1}{2}$, in which case $Q\big(\frac{1}{2}\big) < 0$ and $x^* > \frac{1}{2}$ as $Q(1) > 0$.