Sample code for sunflower simulations with epidermal potential setpoints
(vary parameters as necessary)

```
Remove["Global`*"]
```

Stomatal set points

```
ψle = -.419(* conserved value based on numerical exposed leaf sol, gs varies *)
```

```
-0.419
```

```
ψue  =  -.419
```

```
-0.419
```

Initial values

```
ψrinit = -1.29;
```

```
θvar = θair; (* θair *)
ψvar = ψrinit;  (* ψr *)
```

Temperature

```
θairbase = 273.15 + 25
  (* air temp preserves true mol fraction if change air temp in cuvette*)
```

```
298.15
```

```
θair = 273.15 + 25(* air temp actual after  temp corr *)
```

```
298.15
```

```
θsur = θair; (* for cuvettes *)
```

Soil dependance of ψr

```
ψs = -.2 ;(* stem potential from cov'd leaf in mpa *)
```

```
ψr = ψs
```

```
-0.2
```

Other Environmental parameters

```
R = 8.3145; (*gas constant Joules per mole per Kelvin*)
```

```
Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)
```



```
χa[rh_, θ_] := rh (1.28 (R 298.15)/Patm Exp[-44000/R (1/θ - 1/298.15)])
```

```
RH = .835
```

```
0.835
```

```
χair = χa[RH, θairbase] (* chamber mol fraction *)
```

```
0.0261552
```

```
χsat = χa[1, θairbase] (* chamber mol fraction *)
```

```
0.0313236
```

```
SRsun = 160(*αsr  (1/.45*PARout*10^(-6) * 2.35*10^5 ) (1+r)-AE -FE*)
(* neglect refelctance *)
(* coeff J mol-1,
from Campbell intro env biophysics p 151: Total E incident is about 2x E in PAR,
then typical leaf abs is about half that amount,
so PAR plus NIR about equal to all E in PAR. OR 2.22 =1/.45 is factor
  mult PAR capmbell but also could subtract ps E at 479 kj per mol co2,
and then fluorescence too. ps here is 20 umol m-2 s-2 so ~10 watts*)
```

```
160
```

Physical quantities

```
Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)
```

```
0.00317308
```

```
Pa = Patm * 10^-6 (* atm pressure in MPa*)
```

```
0.1013
```

```
c[ψ_, θ_] := 1.28 (R 298.15)/Patm Exp[-44000/R (1/θ - 1/298.15)]
  Exp[((ψ + Pa - Po) 18.07)/(R θ)] (*mol fraction for psi in MPa*)
```

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

```
0.00184951
```

```
0.000226353
```

```
λ = 44000 - 43 (θvar - 298.15) (* Joules per Mol*)
```

```
44000.
```

```
Dv = Patm/(R θvar) 2.13 * (θvar/273.15)^1.8 *
  10^(-5) (* cDv for mol frac drving force out of leaf *)
```
0.00101901

```
kvh = .026; (* heat cond air J m-2 s-1 K-1*)

σ = 5.670373 * 10^(-8); (*stefan boltzmann J/m2/s/kelvin-4 *)

F = 1; (*view factor radiative from leaf*)
```

Leaf parameters

```
ϵl = 0.96 (* emissivity leaf *)
```
0.96

```
αl = ϵl (* long wave abs leaf *)
```
0.96

```
Al = .7 (* .1263 oakair fraction, say 0.5 just mesophyll*)
Av = 1 - Al; (*Area fraction tissue in leaf from 2011 data 12.63%*)
```
0.7

```
AUl = .7; (* .1263 oakair fraction, say 0.5 just mesophyll*)
AUv = 1 - AUl; (*Area fraction tissue in leaf from 2011 data 12.63%*)

L = 100 × 10^(-6) (*-20 10^(-6)*)
```
$\frac{1}{10\,000}$

```
LU = 100 × 10^(-6)
```
$\frac{1}{10\,000}$

