# Sunflower model (Pieruschka et al. 2010)

## Base case: cold mirror illumination of a leaf in a gas exchange cuvette

Note: Equation numbers reference main text unless otherwise specified.

```
In[368]:= Remove["Global`*"]
```

### Temperature

```
In[369]:= θairbase = 273.15 + 25
            (* air temp preserves true mol fraction if change air temp in cuvette*)
Out[369]= 298.15

In[370]:= θair = 273.15 + 25(* air temp actual after a temp corr *)
Out[370]= 298.15

In[371]:= θsur = θair; (* for cuvette experiments all surfaces assumed at air temp *)
```

### Soil dependance of ψr

```
In[372]:= ψs = -.2 ;(* soil potential (assumed) mpa *)

In[373]:= ψr = ψs (* stem potential (assumed) mpa, not modelling whole plant hydraulics *)
Out[373]= -0.2
```

### Other Environmental parameters

```
In[374]:= R = 8.3145; (*gas constant Joules per mole per Kelvin*)

In[375]:= Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)
```

In[376]:= $\chi a[rh\_, \theta\_] := rh \left(1.28 \frac{R\,298.15}{Patm} \mathrm{Exp}\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right]\right)$

```
In[377]:= RH = .835 (* humidity in chamber free-air stream *)
Out[377]= 0.835

In[378]:= χair = χa[RH, θairbase] (* chamber mol fraction *)
Out[378]= 0.0261552
```



```
In[379]:= χsat = χa[1, θairbase] (* chamber saturated mol fraction *)
Out[379]= 0.0313236

In[380]:= SRsun = 160(* W m-2,
             short wave radiation load from light source per unit leaf area, eq *)
Out[380]= 160
```

### Physical quantities

```
In[381]:= Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)
Out[381]= 0.00317308

In[382]:= Pa = Patm * 10^-6 (* atm pressure in MPa*)
Out[382]= 0.1013
```

In[383]:= $c[\psi\_, \theta\_] := 1.28 \frac{R\,298.15}{Patm} \mathrm{Exp}\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right] \mathrm{Exp}\left[\frac{(\psi + Pa - Po)\ 18.07}{R\ \theta}\right]$ (*mol fraction for ψ in MPa, Eqn. 1.4 *)

```
In[384]:= Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar ;
          (* linearization of dχ/dT 1/K, eqn. 27 Supp. Text S1 *)
          Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar ;
          (* linearization of dχ/dpsi 1/MPa, eqn 26 Supp. Text S1*)

In[385]:= λ = 44 000 - 43 (θvar - 298.15) ; (* Joules per Mol, Latent heat*)
```

In[386]:= $Dv = \frac{Patm}{R\,\theta var} 2.13 * \left(\frac{\theta var}{273.15}\right)^{1.8} * 10^{\wedge}(-5);$
(* cDv for mol frac driving force out of leaf *)

```
In[387]:= kvh = .026;(* Thermal conductivity of air W m-2 K-1*)

In[388]:= σ = 5.670373 * 10^(-8); (*stefan boltzmann W/m2/kelvin-4 *)

In[389]:= F = 1; (*view factor radiative from leaf*)
```

### Leaf parameters

```
In[390]:= ϵl = 0.96;  (* emissivity leaf *)

In[391]:= αl = ϵl ; (* long wave absorptance leaf *)

In[392]:= Al = .2; (* .1263 oakair fraction, say 0.5 just mesophyll*)
          Av = 1 - Al; (*Area fraction tissue in leaf from 2011 data 12.63%*)

In[393]:= AUl = .8;(* .1263 oakair fraction, say 0.5 just mesophyll*)
          AUv = 1 - AUl; (*Area fraction tissue in leaf from 2011 data 12.63%*)

In[395]:= L = 200 × 10^(-6);

In[396]:= LU = 200 × 10^(-6);
```

Check whole leaf air fraction

In[397]:= $$\frac{\text{L Av} + \text{LU AUv}}{\text{L} + \text{LU}} \text{ (*vol weighted area fractions *)}$$

Out[397]= 0.5

In[398]:= `klh = .2;(* Thermal conductivity of the cells, w m-1 K-1*)`

In[399]:= `kl = 1 × 10^(-6); (* hydraulic conductivity of the cells, mol m-1 MPa-1 s-1 *)`

In[400]:= `HA = (0.1); (*mol m-2 MPa-1 s-1, no steady state 1D scale factor *)`

## Absorbed radiation

In[401]:=
```
Q = 0.1 * SRsun / L ; (* w m-3 volumetric heat source,
.1 is fraction of total abs solar rad abs in spongy *)
```

In[402]:=
```
QU = 0.9 * SRsun / L ; (* w m-3 volumetric heat source,
.9 is fraction of total abs solar rad abs in palisade *)
```

## Description of fluxes and relations to env parameters

In[403]:= $$\text{gbl} = \frac{2.13 * \left(\frac{\theta\text{air}}{273.15}\right)^{1.8} * 10\text{^}(-5)\ \text{Patm}}{\text{R}\ \theta\text{air}};\ \text{(* c(tair)*Dv(Tair),}$$
boundary layer molar conductivity, mol m-1 s-1*)

In[404]:= $$\delta = \left(\frac{2.13 * \left(\frac{\theta\text{air}}{273.15}\right)^{1.8} * 10\text{^}(-5)\ \text{Patm}}{\text{R}\ \theta\text{air}}\right) \Big/ 1.42;$$

(* boundary layer thickness, m  eqn. 75 Suppl. text S1 *)

In[405]:= `gbl / δ`

Out[405]= 1.42

In[406]:=
```
(* Li-cor one-sided boundary layer conductance is 1.42 mol/m2/s g_bw,
d=Dv*C_a/g_bw   C_a=40.49 mol/m3*)
```

In[407]:= $$\text{qsl} = \frac{\text{kvh}}{\delta}\ (\theta\text{le} - \theta\text{air});\ \text{(* eqn 1.32 *)}$$

In[408]:= $$\text{qsu} = \frac{\text{kvh}}{\delta}\ (\theta\text{ue} - \theta\text{air});\ \text{(* eqn 1.32 *)}$$

In[409]:= $$\text{qrl} = \sigma\ \epsilon\text{l}\ \theta\text{le}^4 - \text{F}\ \alpha\text{l}\ \left(\sigma\ \theta\text{sur}^4\right);\ \text{(* eqn 1.29 *)}$$

In[410]:= $$\text{qru} = \sigma\ \epsilon\text{l}\ \theta\text{ue}^4 - \text{F}\ \alpha\text{l}\ \left(\sigma\ \theta\text{sur}^4\right);\ \text{(* eqn 1.29 *)}$$

In[411]:= $$\chi[\psi\_,\ \theta\_] := 1.28\ \frac{\text{R}\ 298.15}{\text{Patm}}\ \text{Exp}\left[-\frac{44\,000}{\text{R}}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right]\ \text{Exp}\left[\frac{(\psi + \text{Pa} - \text{Po})\ 18.07}{\text{R}\ \theta}\right]$$

(*for ψ in MPa eqn. 1.4 *)

In[412]:= `χe = χ[ψle, θle]; (* mole fraction at lower epidermis (inside stomata) *)`



In[413]:= `χue = χ[ψue, θue]; (* mole fraction at upper epidermis (inside stomata) *)`

In[414]:= $$\text{Jtran} = \left(\frac{1}{\text{gs}} + \frac{\delta}{\text{gbl}}\right)^{-1} (\chi\text{e} - \chi\text{air});\ \text{(* eqn 1.34 *)}$$

In[415]:= $$\text{UJtran} = \left(\frac{1}{\text{gsu}} + \frac{\delta}{\text{gbl}}\right)^{-1} (\chi\text{ue} - \chi\text{air});\ \text{(* eqn 1.34 *)}$$

In[416]:=
```
ψo = ψr - (Jtran + UJtran) / HA; (* eqn 1.4 *)
(* Describes potential drop from source to vacular plane in leaf *)
```

## Stomatal functions (based on matching total gs for cold mirror initial condition)

In[417]:= `stomalow[ps_] := 0.5363540283333067` + (ps + .9) .6 (* gs max 1.2 *)`

In[418]:= `stomaup[psu_] := 0.17186009317252962` + (psu + 0.45) .6(* gs max 1.2 *)`

## Initial values

In[419]:= `ψrinit = -.9;`

In[420]:= `ψrinitup = -.45`

Out[420]= -0.45

In[421]:=
```
θvar = θair; (* θair *)
ψvar = ψrinit;  (* ψr *)
```

## First iteration

In[423]:= $$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$

(* eqn. 1.10, greater than one favors conduction over latent *)

Out[423]= 2.75973

In[424]:= $$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$

(* eqn 1.12, greater than one favors latent over conduction *)

Out[424]= 1.56827

In[425]:= $$\Pi\text{U}\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$

(* eqn. 1.10,greater than one favors conduction over latent *)

Out[425]= 11.5085

In[426]:= $$\Pi\text{U}\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\ \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$$

(* eqn 1.12, greater than one favors latent over conduction *)

Out[426]= 1.09516

## Check that inverse sums equal one (eqn. 46):

In[427]:= $$\frac{1}{\Pi\psi}+\frac{1}{\Pi\theta}$$

Out[427]= 1.

