```
In[361]:= Remove["Global`*"]
```

Cuvette Solution for Red oak

Parameter values

Fluxes and parameters from data oak

```
In[362]:= Tran = 0.0056; (*.0056 oak moles m-2 s-1*)

In[363]:= θe = 273.15 + 28.83 (*kelvin licor leaf temp*)
          θair = 273.15 + 28.31(*licor air temp*)
Out[363]= 301.98
Out[364]= 301.46

In[365]:= θblk =
           273.15 + 28.125(*licor blk temp 28.125, this temp for radiation in chamber *)
Out[365]= 301.275
```

Physical quantities at 28 C, -1 MPa

```
In[366]:= Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)

In[367]:= λ = 43 837; (* Joules per Mol*)

In[368]:= R = 8.3145; (*gas constant Joules per mole per Kelvin*)
```

In[369]:= $D_v = \frac{Patm}{R\,\theta var}\,2.13 * \left(\frac{\theta var}{273.15}\right)^{1.8} * 10\text{^}(-5)$ (* cDv for mol frac drving force out of leaf *)

Out[369]= $0.0000106815\,\theta var^{0.8}$

```
In[370]:= kvh = .026;(* heat cond air J m-2 s-1 K-1, nobel .0255 20C, .0264 40C*)

In[371]:= Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)
Out[371]= 0.00317308

In[372]:= Pa = Patm * 10^-6 (* atm pressure in MPa*)
Out[372]= 0.1013
```

In[373]:= $c[\psi\_, \theta\_] := 1.28\,\frac{R\,298.15}{Patm}\,Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right] Exp\left[\frac{(\psi + Pa - Po)\;18.07}{R\;\theta}\right]$ (*mol fraction for psi in MPa*)


```
In[374]:= θvar = 302.06 (*update with theta o *)
Out[374]= 302.06

In[375]:= ψvar = -1.688 (* update with psi o *)
Out[375]= -1.688
```

In[376]:= $C_{v\theta} = \partial_\theta\, c[\psi var, \theta]$ /. θ → θvar (* linearization of dχ/dT 1/k *)
$C_{v\psi} = \partial_\psi\, c[\psi, \theta var]$ /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)

```
Out[376]= 0.00226151
Out[377]= 0.000280359

In[378]:= σ = 5.670373 * 10^(-8); (*stefan boltzmann J/m2/s/kelvin-4 *)

In[379]:= F = 0.95; (*emissivity radiative from leaf*)
```

Leaf parameters

```
In[380]:= A1 = .7;(* .1263 oakair fraction, say 0.5 just mesophyll*)
          Av = 1 - A1; (*Area fraction tissue in leaf from 2011 data 12.63%*)

In[382]:= AU1 = .95;(* .1263 oakair fraction, say 0.5 just mesophyll*)
          AUv = 1 - AU1; (*Area fraction tissue in leaf from 2011 data 12.63%*)

In[384]:= L = .72 × 131 × 10^(-6)(*-20 10^(-6)*)
Out[384]= 0.00009432

In[385]:= LU = 1.28 * 131 × 10^(-6)
Out[385]= 0.00016768

In[386]:= klh = .28614;(* heat cond liq phase, based on .25 for whole leaf vogel *)

In[387]:= ktotal = 6.45 × 10^(-7);(* hyd cond leaf mol/m/s/mpa
           1D scaled fit with hA specified  scaled k est on tree*)

In[388]:= kl = (ktotal - .1263 Dv Cvψ) / (1 - .1263)(* hyd cond tissue  *)
```

Out[388]= 6.96509×10^{-7}

```
In[389]:= H = 0.021 (*mol/m2/s/MPa *)
Out[389]= 0.021

In[390]:= HA =
           (0.021) * 0.67(*mol/m2/s/MPa plus steady state 1D scale factor based on Biotout*)
Out[390]= 0.01407

In[391]:= ψo = -1.29 - Tran / HA(* stem potential from cov'
              d leaf in mpa: psio is 'effective potential' at mid vein *)
Out[391]= -1.68801
```

Check whole leaf air fraction

In[392]:= $\frac{L\,A_v + LU\,AU_v}{L + LU}$

Out[392]= 0.14

Description of fluxes and relations to env parameters

In[393]:= $P\theta_u$ = 302.114 (* start with theta e, update with theta u *)

Out[393]= 302.114

In[394]:= $Uh_r = \sigma F \left(P\theta_u{}^2 + \theta_{blk}{}^2\right) (P\theta_u + \theta_{blk})$; (* radiative HTC FOR CHAMBER MEASUREMENTS! For clear sky replace theta blk with sky*)

In[395]:= $h_r = \sigma F \left(\theta_e{}^2 + \theta_{blk}{}^2\right) (\theta_e + \theta_{blk})$; (* radiative HTC FOR CHAMBER MEASUREMENTS! For clear sky replace theta blk with sky*)

In[396]:= δ = 7.27 * 10^(-4); (* in m: cuvette leaf boundary layer*)

In[397]:= (*delta is from licor 1.42 mol/m2/s g_bw, d=Dv*C_a/g_bw, C_a=40.49 mol/m3*)

In[398]:= $\frac{k_{vh}}{\delta}$ (* Licor calc is 39.79 for this parameter, based on Cpv*gbl, or 28*1.42 *)

Out[398]= 35.7634

In[399]:= $q_{sl} = \frac{k_{vh}}{\delta} (\theta_e - \theta_{air})$

Out[399]= 18.597

In[400]:= $q_{su} = \frac{k_{vh}}{\delta} (P\theta_u - \theta_{air})$

(* ITERATIVE CORR approximate as using lower leaf temp so check error *)

Out[400]= 23.3893

In[401]:= $q_{rl} = h_r (\theta_e - \theta_{blk})$

Out[401]= 4.16867

In[402]:= $q_{ru} = Uh_r (P\theta_u - \theta_{blk})$

(* ITERATIVE CORR approximate as using lower leaf temp so check error *)

Out[402]= 4.96432

In[403]:= $qTotal = q_{su} + q_{sl} + q_{rl} + q_{ru} + \lambda\,Tran$

Out[403]= 296.606

In[404]:= $\lambda\,Tran$

Out[404]= 245.487

In[405]:= Q = 0.2 * qTotal / L (* w m-3 volumetric heat source, .2 is fraction of total abs solar rad abs in spongy *)

Out[405]= 628 936.

In[406]:= QU = 0.8 * qTotal / LU

Out[406]= 1.41511×10^6

In[407]:= Q L (* total solar in lower half of leaf*)

Out[407]= 59.3213

In[408]:= QU LU

Out[408]= 237.285

In[409]:= Q L + QU LU

Out[409]= 296.606

In[410]:=

Solution lower potential field

In[411]:= $\Pi_\psi = 1 + \frac{A_1\,k_{1h} + A_v\,k_{vh}}{\lambda\,A_v\,D_v\,C_{v\theta}} + \frac{A_1\,k_{1h} + A_v\,k_{vh}}{\lambda\,A_1\,k_1}\,\frac{C_{v\psi}}{C_{v\theta}}$

Out[411]= 9.00223

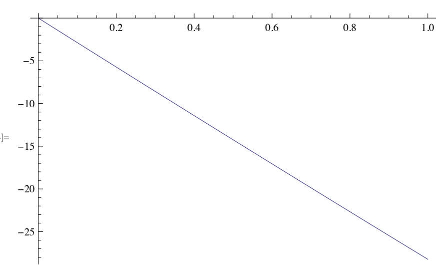
In[412]:= $\alpha = \left(\frac{(q_{sl} + q_{rl})\,L}{Q\,L^2} - \frac{Tran\,L}{Q\,L^2}\,\frac{A_1\,k_{1h} + A_v\,k_{vh}}{A_v\,D_v\,C_{v\theta}}\right)$