Check whole leaf air fraction

```
(L Av + LU AUv)/(L + LU) (*vol weighted area fractions *)
```
0.3

```
klh = .2; (* .28614 heat cond tissue*)

kl =
 1 × 10^(-6) (* hyd cond tissue recon from on tree hydration (leaf at air temp)*)
```
$\frac{1}{1\,000\,000}$

```
HA = (0.1) (*mol/m2/s/MPa from vein cutting, no steady state 1D scale factor *)
```
0.1

Absorbed radiation

```
Q = 0.1 * SRsun / L (* w m-3 volumetric heat source,
   .2 is fraction of total abs solar rad abs in spongy *)
```
160 000.

```
QU = 0.9 * SRsun / LU
```
1.44×10^6

```
Q L (* total solar in lower half of leaf*)
```
16.

```
QU LU
```
144.

```
Q L + QU LU - SRsun
```
0.

Description of fluxes and relations to env parameters

```
gbl = (2.13 * (θair/273.15)^1.8 * 10^(-5) Patm)/(R θair)
  (* c(tair)*Dv(Tair), boundary layer molar conductivity *)
```
0.00101901

```
δ = ((2.13 * (θair/273.15)^1.8 * 10^(-5) Patm)/(R θair)) / 1.42
```
0.000717613

```
gbl / δ
```
1.42

```
(*delta is from licor 1.42 mol/m2/s g_bw, d=Dv*C_a/g_bw, C_a=40.49 mol/m3*)

kvha = 28 * 1.42 * δ; (* ext heat cond air berry*)

qsl = kvh/δ (θle - θair);

qsu = kvh/δ (θue - θair);

qrl = σ ϵl θle^4 - F αl (σ θsur^4);

qru = σ ϵl θue^4 - F αl (σ θsur^4);
```

$$\chi[\psi\_,\ \theta\_] := 1.28\ \frac{R\ 298.15}{Patm}\ Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]\ Exp\left[\frac{(\psi+Pa-Po)\ 18.07}{R\ \theta}\right]$$

(*for psi in MPa*)

$\chi e = \chi[\psi le,\ \theta le];$

$\chi ue = \chi[\psi ue,\ \theta ue];$

$$Jtran = \left(\frac{1}{gs}+\frac{\delta}{gbl}\right)^{-1}(\chi e - \chi air);$$

$$UJtran = \left(\frac{1}{gsu}+\frac{\delta}{gbl}\right)^{-1}(\chi ue - \chi air);$$

$\psi o = \psi r - (Jtran + UJtran)\ /\ HA$

$$-0.2-10.\left(\frac{-0.0261552+0.0313236\ e^{-5291.96\left(-0.00335402+\frac{1}{\theta le}\right)-\frac{0.697357}{\theta le}}}{0.704225+\frac{1}{gs}}+\frac{-0.0261552+0.0313236\ e^{-5291.96\left(-0.00335402+\frac{1}{\theta ue}\right)-\frac{0.697357}{\theta ue}}}{0.704225+\frac{1}{gsu}}\right)$$

First iteration

$$\Pi\psi = 1+\frac{Al\ klh+Av\ kvh}{\lambda\ Av\ Dv\ Cv\theta}+\frac{Al\ klh+Av\ kvh}{\lambda\ Al\ kl}\ \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

7.52835

$$\Pi\theta = 1+\frac{\lambda\ Al\ kl}{Al\ klh+Av\ kvh}\ \frac{Av\ Dv\ Cv\theta}{Al\ kl+Av\ Dv\ Cv\psi}$$ (* greater than one favors latent over conduction *)

1.15318

$$\Pi U\psi = 1+\frac{AUl\ klh+AUv\ kvh}{\lambda\ AUv\ Dv\ Cv\theta}+\frac{AUl\ klh+AUv\ kvh}{\lambda\ AUl\ kl}\ \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

7.52835

$$\Pi U\theta = 1+\frac{\lambda\ AUl\ kl}{AUl\ klh+AUv\ kvh}\ \frac{AUv\ Dv\ Cv\theta}{AUl\ kl+AUv\ Dv\ Cv\psi}$$ (* greater than one favors latent over conduction *)