In[428]:= $$\frac{1}{\Pi U\psi}+\frac{1}{\Pi U\theta}$$

Out[428]= 1.

## Global energy conservation

In[429]:= SRsun == qsu + qru + qrl + qsl + λ Jtran + λ UJtran; (* eqn 1.39 *)

## Solution lower thermal field

In[430]:= $$\theta l[x\_] := \theta o+\left(-\frac{x^2}{2\,L^2}+\frac{x}{L}\right)\frac{Q\,L^2}{\Pi\theta\,(Al\;klh+Av\;kvh)}+\left(-\frac{(qsl+qrl)\,L}{\Pi\theta\,(Al\;klh+Av\;kvh)}-\frac{\lambda\,Al\;kl\;Jtran\;L}{\Pi\theta\,(Al\;klh+Av\;kvh)}\;\frac{1}{(Al\;kl+Av\;Dv\;Cv\psi)}\right)\frac{x}{L}\;\text{(* eqn 1.28 *)}$$

## Solution upper thermal field

In[431]:= $$\theta U[x\_] := \theta o+\left(-\frac{x^2}{2\,LU^2}+\frac{x}{LU}\right)\frac{QU\,LU^2}{\Pi U\theta\,(AUl\;klh+AUv\;kvh)}+\left(-\frac{(qsu+qru)\,LU}{\Pi U\theta\,(AUl\;klh+AUv\;kvh)}-\frac{\lambda\,AUl\;kl\;UJtran\;LU}{\Pi U\theta\,(AUl\;klh+AUv\;kvh)}\;\frac{1}{(AUl\;kl+AUv\;Dv\;Cv\psi)}\right)\frac{x}{LU}\;\text{(* eqn 1.28 *)}$$

## Solution upper potential field

In[432]:= $$\psi U[x\_] := \psi o+\left(\frac{x^2}{2\,LU^2}-\frac{x}{LU}\right)\frac{QU\,LU^2}{\Pi U\psi\;AUl\;kl\;\lambda}+\left(\frac{(qsu+qru)\,LU}{\Pi U\psi\;AUl\;kl\;\lambda}-\frac{UJtran\;LU}{\Pi U\psi\;AUl\;kl\;\lambda}\;\frac{AUl\;klh+AUv\;kvh}{AUv\;Dv\;Cv\theta}\right)\frac{x}{LU}\;\text{(* eqn 1.27 *)}$$

## Solution lower potential field

In[433]:= $$\psi l[x\_] := \psi o+\left(\frac{x^2}{2\,L^2}-\frac{x}{L}\right)\frac{Q\,L^2}{\Pi\psi\;Al\;kl\;\lambda}+\left(\frac{(qsl+qrl)\,L}{\Pi\psi\;Al\;kl\;\lambda}-\frac{Jtran\;L}{\Pi\psi\;Al\;kl\;\lambda}\;\frac{Al\;klh+Av\;kvh}{Av\;Dv\;Cv\theta}\right)\frac{x}{L}\;\text{(* eqn 1.27 *)}$$



## Solve system

In[434]:= $$sol = \text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue}==1,\;\frac{\theta l[L]}{\theta le}==1,\;SRsun==qsu+qru+qrl+qsl+\lambda\,Jtran+\lambda\,UJtran,\;\frac{\psi l[L]}{\psi le}==1,\;\frac{\psi U[LU]}{\psi ue}==1,\;\frac{stomalow[\psi le]}{gs}==1,\;\frac{stomaup[\psi ue]}{gsu}==1\right\},\right.$$
$$\left.\{\{\theta o,\theta air\},\{\theta ue,\theta air\},\{\theta le,\theta air\},\{\psi le,\psi rinit\},\{\psi ue,\psi rinitup\},\{gs,.3\},\{gsu,.3\}\},\;\text{PrecisionGoal}\to 4\right]$$

Out[434]= {θo → 298.521, θue → 298.541, θle → 298.413, ψle → -0.906995, ψue → -0.45013, gs → 0.532157, gsu → 0.171782}

### update seed values

In[435]:= θvar = θo /. sol
ψvar = ψo /. sol

Out[435]= 298.521

Out[436]= -0.230118

## Second iteration

In[437]:= $Cv\theta = \partial_\theta\, c[\psi var, \theta]\;/.\;\theta\to\theta var$ ; (* linearization of dχ/dT 1/k *)
$Cv\psi = \partial_\psi\, c[\psi, \theta var]\;/.\;\psi\to\psi var$ ; (* linearization of dχ/dpsi 1/mpa *)

In[438]:= $$\Pi\psi = 1+\frac{Al\;klh+Av\;kvh}{\lambda\;Av\;Dv\;Cv\theta}+\frac{Al\;klh+Av\;kvh}{\lambda\;Al\;kl}\;\frac{Cv\psi}{Cv\theta}\;\text{(* greater than one favors conduction over latent *)}$$

Out[438]= 2.73893

In[439]:= $$\Pi\theta = 1+\frac{\lambda\;Al\;kl}{Al\;klh+Av\;kvh}\;\frac{Av\;Dv\;Cv\theta}{Al\;kl+Av\;Dv\;Cv\psi}\;\text{(* greater than one favors latent over conduction *)}$$

Out[439]= 1.57507

In[440]:= $$\Pi U\psi = 1+\frac{AUl\;klh+AUv\;kvh}{\lambda\;AUv\;Dv\;Cv\theta}+\frac{AUl\;klh+AUv\;kvh}{\lambda\;AUl\;kl}\;\frac{Cv\psi}{Cv\theta}\;\text{(* greater than one favors conduction over latent *)}$$

Out[440]= 11.2662

In[441]:= $$\Pi U\theta = 1+\frac{\lambda\;AUl\;kl}{AUl\;klh+AUv\;kvh}\;\frac{AUv\;Dv\;Cv\theta}{AUl\;kl+AUv\;Dv\;Cv\psi}\;\text{(* greater than one favors latent over conduction *)}$$

Out[441]= 1.09741

In[442]:= $$\text{sol} = \text{FindRoot}\Big[\Big\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ \text{SRsun} == qsu + qru + qrl + qsl + \lambda\,\text{Jtran} + \lambda\,\text{UJtran},\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\psi U[LU]}{\psi ue} == 1,\ \frac{\text{stomalow}[\psi le]}{gs} == 1,\ \frac{\text{stomaup}[\psi ue]}{gsu} == 1\Big\},$$
$$\{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi rinit\}, \{\psi ue, \psi rinitup\}, \{gs, .3\}, \{gsu, .3\}\}, \text{PrecisionGoal} \to 4\Big]$$

Out[442]= {θo → 298.515, θue → 298.536, θle → 298.409, ψle → -0.899661, ψue → -0.450011, gs → 0.536557, gsu → 0.171854}

### update seed values

In[443]:=
```
θvar = θo /. sol
ψvar = ψo /. sol
```

Out[443]= 298.515

Out[444]= -0.230213

## Third iteration

In[445]:=
```
Cvθ = ∂θ c[ψvar, θ] /. θ → θvar; (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar] /. ψ → ψvar; (* linearization of dχ/dpsi 1/mpa *)
```

In[446]:= $$\Pi\psi = 1 + \frac{Al\ klh + Av\ kvh}{\lambda\ Av\ Dv\ Cv\theta} + \frac{Al\ klh + Av\ kvh}{\lambda\ Al\ kl}\ \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

Out[446]= 2.73917

In[447]:= $$\Pi\theta = 1 + \frac{\lambda\ Al\ kl}{Al\ klh + Av\ kvh}\ \frac{Av\ Dv\ Cv\theta}{Al\ kl + Av\ Dv\ Cv\psi}$$ (* greater than one favors latent over conduction *)

Out[447]= 1.57499

In[448]:= $$\Pi U\psi = 1 + \frac{AUl\ klh + AUv\ kvh}{\lambda\ AUv\ Dv\ Cv\theta} + \frac{AUl\ klh + AUv\ kvh}{\lambda\ AUl\ kl}\ \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

Out[448]= 11.269

In[449]:= $$\Pi U\theta = 1 + \frac{\lambda\ AUl\ kl}{AUl\ klh + AUv\ kvh}\ \frac{AUv\ Dv\ Cv\theta}{AUl\ kl + AUv\ Dv\ Cv\psi}$$ (* greater than one favors latent over conduction *)

Out[449]= 1.09738



In[450]:= $$\text{sol} = \text{FindRoot}\Big[\Big\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ \text{SRsun} == qsu + qru + qrl + qsl + \lambda\,\text{Jtran} + \lambda\,\text{UJtran},\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\psi U[LU]}{\psi ue} == 1,\ \frac{\text{stomalow}[\psi le]}{gs} == 1,\ \frac{\text{stomaup}[\psi ue]}{gsu} == 1\Big\},$$
$$\{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi rinit\}, \{\psi ue, \psi rinitup\}, \{gs, .3\}, \{gsu, .3\}\}, \text{PrecisionGoal} \to 4\Big]$$

Out[450]= {θo → 298.516, θue → 298.536, θle → 298.409, ψle → -0.899748, ψue → -0.450012, gs → 0.536505, gsu → 0.171853}