Out[412]= -27.7366

In[413]:= $\Psi[X\_] := \frac{X^2}{2} - X + \alpha X$

In[414]:= Plot[Ψ[X], {X, 0, 1}]

Out[414]=



In[415]:= $\text{PsiAverageLower} = \psi_o + \frac{Q\,L^2}{\Pi_\psi\,A_l\,k_l\,\lambda} * \int_0^1 \Psi[X]\,dX$

Out[415]= -2.101

Putting back dimensions to find potential of lower epidermis

In[416]:= $a = \left(\frac{(q_{sl} + q_{rl})\,L}{\Pi_\psi\,A_l\,k_l\,\lambda} - \frac{\text{Tran}\,L}{\Pi_\psi\,A_l\,k_l\,\lambda}\,\frac{A_l\,k_{lh} + A_v\,k_{vh}}{A_v\,D_v\,C_{v\theta}}\right)$

Out[416]= -0.806588

In[417]:= $aa = \frac{Q\,L^2}{\Pi_\psi\,A_l\,k_l\,\lambda}$

Out[417]= 0.0290803

In[418]:= $\psi[x\_] := \psi_o + \left(\frac{x^2}{2\,L^2} - \frac{x}{L}\right)\,aa + a\,\frac{x}{L}$

In[419]:= $\psi_e = \psi[L]$

Out[419]= -2.50914

In[420]:= ψ_o

Out[420]= -1.68801

check averaging of potential in lower part of leaf

In[421]:= $\frac{1}{L}\int_0^L \psi[x]\,dx$

Out[421]= -2.101

Solution lower thermal field

In[422]:= $\Pi_\theta = 1 + \frac{\lambda\,A_l\,k_l}{A_l\,k_{lh} + A_v\,k_{vh}}\,\frac{A_v\,D_v\,C_{v\psi}}{A_l\,k_l + A_v\,D_v\,C_{v\psi}}\,\frac{C_{v\theta}}{C_{v\psi}}$

Out[422]= 1.12497

In[423]:= $\frac{1}{\Pi_\theta} + \frac{1}{\Pi_\psi}$ (* test should equal 1 *)

Out[423]= 1.

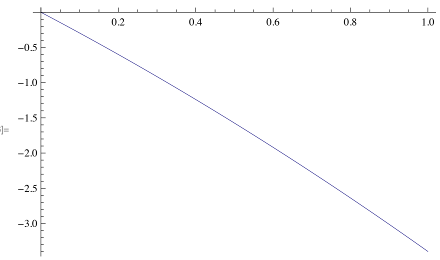
In[424]:= $\beta = -\frac{(q_{sl} + q_{rl})\,L}{Q\,L^2} - \frac{\lambda\,A_l\,k_l\,\text{Tran}\,L}{(A_l\,k_l + A_v\,D_v\,C_{v\psi})\,Q\,L^2}$

Out[424]= -3.89783

In[425]:= $\Theta[X\_] := -\frac{X^2}{2} + X + \beta\,X$

In[426]:= Plot[Θ[X], {X, 0, 1}]

Out[426]=



Putting back dimensions to find temperature at vasc plane

In[427]:= $bb = \frac{Q\,L^2}{\Pi_\theta\,(A_l\,k_{lh} + A_v\,k_{vh})}$

Out[427]= 0.0239005

In[428]:= $b = \left(-\frac{(q_{sl} + q_{rl})\,L}{\Pi_\theta\,(A_l\,k_{lh} + A_v\,k_{vh})} - \frac{\lambda\,A_l\,k_l\,\text{Tran}\,L}{\Pi_\theta\,(A_l\,k_{lh} + A_v\,k_{vh})}\,\frac{1}{(A_l\,k_l + A_v\,D_v\,C_{v\psi})}\right)$

Out[428]= -0.0931602

In[429]:= $d\theta[x\_] := \left(-\frac{x^2}{2\,L^2} + \frac{x}{L}\right)\,bb + b\,\frac{x}{L}$ (* difference from vasc temp *)

In[430]:= $d\theta[L]$

Out[430]= -0.0812099

In[431]:= $\theta_o = \theta_e - d\theta[L]$

Out[431]= 302.061

Solution upper potential field

In[432]:= $\Pi U_\psi = 1 + \frac{AU_l\,k_{lh} + AU_v\,k_{vh}}{\lambda\,AU_v\,D_v\,C_{v\theta}} + \frac{AU_l\,k_{lh} + AU_v\,k_{vh}}{\lambda\,AU_l\,k_l}\,\frac{C_{v\psi}}{C_{v\theta}}$

Out[432]= 55.6804

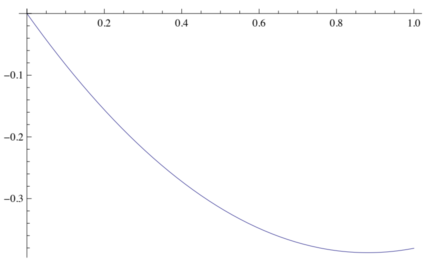
In[433]:= $\frac{(q_{su} + q_{ru})}{QU\,LU}$

Out[433]= 0.119492

In[434]:= $$\Psi U[X\_] := \frac{X^2}{2} - X + \left(\frac{(q_{su} + q_{ru})}{QU\ LU}\right) X$$

In[435]:= `Plot[ΨU[X], {X, 0, 1}]`

Out[435]=



Putting back dimensions to find potential of upper epidermis

In[436]:= $$\psi U[x\_] := \psi_o + \left(\frac{x^2}{2\ LU^2} - \frac{x}{LU}\right) \frac{QU\ LU^2}{\Pi U_\psi\ AU_1\ k_1\ \lambda} + \left(\frac{(q_{su} + q_{ru})\ LU}{\Pi U_\psi\ AU_1\ k_1\ \lambda}\right) \frac{x}{LU}$$

In[437]:= $\psi_u = \psi U[LU]$

Out[437]= -1.69738

In[438]:= $$\text{PsiAverageUpper} = \frac{1}{LU} \int_0^{LU} \psi U[x]\, dx$$

Out[438]= -1.69475

Average leaf water potential as measured by pressure chamber

In[439]:= $$\text{PsiAverageLeaf} = \frac{\text{PsiAverageUpper LU } AU_1 + \text{PsiAverageLower L } A_1}{AU_1\ LU + A_1\ L}$$

```
(* liq vol weighted avg of 5 leaves -1.73, on tree analysis 2009.xcl*)
```

Out[439]= -1.81379

In[440]:= $\text{ScaledPsiAverageLeaf} = 0.65 * (-1.29 - \psi_e) + \psi_e$

```
(* cf 0.65 for oak (scaled k est on tree and leaf paper): given true psie
   and psir as defined in numeric simulation gives average potential in
   domain. Justified if 1D steady sol is scaled to 3d as it is here *)
```

Out[440]= -1.7167

Solution upper thermal field

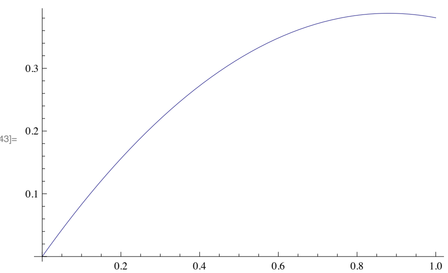
In[441]:= $$\Pi U_\theta = 1 + \frac{\lambda\ AU_1\ k_1}{AU_1\ k_{1h} + AU_v\ k_{vh}}\ \frac{AU_v\ D_v\ C_{v\psi}}{AU_1\ k_1 + AU_v\ D_v\ C_{v\psi}}\ \frac{C_{v\theta}}{C_{v\psi}}$$