1.15318

Global energy conservation

$SRsun == qsu + qru + qrl + qsl + \lambda\ Jtran + \lambda\ UJtran;$

Solution lower thermal field

$$\theta l[x\_] := \theta o + \left(-\frac{x^2}{2\ L^2}+\frac{x}{L}\right)\frac{Q\ L^2}{\Pi\theta\ (Al\ klh+Av\ kvh)}+\left(-\frac{(qsl+qrl)\ L}{\Pi\theta\ (Al\ klh+Av\ kvh)}-\frac{\lambda\ Al\ kl\ Jtran\ L}{\Pi\theta\ (Al\ klh+Av\ kvh)}\ \frac{1}{(Al\ kl+Av\ Dv\ Cv\psi)}\right)\frac{x}{L}$$

Solution upper thermal field

$$\theta U[x\_] := \theta o + \left(-\frac{x^2}{2\ LU^2}+\frac{x}{LU}\right)\frac{QU\ LU^2}{\Pi U\theta\ (AUl\ klh+AUv\ kvh)}+\left(-\frac{(qsu+qru)\ LU}{\Pi U\theta\ (AUl\ klh+AUv\ kvh)}-\frac{\lambda\ AUl\ kl\ UJtran\ LU}{\Pi U\theta\ (AUl\ klh+AUv\ kvh)}\ \frac{1}{(AUl\ kl+AUv\ Dv\ Cv\psi)}\right)\frac{x}{LU}$$

Solution upper potential field

$$\psi U[x\_] := \psi o + \left(\frac{x^2}{2\ LU^2}-\frac{x}{LU}\right)\frac{QU\ LU^2}{\Pi U\psi\ AUl\ kl\ \lambda}+\left(\frac{(qsu+qru)\ LU}{\Pi U\psi\ AUl\ kl\ \lambda}-\frac{UJtran\ LU}{\Pi U\psi\ AUl\ kl\ \lambda}\ \frac{AUl\ klh+AUv\ kvh}{AUv\ Dv\ Cv\theta}\right)\frac{x}{LU}$$

Solution lower potential field

$$\psi l[x\_] := \psi o + \left(\frac{x^2}{2\ L^2}-\frac{x}{L}\right)\frac{Q\ L^2}{\Pi\psi\ Al\ kl\ \lambda}+\left(\frac{(qsl+qrl)\ L}{\Pi\psi\ Al\ kl\ \lambda}-\frac{Jtran\ L}{\Pi\psi\ Al\ kl\ \lambda}\ \frac{Al\ klh+Av\ kvh}{Av\ Dv\ Cv\theta}\right)\frac{x}{L}$$

Solve system

$$sol = FindRoot\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu+qru+qrl+qsl+\lambda\ Jtran+\lambda\ UJtran,\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\psi U[LU]}{\psi ue} == 1\right\},\ \{\{\theta o,\ \theta air\},\ \{\theta ue,\ \theta air\},\ \{\theta le,\ \theta air\},\ \{gs,\ .3\},\ \{gsu,\ .3\}\},\ PrecisionGoal \to 4\right]$$

$\{\theta o \to 298.443,\ \theta ue \to 298.444,\ \theta le \to 298.402,\ gs \to 0.382073,\ gsu \to 0.31033\}$

update seed values

```
θvar = θo /. sol
ψvar = ψo /. sol
```

298.443

-0.231144

Second iteration

```
Cvθ = ∂θ c[ψvar, θ] /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar] /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00189208

0.000231891

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

7.39552

$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

1.15636

$$\Pi U\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

7.39552

$$\Pi U\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\ \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

1.15636

$$\text{sol} = \text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ \text{SRsun} == \text{qsu} + \text{qru} + \text{qrl} + \text{qsl} + \lambda\ \text{Jtran} + \lambda\ \text{UJtran},\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\psi U[LU]}{\psi ue} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{gs, .3\}, \{gsu, .3\}\},\ \text{PrecisionGoal} \to 4\right]$$

{θo → 298.439, θue → 298.44, θle → 298.398, gs → 0.384842, gsu → 0.3113}

update seed values