### update seed values

In[451]:=
```
θvar = θo /. sol
ψvar = ψo /. sol
```

Out[451]= 298.516

Out[452]= -0.230212

## Fourth iteration

In[453]:=
```
Cvθ = ∂θ c[ψvar, θ] /. θ → θvar; (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar] /. ψ → ψvar; (* linearization of dχ/dpsi 1/mpa *)
```

In[454]:= $$\Pi\psi = 1 + \frac{Al\ klh + Av\ kvh}{\lambda\ Av\ Dv\ Cv\theta} + \frac{Al\ klh + Av\ kvh}{\lambda\ Al\ kl}\ \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

Out[454]= 2.73917

In[455]:= $$\frac{Al\ klh + Av\ kvh}{\lambda\ Av\ Dv\ Cv\theta}$$

Out[455]= 0.891892

In[456]:= $$\frac{Al\ klh + Av\ kvh}{\lambda\ Al\ kl}\ \frac{Cv\psi}{Cv\theta}$$

Out[456]= 0.847276

In[457]:= $$\frac{Cv\psi}{Cv\theta}$$

Out[457]= 0.122588

In[458]:= $$\Pi\theta = 1 + \frac{\lambda\ Al\ kl}{Al\ klh + Av\ kvh}\ \frac{Av\ Dv\ Cv\theta}{Al\ kl + Av\ Dv\ Cv\psi}$$ (* greater than one favors latent over conduction *)

Out[458]= 1.57499

In[459]:= $\Pi U\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}} \frac{\text{Cv}\psi}{\text{Cv}\theta}$ (* greater than one favors conduction over latent *)

Out[459]= 11.269

In[460]:= $\Pi U\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}} \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$
(* greater than one favors latent over conduction *)

Out[460]= 1.09738

In[461]:= sol = FindRoot$\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1, \frac{\theta l[L]}{\theta le} == 1, \text{SRsun} == \text{qsu} + \text{qru} + \text{qrl} + \text{qsl} + \lambda\ \text{Jtran} + \lambda\ \text{UJtran}, \frac{\psi l[L]}{\psi le} == 1, \frac{\psi U[LU]}{\psi ue} == 1, \frac{\text{stomalow}[\psi le]}{gs} == 1, \frac{\text{stomaup}[\psi ue]}{gsu} == 1\right\},\right.$
{{θo, θair}, {θue, θair}, {θle, θair}, {ψle, ψrinit},
{ψue, ψrinitup}, {gs, .3}, {gsu, .3}}, PrecisionGoal → 4]

Out[461]= {θo → 298.516, θue → 298.536, θle → 298.409,
ψle → -0.899747, ψue → -0.450012, gs → 0.536506, gsu → 0.171853}

## check convergence

In[462]:= θvar - θo /. sol

Out[462]= $7.24439 \times 10^{-7}$

In[463]:= ψvar - ψo /. sol

Out[463]= $1.31368 \times 10^{-8}$

## Results

In[464]:= MolFracs =
{(χe - χair), (χue - χair), χe, χue, χair} /. sol (* target at Q160 is .0054 *)

Out[464]= {0.00546984, 0.00581404, 0.031625, 0.0319692, 0.0261552}

In[465]:= Temps = {θair, θsur, θue, θo, θle, θle - θair} /. sol

Out[465]= {298.15, 298.15, 298.536, 298.516, 298.409, 0.259177}

In[466]:= Potentials = {ψr, ψo, ψue, ψle} /. sol

Out[466]= {-0.2, -0.230212, -0.450012, -0.899747}

In[467]:= gs /. sol (* target at Q160 is .7 *)

Out[467]= 0.536506

In[468]:= gsu /. sol

Out[468]= 0.171853



In[469]:= totalgs = gs + gsu /. sol

Out[469]= 0.708359

## Fluxes

In[470]:= qsu /. sol (*Sensible flux upper*)

Out[470]= 13.9944

In[471]:= qru /. sol (*Radiative flux upper*)

Out[471]= 2.23337

In[472]:= qsl /. sol (*Sensible flux lower*)

Out[472]= 9.39029

In[473]:= qrl /. sol (*Radiative flux lower*)

Out[473]= 1.49765

In[474]:= latentlow = λ Jtran /. sol

Out[474]= 93.6814

In[475]:= latentup = λ UJtran /. sol

Out[475]= 39.2029

In[476]:= totallatent = latentlow + latentup

Out[476]= 132.884

In[477]:= tranlow = Jtran /. sol (* target at Q160 is .003 *)

Out[477]= 0.00212988

In[478]:= tranup = UJtran /. sol

Out[478]= 0.000891293

In[479]:= totaltran = tranlow + tranup

Out[479]= 0.00302118

## Matching Q160

In[480]:= totaltran (* expect about .0031 *)

Out[480]= 0.00302118

In[481]:= blMFdrop = totaltran / 2.84

Out[481]= 0.00106379

In[482]:= χsurf = χair + blMFdrop (* app mol frac at leaf surface *)

Out[482]= 0.027219

In[483]:= `appgradientLS = χe - χsurf /. sol (* app MF grad at leaf surf exp .0044 *)`

Out[483]= 0.00440604

In[484]:= $appgst = \left(\frac{(\chi e - \chi air)}{totaltran} - \frac{1}{2.84}\right)^{-1}$ /. sol(* 0.7 target *)

Out[484]= 0.68569

## Evaporation distribution percent of totals

In[485]:= $Liquidtolower = \frac{-Al\ kl\ \partial_x \psi l[x]\ /.\ x \to L}{Jtran + UJtran}$ /. sol (* eqn 1.41 *)

Out[485]= 0.199635

In[486]:= $Liquidtoupper = \frac{-AUl\ kl\ \partial_x \psi U[x]\ /.\ x \to LU}{Jtran + UJtran}$ /. sol (* eqn 1.41 *)

Out[486]= 0.242932

In[487]:= VaportoLower =
(-Av Dv Cvψ ( $\partial_x$ ψl[x] /. x → L) - Av Dv Cvθ ( $\partial_x$ θl[x] /. x → L)) / (Jtran + UJtran) /.
sol

Out[487]= 0.50535

In[488]:= VaportoUpper = (-AUv Dv Cvψ ( $\partial_x$ ψU[x] /. x → LU) - AUv Dv Cvθ ( $\partial_x$ θU[x] /. x → LU)) /
(Jtran + UJtran) /. sol
(*Sum of VaportoUpper and VaportoLower is given by eqn 1.44)

In[488]:= (Liquidtolower + Liquidtoupper + VaportoLower + VaportoUpper)

Out[488]= 0.947917 + VaportoUpper

### Condensation on lower epidermis if following is larger than one:

In[489]:= $\frac{(qsl + qrl)}{Jtran\ \lambda}\ \frac{\lambda\ Av\ Dv\ Cv\theta}{Al\ klh + Av\ kvh}$ /. sol (* eqn 1.42 *)

Out[489]= 0.130311

### Condensation on upper epidermis if following is larger than one:

In[490]:= $\frac{(qsu + qru)}{UJtran\ \lambda}\ \frac{\lambda\ AUv\ Dv\ Cv\theta}{AUl\ klh + AUv\ kvh}$ /. sol (* eqn 1.42 *)

Out[490]= 0.0427033

## Leaf water potential and apparent conductance

In[491]:= $LowerAvgPotential = \frac{1}{L}\int_0^L \psi l[x]\,dx$ /. sol

Out[491]= -0.576046



In[492]:= $UpperAvgPotential = \frac{1}{LU}\int_0^{LU} \psi U[x]\,dx$ /. sol

Out[492]= -0.346164

In[493]:= $LWP = \frac{L\ LowerAvgPotential + LU\ UpperAvgPotential}{L + LU}$ (* eqn 1.45 *)

Out[493]= -0.461105

### Naive view of gradients

In[494]:= $Kleaf = \frac{Jtran}{\psi r - LWP}$ /. sol (* Apparent conductance, mol m-2 s-1 MPa-1*)

Out[494]= 0.00815719

## Analysis

### Analyze proportion peristomatal at Lower Epidermis

In[495]:= $ProportionPeristomatal = \frac{-Al\ kl\ \partial_x \psi l[x]\ /.\ x \to L}{Jtran + UJtran}$ /. sol

Out[495]= 0.199635

In[496]:= $PropPeri = \frac{-(qsl + qrl)}{\Pi\psi\ Jtran\ \lambda} + \left(1 + \frac{\lambda\ Av\ Dv\ Cv\theta}{Al\ klh + Av\ kvh} + \frac{Av\ Dv\ Cv\psi}{Al\ kl}\right)^{-1}$ /. sol

Out[496]= 0.283177

In[497]:= $\frac{-(qsl + qrl)}{\Pi\psi\ Jtran\ \lambda}$ /. sol

Out[497]= -0.04243

In[498]:= $\left(1 + \frac{\lambda\ Av\ Dv\ Cv\theta}{Al\ klh + Av\ kvh} + \frac{Av\ Dv\ Cv\psi}{Al\ kl}\right)^{-1}$ /. sol

Out[498]= 0.325607

### Evaporation in lower

In[499]:= $ProportionEvaporationLower = \frac{Al\ kl\ \psi l''[x]\ L}{Jtran + UJtran}$ /. sol