Out[441]= 1.01829

In[442]:= $$\Theta U[X\_] := -\frac{X^2}{2} + X - \left(\frac{(q_{su} + q_{ru})}{QU\ LU}\right) X$$

In[443]:= `Plot[ΘU[X], {X, 0, 1}]`

Out[443]=



In[444]:= $$\theta_u = \theta_o + \Theta U[1] \frac{QU\ LU^2}{\Pi U_\theta\ (AU_1\ k_{1h} + AU_v\ k_{vh})}$$

Out[444]= 302.116

In[445]:= $$\text{peak} = \theta_o + \Theta U[.85] \frac{QU\ LU^2}{\Pi U_\theta\ (AU_1\ k_{1h} + AU_v\ k_{vh})}$$

Out[445]= 302.117

In[446]:= $$\theta U[x\_] := \theta_o + \left(-\frac{x^2}{2\ LU^2} + \frac{x}{LU}\right) \frac{QU\ LU^2}{\Pi U_\theta\ (AU_1\ k_{1h} + AU_v\ k_{vh})} - \left(\frac{(q_{su} + q_{ru})\ LU}{\Pi U_\theta\ (AU_1\ k_{1h} + AU_v\ k_{vh})}\right) \frac{x}{LU}$$

In[447]:= $\theta U[LU]$

Out[447]= 302.116

Chek that difference between lower and upper epidermal temps small

In[448]:= $\theta_e - \theta_u$

Out[448]= -0.135644

In[449]:= $$\frac{h_r\ (\theta_u - \theta_{blk})}{q_{ru}}$$

Out[449]= 1.00129

In[450]:= $\frac{\frac{k_{vh}}{\delta}(\theta_u - \theta_{air})}{q_{su}}$

Out[450]= 1.00251

Evaluate fluxes

Analyze proportion peristomatal at Lower Epidermis

In[451]:= ProportionPeristomatal = $\frac{-A_l\, k_l\, \partial_x \psi[x]\ /.\ x \to L}{\text{Tran}}$

Out[451]= 0.744534

In[452]:= PropPeri = $\frac{-(q_{sl} + q_{rl})}{\Pi_\psi\ \text{Tran}\ \lambda} + \left(1 + \frac{\lambda A_v D_v C_{v\theta}}{A_l k_{lh} + A_v k_{vh}} + \frac{A_v D_v C_{v\psi}}{A_l k_l}\right)^{-1}$

Out[452]= 0.744534

In[453]:= $\frac{-(q_{sl} + q_{rl})}{\Pi_\psi\ \text{Tran}\ \lambda}$

Out[453]= -0.0103015

In[454]:= ProportionInternalVapor = $\frac{-A_v D_v C_{v\psi}(\partial_x \psi[x]\ /.\ x \to L) - A_v D_v C_{v\theta}(\partial_x d\theta[x]\ /.\ x \to L)}{\text{Tran}}$

Out[454]= 0.255466

In[455]:= ProportionPeristomatal + ProportionInternalVapor

Out[455]= 1.

Peristomatal small if either of these two quantities are large (or sensible plus radiative >> latent):

In[456]:= $\frac{\lambda A_v D_v C_{v\theta}}{A_l k_{lh} + A_v k_{vh}}$

Out[456]= 0.147163

In[457]:= $\frac{A_v D_v C_{v\psi}}{A_l k_l}$

Out[457]= 0.17763

Condensation on lower epidermis if following is larger than one:

In[458]:= $\frac{(q_{sl} + q_{rl})}{\text{Tran}\ \lambda}\ \frac{\lambda A_v D_v C_{v\theta}}{A_l k_{lh} + A_v k_{vh}}$

Out[458]= 0.0136474

In[459]:= $\frac{(q_{sl} + q_{rl})}{\text{Tran}\ \lambda}$

Out[459]= 0.0927366

In[460]:= $\frac{\lambda A_v D_v C_{v\theta}}{A_v k_{vh}}$

Out[460]= 3.92619

In[461]:= VaporFluxintoLower = $\frac{-A_v D_v C_{v\psi}(\partial_x \psi[x]\ /.\ x \to 0) - A_v D_v C_{v\theta}(\partial_x d\theta[x]\ /.\ x \to 0)}{\text{Tran}}$

Out[461]= 0.228623

In[462]:= LiquidFluxintoLower = $\frac{-A_l\, k_l\, \partial_x \psi[x]\ /.\ x \to 0}{\text{Tran}}$

Out[462]= 0.771377

In[463]:= ProportionEvaporationLower = $\frac{A_l\, k_l\, \psi''[x]\, L}{\text{Tran}}$

Out[463]= 0.026843

In[464]:= LiquidFluxintoLower - ProportionPeristomatal - ProportionEvaporationLower

Out[464]= -7.28584×10^{-17}

In[465]:= ProportionEvaporationUpper = $\frac{AU_l\, k_l\, \psi U''[x]\, LU}{\text{Tran}}$

Out[465]= 0.0173596

In[466]:= VaporFluxintoUpper = $\frac{-AU_v D_v C_{v\psi}(\partial_x \psi U[x]\ /.\ x \to 0) - AU_v D_v C_{v\theta}(\partial_x \theta U[x]\ /.\ x \to 0)}{\text{Tran}}$

Out[466]= -0.0152853

In[467]:= CondensingVaporFluxUpper = $\frac{-\lambda AU_v D_v C_{v\psi}(\partial_x \psi U[x]\ /.\ x \to LU) - \lambda AU_v D_v C_{v\theta}(\partial_x \theta U[x]\ /.\ x \to LU)}{(q_{su} + q_{ru})}$

Out[467]= 0.0179597

In[468]:= Perivacular = VaporFluxintoLower + VaporFluxintoUpper

Out[468]= 0.213338

In[469]:= ProportionPeristomatal + Perivacular + (- VaporFluxintoUpper) + ProportionEvaporationLower

Out[469]= 1.

In[470]:= 1 - .736 - .015 - .027

Out[470]= 0.222

In[471]:= **LiquidFluxintoUpper** $= \dfrac{-AU_1\, k_1\ \partial_x\, \psi U[x]\ /.\ x \to 0}{Tran}$

Out[471]= 0.0152853

Analyze how much of transpiration stream evaporates in upper part (equal to liquid flux in)

In[472]:= **LIqFluxinUpper** $= \dfrac{QU\ LU}{\Pi U_\psi\ Tran\ \lambda} - \left(\dfrac{(q_{su} + q_{ru})}{\Pi U_\psi\ Tran\ \lambda}\right)$

Out[472]= 0.0152853

In[473]:= **VaporFluxintoUpper + LiquidFluxintoUpper**

Out[473]= -1.73472×10^{-18}

In[474]:= $\dfrac{-(AU_1\, k_{1h} + AU_v\, k_{vh})\ \theta U'[0]}{QU\ LU}$

Out[474]= -0.864695

Check Energy balance of upper part

In[475]:= $\dfrac{-(AU_1\, k_{1h} + AU_v\, k_{vh})\ \theta U'[0]}{1} - (q_{su} + q_{ru}) + \lambda\ (AU_1\, k_1\ \partial_x\, \psi U[x]\ /.\ x \to 0) + QU\ LU$

Out[475]= 0.

Check Energy balance at vascular plane

In[476]:= **DivVaporEnergy** $= \lambda\ ((-A_v\, D_v\, C_{v\psi}\ (\partial_x\, \psi[x]\ /.\ x \to 0) - A_v\, D_v\, C_{v\theta}\ (\partial_x\, d\theta[x]\ /.\ x \to 0)) +$
$(-AU_v\, D_v\, C_{v\psi}\ (\partial_x\, \psi U[x]\ /.\ x \to 0) - AU_v\, D_v\, C_{v\theta}\ (\partial_x\, \theta U[x]\ /.\ x \to 0)))$

Out[476]= 52.3718

In[477]:= **DivConduction** $= -(AU_1\, k_{1h} + AU_v\, k_{vh})\ \theta U'[0] - (A_1\, k_{1h} + A_v\, k_{vh})\ d\theta'[0]$

Out[477]= -52.3718

In[478]:= ψ_o

Out[478]= -1.68801

In[479]:= θ_o

Out[479]= 302.061

In[480]:= **Biotout** $= \dfrac{H\ L\ (\psi_o - \psi_e)}{(A_1\, k_1 + A_v\, D_v\, C_{v\psi})\ (\psi_o - \psi_e) + A_v\, D_v\, C_{v\theta}\ (\theta_o - \theta_e)}$

Out[480]= 3.07923

In[481]:= $(A_1\, k_1 + A_v\, D_v\, C_{v\psi})\ (\psi_o - \psi_e) + A_v\, D_v\, C_{v\theta}\ (\theta_o - \theta_e)$

Out[481]= 5.28192×10^{-7}

In[482]:= **sf = (.151 Biotout + 1.03) ^ (-1)**

Out[482]= 0.668913

In[483]:= **nondnumericpsiavg** $= .721 - \dfrac{.219}{Biotout}$

Out[483]= 0.649878

Plots

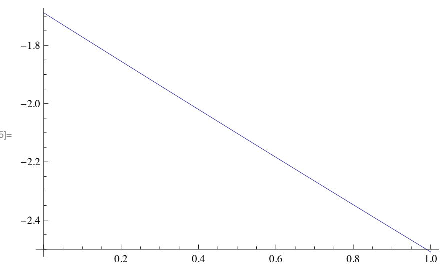
Lower Potential

In[484]:= **psiL** $= \psi[x]\ /.\ x \to L\ X$

Out[484]= $-1.68801 - 0.806588\ X + 0.0290803\ (-1.\ X + 0.5\ X^2)$

In[485]:= **Plot[psiL, {X, 0, 1}]**

Out[485]=

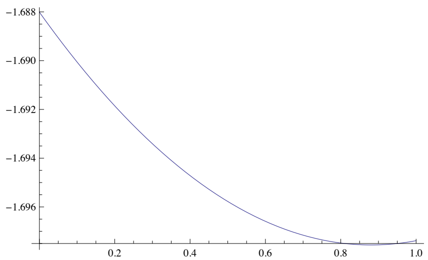


Upper Potential

In[486]:= **psiU** $= \psi U[x]\ /.\ x \to LU\ X$

Out[486]= $-1.68801 + 0.00294372\ X + 0.0246354\ (-1.\ X + 0.5\ X^2)$

In[487]:= `Plot[psiU, {X, 0, 1}]`

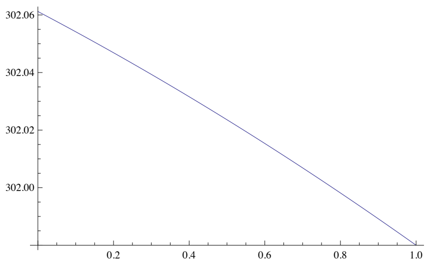
Out[487]=


Lower Thermal

In[488]:= `thetaL = θo + dθ[x] /. x → L X`

Out[488]= 302.061 - 0.0931602 X + 0.0239005 (1. X - 0.5 X^2)

In[489]:= `Plot[thetaL, {X, 0, 1}]`

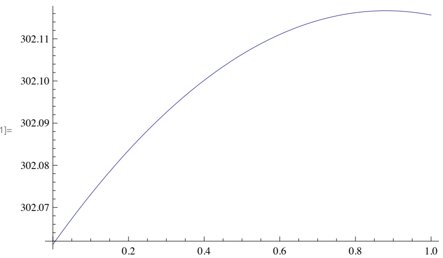
Out[489]=


Upper Thermal

In[490]:= `thetaU = θU[x] /. x → LU X`

Out[490]= 302.061 - 0.017094 X + 0.143056 (1. X - 0.5 X^2)

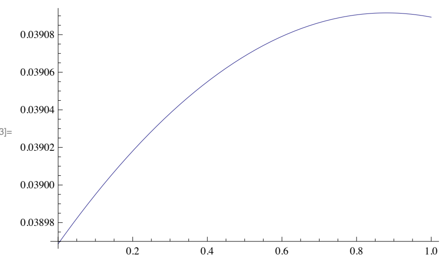
In[491]:= `Plot[thetaU, {X, 0, 1}]`

Out[491]=


Upper mol fraction

In[492]:= `cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;`

In[493]:= `Plot[cU, {X, 0, 1}]`

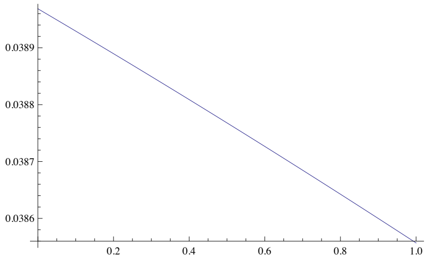
Out[493]=


Lower mol fraction

In[494]:= `cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;`

In[495]:=
```
Plot[cL, {X, 0, 1}]
```

Out[495]=



Export plot data

In[496]:=
```
position = Range[-1, 1, .01];
```

In[497]:=
```
outpos = Table[0, {Length[position]}];
```

In[498]:=
```
potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]
```

In[505]:= ψ_o

Out[505]= -1.68801

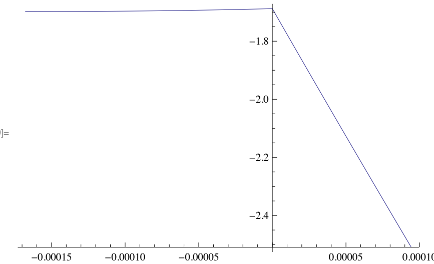
In[506]:=
```
opdata = Transpose[{outpos, potential}];
```

In[507]:=
```
otdata = Transpose[{outpos, temperature}];
```

In[508]:=
```
ocdata = Transpose[{outpos, vapor}];
```

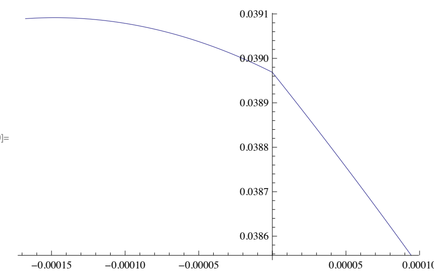

In[509]:=
```
ListPlot[opdata, Joined → True]
```

Out[509]=



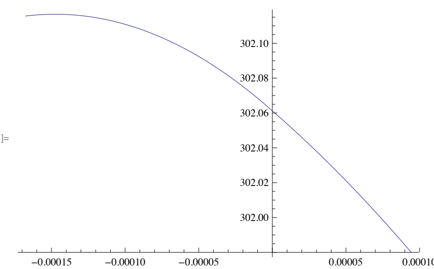
In[510]:=
```
ListPlot[ocdata, Joined → True]
```

Out[510]=



In[511]:=
```
ListPlot[otdata, Joined → True]
```

Out[511]=



In[512]:=
```
(*SetDirectory[ToFileName[NotebookDirectory[]]]*)
```

In[513]:=
```
(*outfile="oak_cuvette_"*)
```

In[514]:=
```
(*Export[outfile<>"potential.xls",opdata]*)
```

```
In[515]:= (*Export[outfile<>"vapor.xls",ocdata]*)

In[516]:= (*Export[outfile<>"temperature.xls",otdata]*)

Remove["Global`*"]
```

Exposed leaf solution for Red Oak

Parameter values

Temp and potential for variable physical quantities: Update for approximate leaf temperature

```
θvar = 302.22; (* θair *)
ψvar = -1.69;  (* ψr *)

vsf = .669 (* vasc scale factor steady
  state based on geometry of veis and eff biot number*)
0.669
```