Out[499]= 0.0439569

In[500]:= $\frac{Q\ L}{\Pi\psi\ \lambda\ Jtran}$ /. sol (*integrating evap 2nd derv over L *)

Out[500]= 0.0623516

## Analyze evaporation in upper part of leaf

In[501]:= $\texttt{ProportionEvaporationUpper} = \dfrac{\texttt{AUl kl}\ \psi\texttt{U''[x] LU}}{\texttt{Jtran + UJtran}}\ \texttt{/. sol}$

Out[501]= 0.096162

In[502]:=
```
VaportoUpperEpidermis =
  (-AUv Dv Cvψ (∂x ψU[x] /. x → LU) - AUv Dv Cvθ (∂x θU[x] /. x → LU)) /
    (Jtran + UJtran) /. sol
```

Out[502]= 0.0520832

In[503]:= $\texttt{LiquidtoUpperEpidermis} = \dfrac{-\texttt{AUl kl}\ \partial_x \psi\texttt{U[x] /. x} \to \texttt{LU}}{\texttt{Jtran + UJtran}}\ \texttt{/. sol}$

Out[503]= 0.242932

In[504]:= $\dfrac{\lambda}{\texttt{QU LU}}\ \dfrac{1}{1}\texttt{(-AUv Dv Cv}\psi\ (\partial_x \psi\texttt{U[x] /. x} \to \texttt{LU) - AUv Dv Cv}\theta\ (\partial_x \theta\texttt{U[x] /. x} \to \texttt{LU)) /. sol}$

Out[504]= 0.0480628

In[505]:= $\dfrac{\lambda}{\texttt{qsu + qru}}\ \dfrac{1}{1}\texttt{(-AUv Dv Cv}\psi\ (\partial_x \psi\texttt{U[x] /. x} \to \texttt{LU) - AUv Dv Cv}\theta\ (\partial_x \theta\texttt{U[x] /. x} \to \texttt{LU)) /. sol}$

Out[505]= 0.426494

In[506]:= `qru /. sol`

Out[506]= 2.23337

## Perivascular vapor

In[507]:=
```
VaporFluxintoLower =
  (-Av Dv Cvψ (∂x ψl[x] /. x → 0) - Av Dv Cvθ (∂x θl[x] /. x → 0)) / (Jtran + UJtran) /.
   sol
```

Out[507]= 0.461393

In[508]:=
```
VaporFluxintoUpper =
  (-AUv Dv Cvψ (∂x ψU[x] /. x → 0) - AUv Dv Cvθ (∂x θU[x] /. x → 0)) /
    (Jtran + UJtran) /. sol
```

Out[508]= -0.0440788

In[509]:= `PerivascularEvap = VaporFluxintoLower + VaporFluxintoUpper`

Out[509]= 0.417314

In[510]:=
```
ProportionPeristomatal + PerivascularEvap + ProportionEvaporationUpper +
  LiquidtoUpperEpidermis + ProportionEvaporationLower (* Check=1 *)
```

Out[510]= 1.



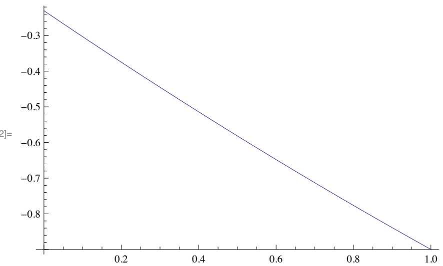
## Plots

### Lower Potential

In[511]:= `psiL = ψl[x] /. sol /. x → L X`

Out[511]= $-0.230212 - 0.603134\,X + 0.132802\left(-X + \frac{X^2}{2}\right)$

In[512]:= `Plot[psiL, {X, 0, 1}]`

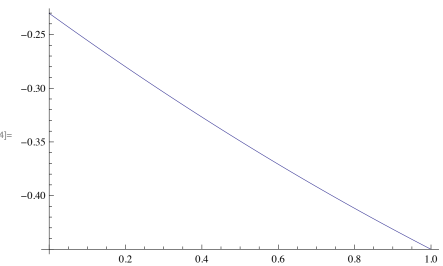
Out[512]=



### Upper Potential

In[513]:= `psiU = ψU[x] /. sol /. x → LU X`

Out[513]= $-0.230212 - 0.183485\,X + 0.0726306\left(-X + \frac{X^2}{2}\right)$

In[514]:= `Plot[psiU, {X, 0, 1}]`

Out[514]=


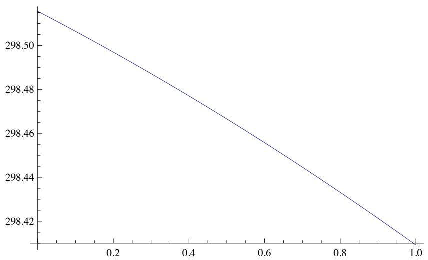
## Lower Thermal

In[515]:=
```
thetaL = θl[x] /. sol /. x → L X
```

Out[515]= $298.516 - 0.12308\, X + 0.0334171 \left(X - \frac{X^2}{2}\right)$

In[516]:=
```
Plot[thetaL, {X, 0, 1}]
```

Out[516]=



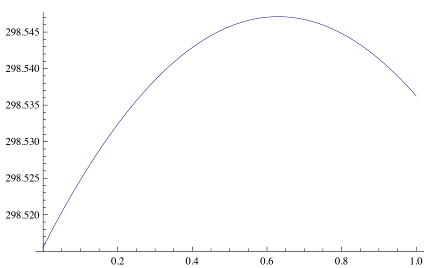

## Upper Thermal

In[517]:=
```
thetaU = θU[x] /. sol /. x → LU X
```

Out[517]= $298.516 - 0.0587283\, X + 0.158864 \left(X - \frac{X^2}{2}\right)$

In[518]:=
```
Plot[thetaU, {X, 0, 1}]
```

Out[518]=



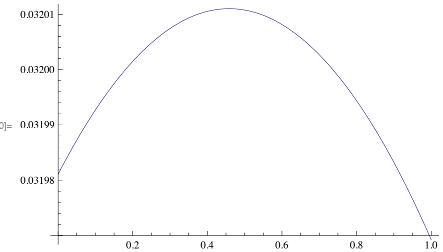

## Upper Vapor Concentration

In[519]:=
```
cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;
```



In[520]:=
```
Plot[cU, {X, 0, 1}]
```

Out[520]=

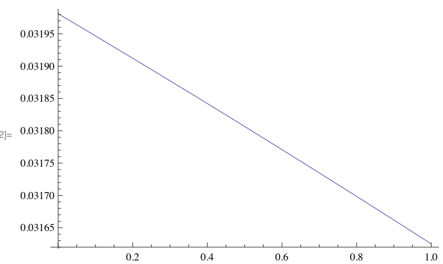



## Lower Vapor Concentration

In[521]:=
```
cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;
```

In[522]:=
```
Plot[cL, {X, 0, 1}]
```

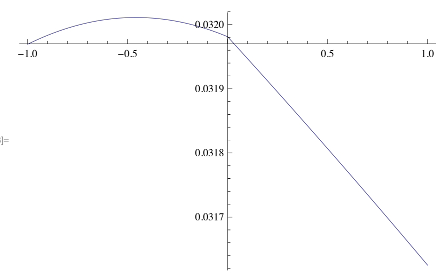
Out[522]=




## Whole leaf vapor

In[523]:=
```
Show[Plot[cU /. X → -z, {z, 0, -1}], Plot[cL, {X, 0, 1}], PlotRange → All]
```

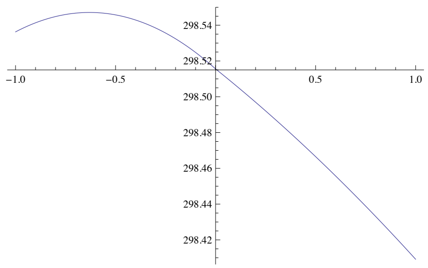
Out[523]=



## Whole leaf temperature

In[524]:=
```
Show[Plot[thetaU /. X → -z, {z, 0, -1}], Plot[thetaL, {X, 0, 1}], PlotRange → All]
```

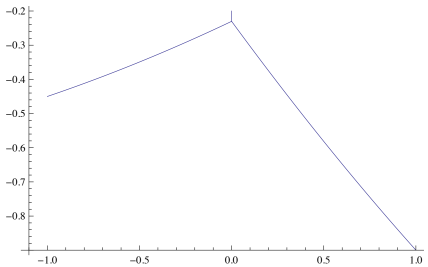
Out[524]=




## Whole leaf potential

In[525]:=
```
Show[Plot[psiU /. X → -z, {z, 0, -1}], Plot[psiL, {X, 0, 1}],
 ListPlot[{{0, ψr}, {0, ψo /. sol}}, Joined → True],
 AxesOrigin → {-1.1, ψle /. sol}, PlotRange → All]
```

Out[525]=




## Export plot data

In[526]:=
```
position = Range[-1, 1, .01];
```

In[527]:=
```
outpos = Table[0, {Length[position]}];
```



In[528]:=
```
potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]
```

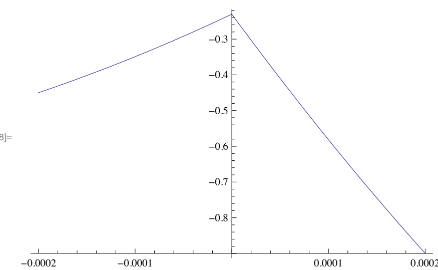
In[535]:=
```
opdata = Transpose[{outpos, potential}];
```