Environmental parameters

```
R = 8.3145; (*gas constant Joules per mole per Kelvin*)

Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)

θair = 273.15 + 28.31(* air temp *)
301.46

θsur = θair;
```

$$\chi a[rh\_, \theta\_] := rh \left(1.28 \frac{R\,298.15}{Patm} Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right]\right)$$

```
χair = χa[.437, θair] (* From Licor matching external conditions *)
0.0166338

PARout = 1700 (*PARo umol m-2 s-1*)
1700

αsr = 0.4(* absorbance over all solar PAR+NIR, 400 to 3000 nm, range .4 to .6 *)
0.4

r = 0.15 (* reflectance of surroundings to solar radiation Nobel *)
0.15
```



```
Assimilation = 19.7 × 10^-6(* mol m-2 s-1*)
0.0000197

AE = 479 000 * Assimilation (* 479 kJ per mole co2 fixed, J m-2 s-1  *)
```

$$FE = .01\,\alpha sr\left(\frac{1}{.45} * PARout * 10^{\wedge}(-6) * 2.35 * 10^{\wedge}5\right)(1 + r)$$

```
  (* J m-2 s-1 Energy lost to fluorescence, 3% of abs vis *)
9.4363

4.08378
```

$$SRsun = \alpha sr\left(\frac{1}{.45} * PARout * 10^{\wedge}(-6) * 2.35 * 10^{\wedge}5\right)(1 + r) - AE - FE$$

```
(* neglect refelctance *)
(* coeff J mol-1,
from Campbell intro env biophysics p 151: Total E incident is about 2x E in PAR,
then typical leaf abs is about half that amount,
so PAR plus NIR about equal to all E in PAR. OR 2.22 =1/.45 is factor
  mult PAR capmbell but also could subtract ps E at 479 kj per mol co2,
and then fluorescence too. ps here is 20 umol m-2 s-2 so ~10 watts*)
394.858

N[479 * 20 * 10^-3]
9.58

windspeed = 3.86(* Logan airport mean for day m s-1*)
3.86
```

Physical quantities: Update for approximate leaf temperature

```
Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)
0.00317308

Pa = Patm * 10^-6 (* atm pressure in MPa*)
0.1013
```

$$c[\psi\_, \theta\_] := 1.28 \frac{R\,298.15}{Patm} Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right] Exp\left[\frac{(\psi + Pa - Po)\ 18.07}{R\ \theta}\right] \text{(*mol fraction for psi in MPa*)}$$

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
0.00228015

0.000282819
```

```
λ = 44 000 - 43 (θvar - 298.15) (* Joules per Mol*)
```
43 825.

$$\left(1.28\ \frac{R\ 298.15}{Patm}\ 18.07\right) \Big/ (R\ 298.15)$$
```
(c[ψvar, θvar] 18.07) / (R θvar)
```
0.000228328

0.000282819

$$Dv = \frac{Patm}{R\ \theta var}\ 2.13 * \left(\frac{\theta var}{273.15}\right)^{1.8} *$$
```
  10^(-5) (* cDv for mol frac drving force out of leaf *)
```
0.00103012

```
kvh = .026; (* heat cond air J m-2 s-1 K-1*)
```

```
σ = 5.670373 * 10^(-8); (*stefan boltzmann J/m2/s/kelvin-4 *)
```

```
F = 1; (*view factor radiative from leaf*)
```

$$\epsilon ac = 1.72\left(\frac{Patm\ \chi a[.437,\ \theta air]}{1000\ \theta air}\right)^{1/7}$$
```
  (*.32 rh logan airport,vapor pressure is in kpa, campbell p 162*)
```
0.819831

```
cloud = 0 (* fraction sky cloudy *)
```
0

```
ϵa = (1 - .84 cloud) ϵac + .84 cloud (* campbell p 162*)
```
0.819831

Leaf parameters

```
ϵl = 0.96 (* emissivity leaf *)
```
0.96

```
αl = ϵl (* long wave abs leaf *)
```
0.96

```
gs = 0.3(* stomatal conductance mol m-2 s-1 , from cuvette data *)
```
0.3

```
charlength = .1 (* length normal to wind, m *)
```
0.1

$$\delta = 4\left(\frac{charlength}{windspeed}\right)^{1/2} * 10\hat{\ }(-3)\ (\text{* in m: cuvette leaf boundary layer*})$$
0.000643823

```
Al = .7(* .1263 oakair fraction, say 0.5 just mesophyll*)
Av = 1 - Al; (*Area fraction tissue in leaf from 2011 data 12.63%*)
```
0.7

```
AUl = .95;(* .1263 oakair fraction, say 0.5 just mesophyll*)
AUv = 1 - AUl; (*Area fraction tissue in leaf from 2011 data 12.63%*)
```

```
L = .72 × 131 × 10^(-6) (*-20 10^(-6)*)
```
0.00009432

```
LU = 1.28 * 131 × 10^(-6)
```
0.00016768

Check whole leaf air fraction

$$\frac{L\ Av + LU\ AUv}{L + LU}\ (\text{*vol weighted area fractions *})$$
0.14

```
klh = .28614;(* heat cond tissue*)
```

```
ktotal = 6.45 × 10^(-7);(* hyd cond leaf mol/m/s/mpa *)
```

```
kl = (ktotal - .1263 Dv Cvψ) / (1 - .1263) (* hyd cond tissue  *)
```
6.96124×10^{-7}

```
H = .021;
```

```
HA = (0.021) * vsf(*mol/m2/s/MPa from vein cutting,
  no steady state 1D scale factor *)
```
0.014049

```
ψr = -1.29 ;(* stem potential from cov'd leaf in mpa *)
```

Absorbed radiation

```
Q = 0.2 * SRsun / L (* w m-3 volumetric heat source,
   .2 is fraction of total abs solar rad abs in spongy *)
```
837 272.

```
QU = 0.8 * SRsun / LU
```
1.88386×10^{6}

```
Q L (* total solar in lower half of leaf*)
```
78.9715

QU LU

315.886

Q L + QU LU - SRsun

0.

Description of fluxes and relations to env parameters

(*delta is from licor 1.42 mol/m2/s g_bw, d=Dv*C_a/g_bw, C_a=40.49 mol/m3*)

$$\mathrm{qsl} = \frac{\mathrm{kvh}}{\delta}\,(\theta\mathrm{le} - \theta\mathrm{air});$$

$$\mathrm{qsu} = \frac{\mathrm{kvh}}{\delta}\,(\theta\mathrm{ue} - \theta\mathrm{air});$$

$$\mathrm{qrl} = \sigma\,\epsilon\mathrm{l}\,\theta\mathrm{le}^4 - \mathrm{F}\,\alpha\mathrm{l}\,\left(\sigma\,\theta\mathrm{sur}^4\right);$$

$$\mathrm{qru} = \sigma\,\epsilon\mathrm{l}\,\theta\mathrm{ue}^4 - \mathrm{F}\,\alpha\mathrm{l}\,\left(\sigma\,\epsilon\mathrm{a}\,\theta\mathrm{air}^4\right);$$

$$\chi[\psi\_, \theta\_] := 1.28\,\frac{\mathrm{R}\,298.15}{\mathrm{Patm}}\,\mathrm{Exp}\left[-\frac{44\,000}{\mathrm{R}}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right]\,\mathrm{Exp}\left[\frac{(\psi + \mathrm{Pa} - \mathrm{Po})\ 18.02}{\mathrm{R}\ \theta}\right]$$

(*for psi in MPa*)

$$\chi\mathrm{e} = \chi[\psi\mathrm{le}, \theta\mathrm{le}];$$

$$\mathrm{gbl} = \frac{2.13 * \left(\frac{\theta\mathrm{air}}{273.15}\right)^{1.8} * 10\,\hat{}\,(-5)\ \mathrm{Patm}}{\mathrm{R}\,\theta\mathrm{air}}$$

(* c(tair)*Dv(Tair), boundary layer molar conductivity *)

0.00102805

gbl / δ

1.59679

$$\mathrm{Jtran} = \left(\frac{1}{\mathrm{gs}} + \frac{\delta}{\mathrm{gbl}}\right)^{-1}(\chi\mathrm{e} - \chi\mathrm{air});$$

$$\psi\mathrm{o} = \psi\mathrm{r} - \mathrm{Jtran} / \mathrm{HA};$$

$$-1.29 - 17.9765\left(-0.0166338 + 0.0313236\,e^{-5291.96\left(-0.00335402+\frac{1}{\theta\mathrm{le}}\right)+\frac{2.1673\,(0.0981269+\psi\mathrm{le})}{\theta\mathrm{le}}}\right)$$

Dimensionless parameters

$$\Pi\psi = 1 + \frac{\mathrm{Al\ klh + Av\ kvh}}{\lambda\,\mathrm{Av\ Dv\ Cv}\theta} + \frac{\mathrm{Al\ klh + Av\ kvh}}{\lambda\,\mathrm{Al\ kl}}\,\frac{\mathrm{Cv}\psi}{\mathrm{Cv}\theta}$$ **(* greater than one favors conduction over latent *)**

8.9473

$$\Pi\theta = 1 + \frac{\lambda\,\mathrm{Al\ kl}}{\mathrm{Al\ klh + Av\ kvh}}\,\frac{\mathrm{Av\ Dv\ Cv}\theta}{\mathrm{Al\ kl + Av\ Dv\ Cv}\psi}$$

(* greater than one favors latent over conduction *)

1.12583

$$\frac{1}{\Pi\theta} + \frac{1}{\Pi\psi}$$ **(* test should equal 1 *)**

1.

$$\Pi\mathrm{U}\psi = 1 + \frac{\mathrm{AUl\ klh + AUv\ kvh}}{\lambda\,\mathrm{AUv\ Dv\ Cv}\theta} + \frac{\mathrm{AUl\ klh + AUv\ kvh}}{\lambda\,\mathrm{AUl\ kl}}\,\frac{\mathrm{Cv}\psi}{\mathrm{Cv}\theta}$$ **(* greater than one favors conduction over latent *)**

55.2365

$$\Pi\mathrm{U}\theta = 1 + \frac{\lambda\,\mathrm{AUl\ kl}}{\mathrm{AUl\ klh + AUv\ kvh}}\,\frac{\mathrm{AUv\ Dv\ Cv}\theta}{\mathrm{AUl\ kl + AUv\ Dv\ Cv}\psi}$$

(* greater than one favors latent over conduction *)

1.01844

$$\frac{1}{\Pi\mathrm{U}\theta} + \frac{1}{\Pi\mathrm{U}\psi}$$ **(* test should equal 1 *)**

1.

Global energy conservation

$$\mathrm{SRsun} == \mathrm{qsu} + \mathrm{qru} + \mathrm{qrl} + \mathrm{qsl} + \lambda\,\mathrm{Jtran}\ ;$$

Solution lower thermal field

$$\theta\mathrm{l}[\mathrm{x}\_] := \theta\mathrm{o} + \left(-\frac{\mathrm{x}^2}{2\,\mathrm{L}^2} + \frac{\mathrm{x}}{\mathrm{L}}\right)\frac{\mathrm{Q\ L}^2}{\Pi\theta\ (\mathrm{Al\ klh + Av\ kvh})} + \left(-\frac{(\mathrm{qsl + qrl})\ \mathrm{L}}{\Pi\theta\ (\mathrm{Al\ klh + Av\ kvh})} - \frac{\lambda\,\mathrm{Al\ kl\ Jtran\ L}}{\Pi\theta\ (\mathrm{Al\ klh + Av\ kvh})}\,\frac{1}{(\mathrm{Al\ kl + Av\ Dv\ Cv}\psi)}\right)\frac{\mathrm{x}}{\mathrm{L}}$$

Solution upper thermal field

$$\theta U[x\_] := \theta o + \left(-\frac{x^2}{2\,LU^2} + \frac{x}{LU}\right) \frac{QU\,LU^2}{\Pi U\theta\,(AUl\,klh + AUv\,kvh)} - \left(\frac{(qsu + qru)\,LU}{\Pi U\theta\,(AUl\,klh + AUv\,kvh)}\right) \frac{x}{LU}$$

Solution upper potential field

$$\psi U[x\_] := \psi o + \left(\frac{x^2}{2\,LU^2} - \frac{x}{LU}\right) \frac{QU\,LU^2}{\Pi U\psi\,AUl\,kl\,\lambda} + \left(\frac{(qsu + qru)\,LU}{\Pi U\psi\,AUl\,kl\,\lambda}\right) \frac{x}{LU}$$

Solution lower potential field

$$\psi l[x\_] := \psi o + \left(\frac{x^2}{2\,L^2} - \frac{x}{L}\right) \frac{Q\,L^2}{\Pi\psi\,Al\,kl\,\lambda} + \left(\frac{(qsl + qrl)\,L}{\Pi\psi\,Al\,kl\,\lambda} - \frac{Jtran\,L}{\Pi\psi\,Al\,kl\,\lambda}\,\frac{Al\,klh + Av\,kvh}{Av\,Dv\,Cv\theta}\right) \frac{x}{L}$$

Solve system

sol =

$$\text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\,Jtran,\ \frac{\psi l[L]}{\psi le} == 1\right\},\right.$$

$$\left.\{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\}, \text{PrecisionGoal} \to 4\right]$$

{θo → 302.215, θue → 302.239, θle → 302.134, ψle → -2.51429}

```
ψue = ψU[LU] /. sol (* Water potential upper epidermis *)
```

-1.6946

Results

```
Temps = {θair, θsur, θue, θo, θle} /. sol
```

{301.46, 301.46, 302.239, 302.215, 302.134}

```
Potentials = {ψr, ψo, ψue, ψle} /. sol
```

{-1.29, -1.69032, -1.6946, -2.51429}

Fluxes

```
qsu /. sol (*Sensible flux upper*)
```

31.4686

```
qru /. sol (*Radiative flux upper*)
```

85.6659


```
qsl /. sol (*Sensible flux lower*)
```

27.2113

```
qrl /. sol (*Radiative flux lower*)
```

4.03302

```
λ Jtran /. sol
```

246.479

```
Jtran /. sol
```

0.00562416

Analyze proportion peristomatal at Lower Epidermis

$$\text{ProportionPeristomatal} = \frac{-Al\,kl\,\partial_x \psi l[x]\ /.\ x \to L}{Jtran}\ /.\ sol$$

0.73898

$$\text{PropPeri} = \frac{-(qsl + qrl)}{\Pi\psi\,Jtran\,\lambda} + \left(1 + \frac{\lambda\,Av\,Dv\,Cv\theta}{Al\,klh + Av\,kvh} + \frac{Av\,Dv\,Cv\psi}{Al\,kl}\right)^{-1}\ /.\ sol$$

0.73898

$$\frac{-(qsl + qrl)}{\Pi\psi\,Jtran\,\lambda}\ /.\ sol$$

-0.0141677

$$\left(1 + \frac{\lambda\,Av\,Dv\,Cv\theta}{Al\,klh + Av\,kvh} + \frac{Av\,Dv\,Cv\psi}{Al\,kl}\right)^{-1}\ /.\ sol$$

0.753147

ProportionInternalVapor =

$$\frac{-Av\,Dv\,Cv\psi\,(\partial_x \psi l[x]\ /.\ x \to L) - Av\,Dv\,Cv\theta\,(\partial_x \theta l[x]\ /.\ x \to L)}{Jtran}\ /.\ sol$$

0.26102