In[536]:=
```
otdata = Transpose[{outpos, temperature}];
```

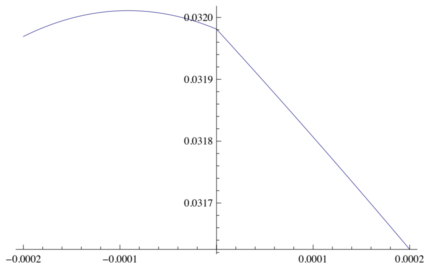
In[537]:=
```
ocdata = Transpose[{outpos, vapor}];
```

In[538]:=
```
ListPlot[opdata, Joined → True]
```

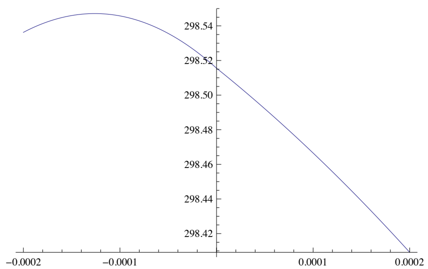
Out[538]=



In[539]:= `ListPlot[ocdata, Joined → True]`

Out[539]=



In[540]:= `ListPlot[otdata, Joined → True]`

Out[540]=



In[541]:= `SetDirectory[ToFileName[NotebookDirectory[]]];`

In[542]:= `(*outfile="sunflower_160_"*)`

In[543]:= `(*Export[outfile<>"potential.xls",opdata]*)`

In[544]:= `(*Export[outfile<>"vapor.xls",ocdata]*)`

In[545]:= `(*Export[outfile<>"temperature.xls",otdata]*)`

# Full mirror illumination of a leaf in gas exchange cuvette

In[546]:= `Remove["Global`*"]`

## Temperature

In[547]:= `θairbase = 273.15 + 25`
`(* air temp preserves true mol fraction if change air temp in cuvette*)`

Out[547]= 298.15

In[548]:= `θair = 273.15 + 25 - .4(* air temp actual after  temp corr *)`

Out[548]= 297.75

In[549]:= `θsur = θair; (* for cuvettes *)`

## Soil dependance of ψr

In[550]:= `ψs = - .2 ;(* stem potential from cov'd leaf in mpa *)`

In[551]:= `ψr = ψs;`

## Other Environmental parameters

In[552]:= `R = 8.3145; (*gas constant Joules per mole per Kelvin*)`

In[553]:= `Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)`

In[554]:= $\chi a[rh\_, \theta\_] := rh \left(1.28\ \frac{R\ 298.15}{Patm}\ Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]\right)$

In[555]:= `RH = .835;`

In[556]:= `χair = χa[RH, θairbase] ; (* chamber mol fraction *)`

In[557]:= `χsat = χa[1, θairbase] ; (* chamber mol fraction *)`

In[558]:= `SRsun = 200;`

### Physical quantities

In[559]:= `Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)`

Out[559]= 0.00317308

In[560]:= `Pa = Patm * 10^-6 (* atm pressure in MPa*)`

Out[560]= 0.1013

In[561]:= $c[\psi\_, \theta\_] := 1.28 \frac{R\,298.15}{Patm} \mathrm{Exp}\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right] \mathrm{Exp}\left[\frac{(\psi + Pa - Po)\ 18.07}{R\ \theta}\right]$ (*mol fraction for psi in MPa*)

In[562]:= $Cv\theta = \partial_\theta\, c[\psi var, \theta]$ /. $\theta \to \theta var$ (* linearization of d$\chi$/dT 1/k *)
$Cv\psi = \partial_\psi\, c[\psi, \theta var]$ /. $\psi \to \psi var$ (* linearization of d$\chi$/dpsi 1/mpa *)

Out[562]= $$0.0313236\, e^{-5291.96\left(-0.00335402+\frac{1}{\theta var}\right)+\frac{2.17331\,(0.0981269+\psi var)}{\theta var}} \left(\frac{5291.96}{\theta var^2} - \frac{2.17331\,(0.0981269 + \psi var)}{\theta var^2}\right)$$

Out[563]= $$\frac{0.0680759\, e^{-5291.96\left(-0.00335402+\frac{1}{\theta var}\right)+\frac{2.17331\,(0.0981269+\psi var)}{\theta var}}}{\theta var}$$

In[564]:= $\lambda = 44\,000 - 43\ (\theta var - 298.15)$ (* Joules per Mol*)

Out[564]= $44\,000 - 43\ (-298.15 + \theta var)$

In[565]:= $Dv = \frac{Patm}{R\,\theta var} 2.13 * \left(\frac{\theta var}{273.15}\right)^{1.8} * 10\,\hat{}\,(-5);$
(* cDv for mol frac drving force out of leaf *)

In[566]:= kvh = .026;(* heat cond air J m-2 s-1 K-1*)

In[567]:= $\sigma = 5.670373 * 10\,\hat{}\,(-8);$ (*stefan boltzmann J/m2/s/kelvin-4 *)

In[568]:= F = 1; (*view factor radiative from leaf*)

## Leaf parameters

In[569]:= $\epsilon l = 0.96$ (* emissivity leaf *)

Out[569]= 0.96

In[570]:= $\alpha l = \epsilon l$ (* long wave abs leaf *)

Out[570]= 0.96

In[571]:= Al = .2;
Av = 1 - Al;

In[573]:= AUl = .8;
AUv = 1 - AUl;

In[575]:= L = 200 × 10^(-6); (*-20 10^(-6)*)

In[576]:= LU = 200 × 10^(-6);

Check whole leaf air fraction

In[577]:= $\frac{L\,Av + LU\,AUv}{L + LU}$ (*vol weighted area fractions *)

Out[577]= 0.5

In[578]:= klh = .2;(* heat cond tissue*)



In[579]:= kl = 1 × 10^(-6); (* hyd cond tissue *)

In[580]:= HA = (0.1); (*mol/m2/s/MPa *)

## Absorbed radiation

In[581]:= Q = 0.1 * SRsun / L ; (* w m-3 volumetric heat source,
.2 is fraction of total abs solar rad abs in spongy *)

In[582]:= QU = 0.9 * SRsun / LU;

900 000.

## Description of fluxes and relations to env parameters

In[583]:= $gbl = \frac{2.13 * \left(\frac{\theta air}{273.15}\right)^{1.8} * 10\,\hat{}\,(-5)\ Patm}{R\,\theta air}$; (* c(tair)*Dv(Tair),
boundary layer molar conductivity *)

In[584]:= $\delta = \left(\frac{2.13 * \left(\frac{\theta air}{273.15}\right)^{1.8} * 10\,\hat{}\,(-5)\ Patm}{R\,\theta air}\right) \Big/ 1.42;$

In[585]:= $gbl / \delta$

Out[585]= 1.42

In[586]:= (*delta is from licor 1.42 mol/m2/s g_bw, d=Dv*C_a/g_bw, C_a=40.49 mol/m3*)

In[587]:= $qsl = \frac{kvh}{\delta}\ (\theta le - \theta air);$

In[588]:= $qsu = \frac{kvh}{\delta}\ (\theta ue - \theta air);$

In[589]:= $qrl = \sigma\,\epsilon l\,\theta le^4 - F\,\alpha l\,\left(\sigma\ \theta sur^4\right);$

In[590]:= $qru = \sigma\,\epsilon l\,\theta ue^4 - F\,\alpha l\,\left(\sigma\ \theta sur^4\right);$

In[591]:= $\chi[\psi\_, \theta\_] := 1.28 \frac{R\,298.15}{Patm} \mathrm{Exp}\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right] \mathrm{Exp}\left[\frac{(\psi + Pa - Po)\ 18.07}{R\ \theta}\right]$
(*for psi in MPa*)

In[592]:= $\chi e = \chi[\psi le, \theta le];$

In[593]:= $\chi ue = \chi[\psi ue, \theta ue];$

In[594]:= $Jtran = \left(\frac{1}{gs} + \frac{\delta}{gbl}\right)^{-1} (\chi e - \chi air);$

In[595]:= $UJtran = \left(\frac{1}{gsu} + \frac{\delta}{gbl}\right)^{-1} (\chi ue - \chi air);$

In[596]:=
```
ψo = ψr - (Jtran + UJtran) / HA;
```

## Stomatal set points

In[597]:=
```
stomalow[ps_] := 0.5363540283333067` + (ps + .9) .6 (*  gs max 1.2 *)
```

In[598]:=
```
stomaup[psu_] := 0.17186009317252962` + (psu + 0.45) .6
```

## Initial values

In[599]:=
```
ψrinit = -.9;
```

In[600]:=
```
ψrinitup = -.45;
```

In[601]:=
```
θvar = θair; (* θair *)
ψvar = ψrinit;  (* ψr *)
```

## First iteration

In[603]:=
$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

Out[603]= 2.77846

In[604]:=
$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

Out[604]= 1.56228

In[605]:=
$$\Pi U\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

Out[605]= 11.727

In[606]:=
$$\Pi U\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\ \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