```
ProportionPeristomatal + ProportionInternalVapor
```

1.

Condensation on lower epidermis if following is larger than one:

$$\frac{(qsl + qrl)}{Jtran\,\lambda}\,\frac{\lambda\,Av\,Dv\,Cv\theta}{Al\,klh + Av\,kvh}\ /.\ sol$$

0.0188113

Origin of vapor internally transported

VaporFluxintoLower =
$$\frac{-\text{Av Dv Cv}\psi\ (\partial_x \psi l[x]\ /.\ x \to 0) - \text{Av Dv Cv}\theta\ (\partial_x \theta l[x]\ /.\ x \to 0)}{\text{Jtran}}\ /.\ \text{sol}$$
0.225211

$$\text{ProportionEvaporationLower} = \frac{\text{Al kl}\ \psi l''[x]\ \text{L}}{\text{Jtran}}\ /.\ \text{sol}$$
0.0358095

$$\frac{\text{Q L}}{\Pi\psi\ \lambda\ \text{Jtran}}\ /.\ \text{sol}\ (*\text{integrating evap 2nd derv over L }*)$$
0.0358095

ProportionPeristomatal + VaporFluxintoLower + ProportionEvaporationLower (* Check=1 *)
1.

Analyze evaporation in upper part of leaf

$$\text{LiquidFluxintoUpper} = \frac{-\text{AUl kl}\ \partial_x \psi U[x]\ /.\ x \to 0}{\text{Jtran}}\ /.\ \text{sol}$$
0.0145984

$$\text{EvapOriginUpper} = \frac{\text{QU LU} - (\text{qsu} + \text{qru})}{\Pi U\psi\ \lambda\ \text{Jtran}}\ /.\ \text{sol}$$
0.0145984

VaporFluxintoUpper =
$$\frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi U[x]\ /.\ x \to 0) - \text{AUv Dv Cv}\theta\ (\partial_x \theta U[x]\ /.\ x \to 0)}{\text{Jtran}}\ /.\ \text{sol}$$
-0.0145984

CondensationUpperEpidermis =
$$\frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi U[x]\ /.\ x \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta U[x]\ /.\ x \to \text{LU})}{1}\ /.\ \text{sol}$$
0.0000483879

$$\text{FluxFromUpperEpidermis} = \frac{-\text{AUl kl}\ \partial_x \psi U[x]\ /.\ x \to \text{LU}}{\text{Jtran}}\ /.\ \text{sol}$$
-0.00860357

$$\frac{\lambda}{\text{QU LU}}\ \frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi U[x]\ /.\ x \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta U[x]\ /.\ x \to \text{LU})}{1}\ /.\ \text{sol}$$
0.00671317

$$\frac{\lambda}{\text{qsu} + \text{qru}}\ \frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi U[x]\ /.\ x \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta U[x]\ /.\ x \to \text{LU})}{1}\ /.\ \text{sol}$$
0.018104

qru /. sol
85.6659

$$\text{ConductiontoUepidermis} = \frac{-(\text{AUl klh} + \text{AUv kvh})\ (\partial_x \theta U[x]\ /.\ x \to \text{LU})}{\text{QU LU}}\ /.\ \text{sol}$$
0.364099

$$\text{ConductiontoUvascplane} = \frac{-(\text{AUl klh} + \text{AUv kvh})\ (\partial_x \theta U[x]\ /.\ x \to 0)}{\text{QU LU}}\ /.\ \text{sol}$$
-0.617797

$$\text{ConductionLvascplane} = \frac{-(\text{Al klh} + \text{Av kvh})\ (\partial_x \theta l[x]\ /.\ x \to 0)}{\text{QU LU}}\ /.\ \text{sol}$$
0.453461

$$\text{ConductionULratio} = \frac{-(\partial_x \theta U[x]\ /.\ x \to 0)}{-(\partial_x \theta l[x]\ /.\ x \to 0)}\ /.\ \text{sol}$$
-1.03801

$$\text{ConductionULratio} = \frac{-(\text{AUl klh} + \text{AUv kvh})\ (\partial_x \theta U[x]\ /.\ x \to 0)}{-(\text{Al klh} + \text{Av kvh})\ (\partial_x \theta l[x]\ /.\ x \to 0)}\ /.\ \text{sol}$$
-1.3624

Leaf water potential and apparent conductance

$$\text{LowerAvgPotential} = \frac{1}{\text{L}} \int_0^{\text{L}} \psi l[x]\, dx\ /.\ \text{sol}$$
-2.10555

$$\text{UpperAvgPotential} = \frac{1}{\text{LU}} \int_0^{\text{LU}} \psi U[x]\, dx\ /.\ \text{sol}$$
-1.69522

$$\text{LWP} = \frac{\text{L LowerAvgPotential} + \text{LU UpperAvgPotential}}{\text{L} + \text{LU}}$$
-1.84294

$$\text{Kleaf} = \frac{\text{Jtran}}{\psi r - \text{LWP}}\ /.\ \text{sol}\ (*\text{ Apparent conductance, mol m-2 s-1 MPa-1}*)$$
0.0101714

```
ApparentLength = (ProportionPeristomatal + 0.5 ProportionEvaporationLower)
 (* Dist from evap site to vasc *)
```

0.756884

Update and re-iterate for approximate leaf temp and potential

$$c[\psi\_, \theta\_] := 1.28\ \frac{R\,298.15}{Patm}\ \mathrm{Exp}\left[-\frac{44\,000}{R}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]\ \mathrm{Exp}\left[\frac{\psi\ 18.07}{R\ \theta}\right]$$

$Cv\psi - \partial_{\psi}\, c[\psi, \theta o]$ /. $\psi \to \psi o$ /. sol

2.82987×10^{-7}

$Cv\theta - \partial_{\theta}\, c[\psi o, \theta]$ /. $\theta \to \theta o$ /. sol

2.14906×10^{-6}

$\theta var - \theta o$ /. sol

0.00536141

$\psi var - \psi o$ /. sol

0.000324769

θo /. sol

302.215

ψo /. sol

-1.69032

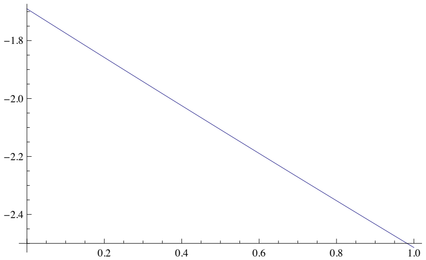
Plots

Lower Potential

psiL = $\psi l[x]$ /. sol /. x → L X

$-1.69032 - 0.804469\,X + 0.038983\,(-1.\,X + 0.5\,X^2)$


```
Plot[psiL, {X, 0, 1}]
```

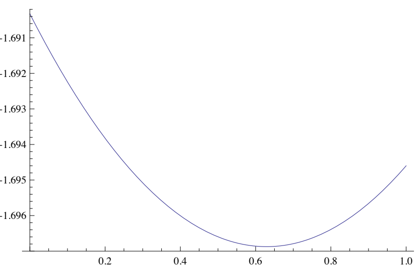


Upper Potential

psiU = $\psi U[x]$ /. sol /. x → LU X

$-1.69032 + 0.0122689\,X + 0.0330867\,(-1.\,X + 0.5\,X^2)$