Out[606]= 1.09322

## Global energy conservation

In[607]:=
```
SRsun == qsu + qru + qrl + qsl + λ Jtran + λ UJtran;
```



## Solution lower thermal field

In[608]:=
$$\theta l[x\_] := \theta o + \left(-\frac{x^2}{2\ L^2} + \frac{x}{L}\right) \frac{Q\ L^2}{\Pi\theta\ (\text{Al klh} + \text{Av kvh})} + \left(-\frac{(\text{qsl} + \text{qrl})\ L}{\Pi\theta\ (\text{Al klh} + \text{Av kvh})} - \frac{\lambda\ \text{Al kl Jtran L}}{\Pi\theta\ (\text{Al klh} + \text{Av kvh})}\ \frac{1}{(\text{Al kl} + \text{Av Dv Cv}\psi)}\right) \frac{x}{L}$$

## Solution upper thermal field

In[609]:=
$$\theta U[x\_] := \theta o + \left(-\frac{x^2}{2\ LU^2} + \frac{x}{LU}\right) \frac{QU\ LU^2}{\Pi U\theta\ (\text{AUl klh} + \text{AUv kvh})} + \left(-\frac{(\text{qsu} + \text{qru})\ LU}{\Pi U\theta\ (\text{AUl klh} + \text{AUv kvh})} - \frac{\lambda\ \text{AUl kl UJtran LU}}{\Pi U\theta\ (\text{AUl klh} + \text{AUv kvh})}\ \frac{1}{(\text{AUl kl} + \text{AUv Dv Cv}\psi)}\right) \frac{x}{LU}$$

## Solution upper potential field

In[610]:=
$$\psi U[x\_] := \psi o + \left(\frac{x^2}{2\ LU^2} - \frac{x}{LU}\right) \frac{QU\ LU^2}{\Pi U\psi\ \text{AUl kl}\ \lambda} + \left(\frac{(\text{qsu} + \text{qru})\ LU}{\Pi U\psi\ \text{AUl kl}\ \lambda} - \frac{\text{UJtran LU}}{\Pi U\psi\ \text{AUl kl}\ \lambda}\ \frac{\text{AUl klh} + \text{AUv kvh}}{\text{AUv Dv Cv}\theta}\right) \frac{x}{LU}$$

## Solution lower potential field

In[611]:=
$$\psi l[x\_] := \psi o + \left(\frac{x^2}{2\ L^2} - \frac{x}{L}\right) \frac{Q\ L^2}{\Pi\psi\ \text{Al kl}\ \lambda} + \left(\frac{(\text{qsl} + \text{qrl})\ L}{\Pi\psi\ \text{Al kl}\ \lambda} - \frac{\text{Jtran L}}{\Pi\psi\ \text{Al kl}\ \lambda}\ \frac{\text{Al klh} + \text{Av kvh}}{\text{Av Dv Cv}\theta}\right) \frac{x}{L}$$

## Solve system

In[612]:=
$$\text{sol} = \text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ \text{SRsun} == \text{qsu} + \text{qru} + \text{qrl} + \text{qsl} + \lambda\ \text{Jtran} + \lambda\ \text{UJtran},\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\psi U[LU]}{\psi ue} == 1,\ \frac{\text{stomalow}[\psi le]}{gs} == 1,\ \frac{\text{stomaup}[\psi ue]}{gsu} == 1\right\},\right.$$
```
{{θo, θair}, {θue, θair}, {θle, θair}, {ψle, ψrinit},
 {ψue, ψrinitup}, {gs, .3}, {gsu, .3}}, PrecisionGoal → 4]
```

Out[612]=
```
{θo → 298.552, θue → 298.572, θle → 298.406,
 ψle → -0.839312, ψue → -0.452081, gs → 0.572767, gsu → 0.170612}
```

## update seed values

In[613]:=
```
θvar = θo /. sol
ψvar = ψo /. sol
```

Out[613]= 298.552

Out[614]= -0.231317

## Second iteration

In[615]:=
```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```
Out[615]= 0.00190297

Out[616]= 0.00023331

In[617]:=
```
Πψ = 1 + (Al klh + Av kvh)/(λ Av Dv Cvθ) +
  (Al klh + Av kvh)/(λ Al kl) Cvψ/Cvθ (* greater than one favors conduction over latent *)
```
Out[617]= 2.73754

In[618]:=
```
Πθ = 1 + (λ Al kl)/(Al klh + Av kvh) (Av Dv Cvθ)/(Al kl + Av Dv Cvψ)
   (* greater than one favors latent over conduction *)
```
Out[618]= 1.57553

In[619]:=
```
ΠUψ = 1 + (AUl klh + AUv kvh)/(λ AUv Dv Cvθ) +
  (AUl klh + AUv kvh)/(λ AUl kl) Cvψ/Cvθ (* greater than one favors conduction over latent *)
```
Out[619]= 11.2499

In[620]:=
```
ΠUθ = 1 + (λ AUl kl)/(AUl klh + AUv kvh) (AUv Dv Cvθ)/(AUl kl + AUv Dv Cvψ)
   (* greater than one favors latent over conduction *)
```
Out[620]= 1.09756

In[621]:=
```
sol = FindRoot[{θU[LU]/θue == 1, θl[L]/θle == 1, SRsun == qsu + qru + qrl + qsl + λ Jtran + λ UJtran,
   ψl[L]/ψle == 1, ψU[LU]/ψue == 1, stomalow[ψle]/gs == 1, stomaup[ψue]/gsu == 1},
  {{θo, θair}, {θue, θair}, {θle, θair}, {ψle, ψrinit},
   {ψue, ψrinitup}, {gs, .3}, {gsu, .3}}, PrecisionGoal → 4]
```
Out[621]= {θo → 298.541, θue → 298.561, θle → 298.398,
 ψle → -0.822721, ψue → -0.451833, gs → 0.582721, gsu → 0.17076}

### update seed values

In[622]:=
```
θvar = θo /. sol
ψvar = ψo /. sol
```
Out[622]= 298.541

Out[623]= -0.231522



## Third iteration

In[624]:=
```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```
Out[624]= 0.00190181

Out[625]= 0.000233159

In[626]:=
```
Πψ = 1 + (Al klh + Av kvh)/(λ Av Dv Cvθ) +
  (Al klh + Av kvh)/(λ Al kl) Cvψ/Cvθ (* greater than one favors conduction over latent *)
```
Out[626]= 2.73806

In[627]:=
```
Πθ = 1 + (λ Al kl)/(Al klh + Av kvh) (Av Dv Cvθ)/(Al kl + Av Dv Cvψ)
   (* greater than one favors latent over conduction *)
```
Out[627]= 1.57536

In[628]:=
```
ΠUψ = 1 + (AUl klh + AUv kvh)/(λ AUv Dv Cvθ) +
  (AUl klh + AUv kvh)/(λ AUl kl) Cvψ/Cvθ (* greater than one favors conduction over latent *)
```
Out[628]= 11.256

In[629]:=
```
ΠUθ = 1 + (λ AUl kl)/(AUl klh + AUv kvh) (AUv Dv Cvθ)/(AUl kl + AUv Dv Cvψ)
   (* greater than one favors latent over conduction *)
```
Out[629]= 1.0975

In[630]:=
```
sol = FindRoot[{θU[LU]/θue == 1, θl[L]/θle == 1, SRsun == qsu + qru + qrl + qsl + λ Jtran + λ UJtran,
   ψl[L]/ψle == 1, ψU[LU]/ψue == 1, stomalow[ψle]/gs == 1, stomaup[ψue]/gsu == 1},
  {{θo, θair}, {θue, θair}, {θle, θair}, {ψle, ψrinit},
   {ψue, ψrinitup}, {gs, .3}, {gsu, .3}}, PrecisionGoal → 4]
```
Out[630]= {θo → 298.541, θue → 298.561, θle → 298.398,
 ψle → -0.822938, ψue → -0.451836, gs → 0.582591, gsu → 0.170758}

### update seed values

In[631]:=
```
θvar = θo /. sol
ψvar = ψo /. sol
```
Out[631]= 298.541

Out[632]= -0.23152

## Fourth iteration

In[633]:= 
```
Cvθ = ∂θ c[ψvar, θ] /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar] /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```
Out[633]= 0.00190182

Out[634]= 0.000233161

In[635]:= $\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\frac{\text{Cv}\psi}{\text{Cv}\theta}$ (* greater than one favors conduction over latent *)

Out[635]= 2.73805

In[636]:= $\frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta}$

Out[636]= 0.890682

In[637]:= $\frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\frac{\text{Cv}\psi}{\text{Cv}\theta}$

Out[637]= 0.847368

In[638]:= $\frac{\text{Cv}\psi}{\text{Cv}\theta}$

Out[638]= 0.122599

In[639]:= $\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$
(* greater than one favors latent over conduction *)

Out[639]= 1.57536

In[640]:= $\Pi U\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\frac{\text{Cv}\psi}{\text{Cv}\theta}$ (* greater than one favors conduction over latent *)

Out[640]= 11.2559

In[641]:= $\Pi U\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$
(* greater than one favors latent over conduction *)

Out[641]= 1.0975



In[642]:= sol = FindRoot[{ $\frac{\theta\text{U[LU]}}{\theta\text{ue}} == 1$, $\frac{\theta\text{l[L]}}{\theta\text{le}} == 1$, SRsun == qsu + qru + qrl + qsl + λ Jtran + λ UJtran, $\frac{\psi\text{l[L]}}{\psi\text{le}} == 1$, $\frac{\psi\text{U[LU]}}{\psi\text{ue}} == 1$, $\frac{\text{stomalow}[\psi\text{le}]}{\text{gs}} == 1$, $\frac{\text{stomaup}[\psi\text{ue}]}{\text{gsu}} == 1$},
{{θo, θair}, {θue, θair}, {θle, θair}, {ψle, ψrinit},
{ψue, ψrinitup}, {gs, .3}, {gsu, .3}}, PrecisionGoal → 4]

Out[642]= {θo → 298.541, θue → 298.561, θle → 298.398,
ψle → -0.822936, ψue → -0.451836, gs → 0.582593, gsu → 0.170758}

### check convergence

In[643]:= θvar - θo /. sol

Out[643]= $1.92132 \times 10^{-6}$

In[644]:= ψvar - ψo /. sol

Out[644]= $3.44654 \times 10^{-8}$

## Results

In[645]:= MolFracs =
{(χe - χair), (χue - χair), χe, χue, χair} /. sol (* target at Q160 is .0054 *)

Out[645]= {0.00546716, 0.0058614, 0.0316224, 0.0320166, 0.0261552}

In[646]:= Temps = {θair, θsur, θue, θo, θle, θle - θair} /. sol

Out[646]= {297.75, 297.75, 298.561, 298.541, 298.398, 0.648344}

In[647]:= Potentials = {ψr, ψo, ψue, ψle} /. sol

Out[647]= {-0.2, -0.23152, -0.451836, -0.822936}

In[648]:= gs /. sol (* target at Q160 is .7 *)

Out[648]= 0.582593

In[649]:= gsu /. sol

Out[649]= 0.170758

In[650]:= totalgs = gs + gsu /. sol

Out[650]= 0.753351

### Fluxes

In[651]:= qsu /. sol (*Sensible flux upper*)

Out[651]= 29.4297

In[652]:= `qru /. sol (*Radiative flux upper*)`

Out[652]= 4.68284

In[653]:= `qsl /. sol (*Sensible flux lower*)`

Out[653]= 23.5155

In[654]:= `qrl /. sol (*Radiative flux lower*)`

Out[654]= 3.73871

In[655]:= `latentlow = λ Jtran /. sol`

Out[655]= 99.3367

In[656]:= `latentup = λ UJtran /. sol`

Out[656]= 39.2965

In[657]:= `totallatent = latentlow + latentup`

Out[657]= 138.633

In[658]:= `tranlow = Jtran /. sol (* target at Q160 is .003 *)`

Out[658]= 0.00225851

In[659]:= `tranup = UJtran /. sol`

Out[659]= 0.000893443

In[660]:= `totaltran = tranlow + tranup`

Out[660]= 0.00315196

## Matching Q160

In[661]:= `totaltran (* expect about .0031 *)`

Out[661]= 0.00315196

In[662]:= `blMFdrop = totaltran / 2.84`

Out[662]= 0.00110984

In[663]:= `χsurf = χair + blMFdrop (* app mol frac at leaf surface *)`

Out[663]= 0.027265

In[664]:= `appgradientLS = χe - χsurf /. sol (* app MF grad at leaf surf exp .0044 *)`

Out[664]= 0.00435732

In[665]:= $appgst = \left(\frac{(\chi e - \chi air)}{totaltran} - \frac{1}{2.84}\right)^{-1}$ `/. sol(* 0.7 *)`

Out[665]= 0.723371

## Evaporation distribution percent of totals

In[666]:= $Liquidtolower = \frac{-Al\ kl\ \partial_x \psi l[x]\ /.\ x \to L}{Jtran + UJtran}$ `/. sol`

Out[666]= 0.16129

In[667]:= $Liquidtoupper = \frac{-AUl\ kl\ \partial_x \psi U[x]\ /.\ x \to LU}{Jtran + UJtran}$ `/. sol`

Out[667]= 0.221918

In[668]:= `VaportoLower =`
`(-Av Dv Cvψ (∂x ψl[x] /. x → L) - Av Dv Cvθ (∂x θl[x] /. x → L)) / (Jtran + UJtran) /.`
`sol`

Out[668]= 0.555253

In[669]:= `VaportoUpper = (-AUv Dv Cvψ (∂x ψU[x] /. x → LU) - AUv Dv Cvθ (∂x θU[x] /. x → LU)) /`
`(Jtran + UJtran) /. sol`

Out[669]= 0.061539

In[670]:= `(Liquidtolower + Liquidtoupper + VaportoLower + VaportoUpper)`

Out[670]= 1.

### Condensation on lower epidermis if following is larger than one:

In[671]:= $\frac{(qsl + qrl)}{Jtran\ \lambda} \frac{\lambda\ Av\ Dv\ Cv\theta}{Al\ klh + Av\ kvh}$ `/. sol`

Out[671]= 0.308036

### Condensation on upper epidermis if following is larger than one:

In[672]:= $\frac{(qsu + qru)}{UJtran\ \lambda} \frac{\lambda\ AUv\ Dv\ Cv\theta}{AUl\ klh + AUv\ kvh}$ `/. sol`

Out[672]= 0.0896749

## Leaf water potential and apparent conductance

In[673]:= $LowerAvgPotential = \frac{1}{L} \int_0^L \psi l[x]\, dx$ `/. sol`

Out[673]= -0.541067

In[674]:= $UpperAvgPotential = \frac{1}{LU} \int_0^{LU} \psi U[x]\, dx$ `/. sol`

Out[674]= -0.349253

In[675]:= $$\text{LWP} = \frac{\text{L LowerAvgPotential} + \text{LU UpperAvgPotential}}{\text{L} + \text{LU}}$$

Out[675]= -0.44516

### Naive view of gradients

In[676]:= $$\text{Kleaf} = \frac{\text{Jtran}}{\psi\text{r} - \text{LWP}}\ \text{/. sol}$$ (* Apparent conductance, mol m-2 s-1 MPa-1*)

Out[676]= 0.00921241

## Analysis

### Analyze proportion peristomatal at Lower Epidermis

In[677]:= $$\text{ProportionPeristomatal} = \frac{-\text{Al kl}\ \partial_x \psi\text{l}[x]\ \text{/. } x \to \text{L}}{\text{Jtran} + \text{UJtran}}\ \text{/. sol}$$

Out[677]= 0.16129