```
Plot[psiU, {X, 0, 1}]
```

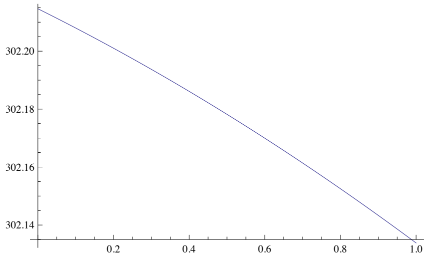


Lower Thermal

thetaL = $\theta l[x]$ /. sol /. x → L X

$302.215 - 0.0967173\,X + 0.0317932\,(1.\,X - 0.5\,X^2)$

```
Plot[thetaL, {X, 0, 1}]
```

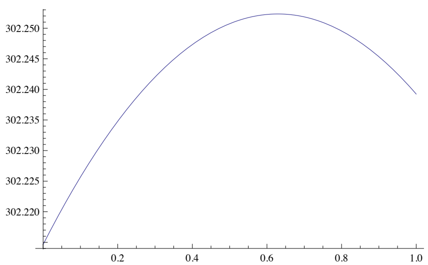


Upper Thermal

```
thetaU = θU[x] /. sol /. x → LU X
```

$302.215 - 0.0706086\,X + 0.190416\,\left(1.\,X - 0.5\,X^2\right)$

```
Plot[thetaU, {X, 0, 1}]
```

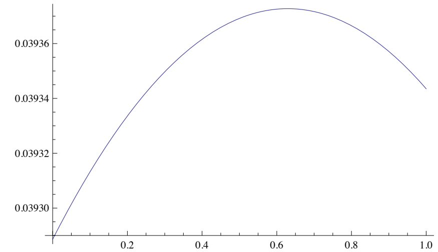


Upper Vapor Concentration

```
cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;
```



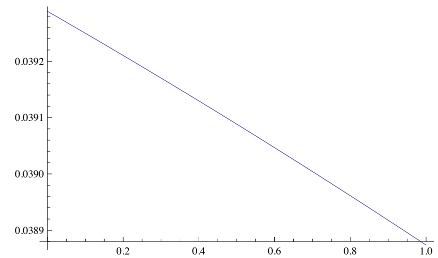
```
Plot[cU, {X, 0, 1}]
```



Lower Vapor Concentration

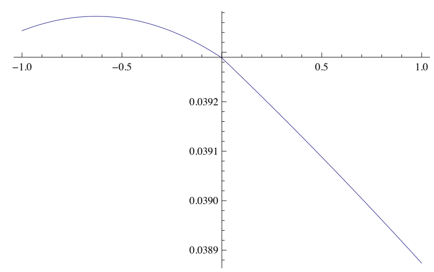
```
cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;
```

```
Plot[cL, {X, 0, 1}]
```



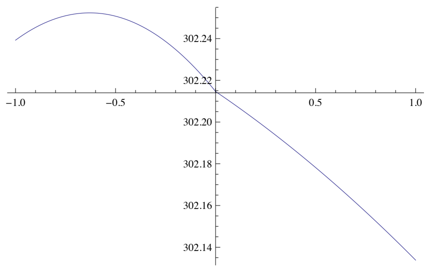
Whole leaf vapor

```
Show[Plot[cU /. X → -z, {z, 0, -1}], Plot[cL, {X, 0, 1}], PlotRange → All]
```



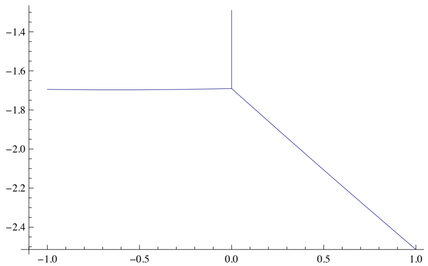
Whole leaf temperature

```
Show[Plot[thetaU /. X → -z, {z, 0, -1}], Plot[thetaL, {X, 0, 1}], PlotRange → All]
```



Whole leaf potential

```
Show[Plot[psiU /. X → -z, {z, 0, -1}], Plot[psiL, {X, 0, 1}],
 ListPlot[{{0, ψr}, {0, ψo /. sol}}, Joined → True],
 AxesOrigin → {-1.1, ψle /. sol}, PlotRange → All]
```



$$\text{Biotout} = \frac{\text{H L } (\psi\text{o} - \psi\text{le})}{(\text{Al kl} + \text{Av Dv Cv}\psi)\ (\psi\text{o} - \psi\text{le}) + \text{Av Dv Cv}\theta\ (\theta\text{o} - \theta\text{le})}\ /.\ \text{sol}$$

3.07658

$$\frac{\text{Av Dv Cv}\psi\ (\psi\text{o} - \psi\text{le})}{\text{Jtran L}}\ /.\ \text{sol}$$

$$\frac{\text{Av Dv Cv}\theta\ (\theta\text{o} - \theta\text{le})}{\text{Jtran L}}\ /.\ \text{sol}$$

0.135757

0.107358


```
sf = (.151 Biotout + 1.03) ^ (-1)
```

0.669092

$$\text{nondnumericpsiavg} = .721 - \frac{.219}{\text{Biotout}}$$

0.649817

```
ScaledPsiAverageLeaf = nondnumericpsiavg * (-1.29 - ψle ) + ψle /. sol
  (* cf 0.65 for oak (scaled k est on tree and leaf paper): given true psie
    and psir as defined in numeric simulation gives average potential in
    domain. Justified if 1D steady sol is scaled to 3d as it is here *)
```

-1.71872

```
kleaf = Jtran / (ψr - ScaledPsiAverageLeaf) /. sol
```

0.0131184

```
θo /. sol
```

302.215

```
ψo /. sol
```

-1.69032

```
Tleaf = 273.15 + 28.83
```

301.98

```
θle - Tleaf /. sol
```

0.153818

```
Jtran /. sol
```

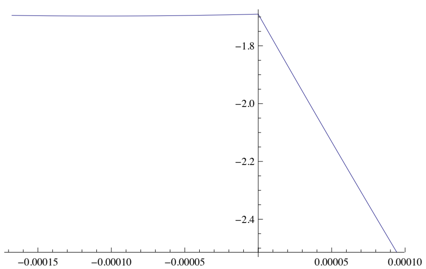
0.00562416

Export plot data

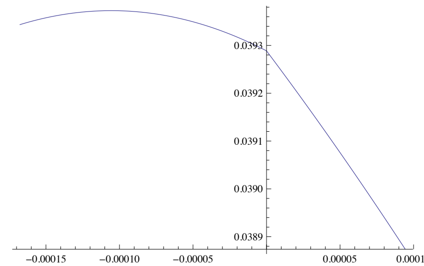
```
position = Range[-1, 1, .01];
```

```
outpos = Table[0, {Length[position]}];
```

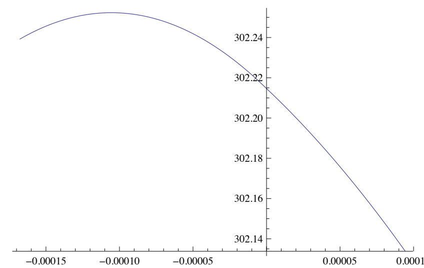
```
potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]

opdata = Transpose[{outpos, potential}];

otdata = Transpose[{outpos, temperature}];

ocdata = Transpose[{outpos, vapor}];

ListPlot[opdata, Joined → True]
```




```
ListPlot[ocdata, Joined → True]
```




```
ListPlot[otdata, Joined → True]
```




```
(*SetDirectory[ToFileName[NotebookDirectory[]]]*)
/Users/Tony/Dropbox/Manuscripts/Active manuscripts/vapor and liquid/mol fraction

(*outfile="oak_numeric_"*)
oak_numeric_

(*Export[outfile<>"potential.xls",opdata]*)
oak_numeric_potential.xls

(*Export[outfile<>"vapor.xls",ocdata]*)
oak_numeric_vapor.xls

(*Export[outfile<>"temperature.xls",otdata]*)
oak_numeric_temperature.xls
```