In[678]:= $$\text{PropPeri} = \frac{-(\text{qsl} + \text{qrl})}{\Pi\psi\ \text{Jtran}\ \lambda} + \left(1 + \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1}\ \text{/. sol}$$

Out[678]= 0.225094

In[679]:= $$\frac{-(\text{qsl} + \text{qrl})}{\Pi\psi\ \text{Jtran}\ \lambda}\ \text{/. sol}$$

Out[679]= -0.100204

In[680]:= $$\left(1 + \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1}\ \text{/. sol}$$

Out[680]= 0.325298

### Evaporation in lower

In[681]:= $$\text{ProportionEvaporationLower} = \frac{\text{Al kl}\ \psi\text{l}''[x]\ \text{L}}{\text{Jtran} + \text{UJtran}}\ \text{/. sol}$$

Out[681]= 0.0526892

In[682]:= $$\frac{\text{Q L}}{\Pi\psi\ \lambda\ \text{Jtran}}\ \text{/. sol}$$ (*integrating evap 2nd derv over L *)

Out[682]= 0.0735324



### Analyze evaporation in upper part of leaf

In[683]:= $$\text{ProportionEvaporationUpper} = \frac{\text{AUl kl}\ \psi\text{U}''[x]\ \text{LU}}{\text{Jtran} + \text{UJtran}}\ \text{/. sol}$$

Out[683]= 0.115352

In[684]:= $$\text{VaportoUpperEpidermis} = (-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[x]\ \text{/. } x \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[x]\ \text{/. } x \to \text{LU})) / (\text{Jtran} + \text{UJtran})\ \text{/. sol}$$

Out[684]= 0.061539

In[685]:= $$\text{LiquidtoUpperEpidermis} = \frac{-\text{AUl kl}\ \partial_x \psi\text{U}[x]\ \text{/. } x \to \text{LU}}{\text{Jtran} + \text{UJtran}}\ \text{/. sol}$$

Out[685]= 0.221918

In[686]:= $$\frac{\lambda}{\text{QU LU}}\ \frac{1}{1}(-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[x]\ \text{/. } x \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[x]\ \text{/. } x \to \text{LU}))\ \text{/. sol}$$

Out[686]= 0.0473963

In[687]:= $$\frac{\lambda}{\text{qsu} + \text{qru}}\ \frac{1}{1}(-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[x]\ \text{/. } x \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[x]\ \text{/. } x \to \text{LU}))\ \text{/. sol}$$

Out[687]= 0.250094

In[688]:= qru /. sol

Out[688]= 4.68284

### Perivascular vapor

In[689]:= $$\text{VaporFluxintoLower} = (-\text{Av Dv Cv}\psi\ (\partial_x \psi\text{l}[x]\ \text{/. } x \to 0) - \text{Av Dv Cv}\theta\ (\partial_x \theta\text{l}[x]\ \text{/. } x \to 0)) / (\text{Jtran} + \text{UJtran})\ \text{/. sol}$$

Out[689]= 0.502564

In[690]:= $$\text{VaporFluxintoUpper} = (-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[x]\ \text{/. } x \to 0) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[x]\ \text{/. } x \to 0)) / (\text{Jtran} + \text{UJtran})\ \text{/. sol}$$

Out[690]= -0.0538129

In[691]:= PerivascularEvap = VaporFluxintoLower + VaporFluxintoUpper

Out[691]= 0.448751

In[692]:= ProportionPeristomatal + PerivascularEvap + ProportionEvaporationUpper + LiquidtoUpperEpidermis + ProportionEvaporationLower (* Check=1 *)

Out[692]= 1.

# Plots

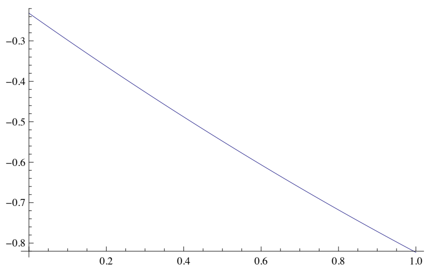
## Lower Potential

In[693]:=
```
psiL = ψl[x] /. sol /. x → L X
```

Out[693]= $-0.23152 - 0.508379\, X + 0.166074 \left(-X + \frac{X^2}{2}\right)$

In[694]:=
```
Plot[psiL, {X, 0, 1}]
```

Out[694]=



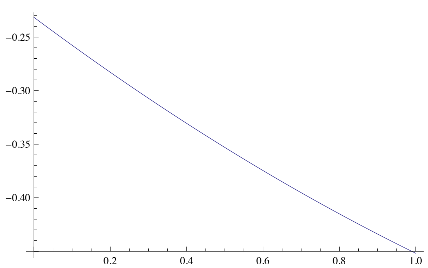

## Upper Potential

In[695]:=
```
psiU = ψU[x] /. sol /. x → LU X
```

Out[695]= $-0.23152 - 0.174869\, X + 0.0908961 \left(-X + \frac{X^2}{2}\right)$

In[696]:=
```
Plot[psiU, {X, 0, 1}]
```

Out[696]=





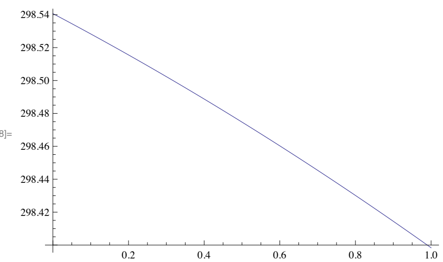

## Lower Thermal

In[697]:=
```
thetaL = θl[x] /. sol /. x → L X
```

Out[697]= $298.541 - 0.163205\, X + 0.0417616 \left(X - \frac{X^2}{2}\right)$

In[698]:=
```
Plot[thetaL, {X, 0, 1}]
```

Out[698]=



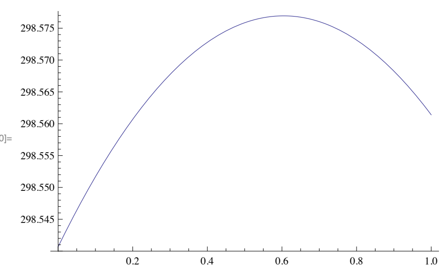

## Upper Thermal

In[699]:=
```
thetaU = θU[x] /. sol /. x → LU X
```

Out[699]= $298.541 - 0.0785445\, X + 0.198557 \left(X - \frac{X^2}{2}\right)$

In[700]:=
```
Plot[thetaU, {X, 0, 1}]
```

Out[700]=

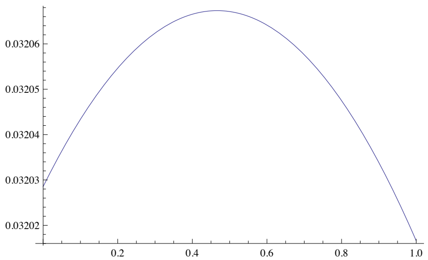



## Upper Vapor Concentration

In[701]:=
```
cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;
```

In[702]:=
```
Plot[cU, {X, 0, 1}]
```

Out[702]=

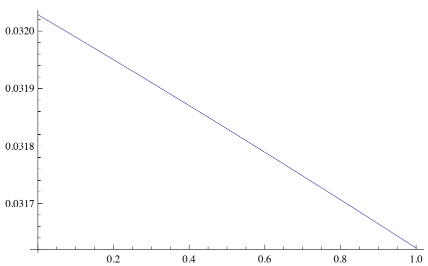



## Lower Vapor Concentration

In[703]:=
```
cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;
```

In[704]:=
```
Plot[cL, {X, 0, 1}]
```

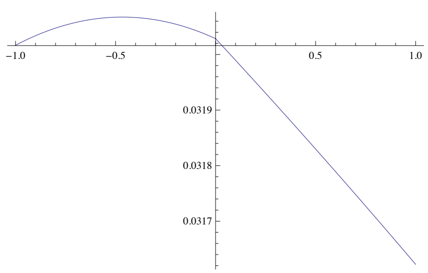
Out[704]=




## Whole leaf vapor

In[705]:=
```
Show[Plot[cU /. X → -z, {z, 0, -1}], Plot[cL, {X, 0, 1}], PlotRange → All]
```

Out[705]=

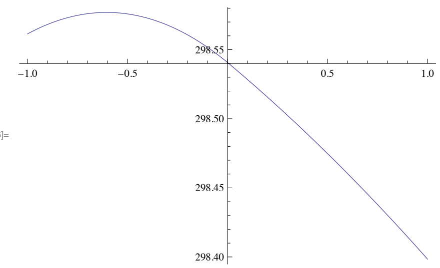





## Whole leaf temperature

In[706]:=
```
Show[Plot[thetaU /. X → -z, {z, 0, -1}], Plot[thetaL, {X, 0, 1}], PlotRange → All]
```

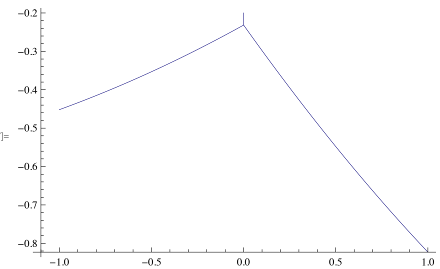
Out[706]=




## Whole leaf potential

In[707]:=
```
Show[Plot[psiU /. X → -z, {z, 0, -1}], Plot[psiL, {X, 0, 1}],
 ListPlot[{{0, ψr}, {0, ψo /. sol}}, Joined → True],
 AxesOrigin → {-1.1, ψle /. sol}, PlotRange → All]
```

Out[707]=




## Export plot data

In[708]:=
```
position = Range[-1, 1, .01];
```

In[709]:=
```
outpos = Table[0, {Length[position]}];
```

In[710]:=
```
potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]
```

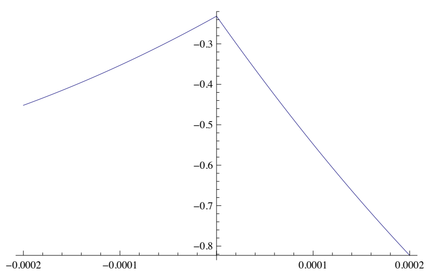
In[717]:=
```
opdata = Transpose[{outpos, potential}];
```

In[718]:=
```
otdata = Transpose[{outpos, temperature}];
```

In[719]:=
```
ocdata = Transpose[{outpos, vapor}];
```

In[720]:=
```
ListPlot[opdata, Joined → True]
```

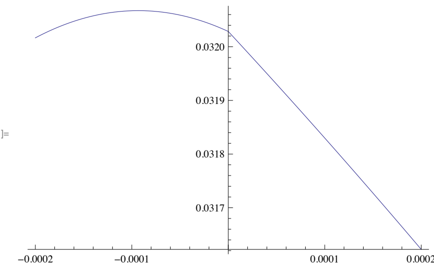
Out[720]=





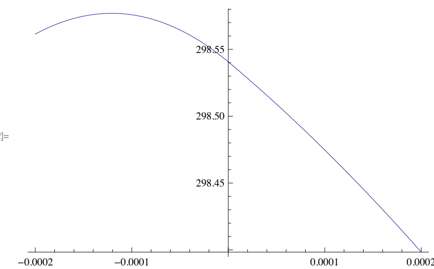

In[721]:=
```
ListPlot[ocdata, Joined → True]
```

Out[721]=




In[722]:=
```
ListPlot[otdata, Joined → True]
```

Out[722]=




In[723]:=
```
(*SetDirectory[ToFileName[NotebookDirectory[]]]*)
```

In[724]:=
```
(*outfile="sunflower_200TC_"*)
```

In[725]:=
```
(*Export[outfile<>"potential.xls",opdata]*)
```

In[726]:=
```
(*Export[outfile<>"vapor.xls",ocdata]*)
```

In[727]:=
```
(*Export[outfile<>"temperature.xls",otdata]*)
```