```
θvar = θo /. sol
ψvar = ψo /. sol
```

298.439

-0.23123

Third iteration

```
Cvθ = ∂θ c[ψvar, θ] /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar] /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00189162

0.000231831

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

7.39693

$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

1.15632

$$\Pi U\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

7.39693

$$\Pi U\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\ \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

1.15632

$$\text{sol} = \text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ \text{SRsun} == \text{qsu} + \text{qru} + \text{qrl} + \text{qsl} + \lambda\ \text{Jtran} + \lambda\ \text{UJtran},\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\psi U[LU]}{\psi ue} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{gs, .3\}, \{gsu, .3\}\},\ \text{PrecisionGoal} \to 4\right]$$

{θo → 298.439, θue → 298.44, θle → 298.398, gs → 0.384812, gsu → 0.311289}

update seed values

```
θvar = θo /. sol
ψvar = ψo /. sol
```

298.439

-0.231229

Fourth iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00189163

0.000231831

$$\Pi\psi = 1 + \frac{Al\ klh + Av\ kvh}{\lambda\ Av\ Dv\ Cv\theta} + \frac{Al\ klh + Av\ kvh}{\lambda\ Al\ kl}\ \frac{Cv\psi}{Cv\theta}$$

(* greater than one favors conduction over latent *)

7.39692

$$\frac{Al\ klh + Av\ kvh}{\lambda\ Av\ Dv\ Cv\theta}$$

5.8088

$$\frac{Al\ klh + Av\ kvh}{\lambda\ Al\ kl}\ \frac{Cv\psi}{Cv\theta}$$

0.588113

$$\frac{Cv\psi}{Cv\theta}$$

0.122557

$$\Pi\theta = 1 + \frac{\lambda\ Al\ kl}{Al\ klh + Av\ kvh}\ \frac{Av\ Dv\ Cv\theta}{Al\ kl + Av\ Dv\ Cv\psi}$$

(* greater than one favors latent over conduction *)

1.15633

$$\Pi U\psi = 1 + \frac{AUl\ klh + AUv\ kvh}{\lambda\ AUv\ Dv\ Cv\theta} + \frac{AUl\ klh + AUv\ kvh}{\lambda\ AUl\ kl}\ \frac{Cv\psi}{Cv\theta}$$

(* greater than one favors conduction over latent *)

7.39692

$$\Pi U\theta = 1 + \frac{\lambda\ AUl\ kl}{AUl\ klh + AUv\ kvh}\ \frac{AUv\ Dv\ Cv\theta}{AUl\ kl + AUv\ Dv\ Cv\psi}$$

(* greater than one favors latent over conduction *)

1.15633

$$sol = FindRoot\Big[\Big\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\ Jtran + \lambda\ UJtran,\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\psi U[LU]}{\psi ue} == 1\Big\},$$

```
{{θo, θair}, {θue, θair}, {θle, θair}, {gs, .3}, {gsu, .3}}, PrecisionGoal → 4]
```

{θo → 298.439, θue → 298.44, θle → 298.398, gs → 0.384812, gsu → 0.311289}

check convergence

```
θvar - θo /. sol
```

5.3918×10^{-7}

```
ψvar - ψo /. sol
```

9.97454×10^{-9}

Results

```
MolFracs =
 {(χe - χair), (χue - χair), χe, χue, χair} /. sol (* target at Q160 is .0054 *)
```

{0.00555939, 0.00563885, 0.0317146, 0.031794, 0.0261552}

```
ψue = ψU[LU] /. sol (* Water potential upper epidermis *)
```

-0.419

```
Temps = {θair, θsur, θue, θo, θle, θle - θair} /. sol
```

{298.15, 298.15, 298.44, 298.439, 298.398, 0.247845}

```
Potentials = {ψr, ψo, ψue, ψle} /. sol
```

{-0.2, -0.231229, -0.419, -0.419}

```
gs /. sol (* target at Q160 is .7 *)
```

0.384812

```
gsu /. sol
```

0.311289

```
totalgs = gs + gsu /. sol
```

0.696102

Fluxes

```
qsu /. sol (*Sensible flux upper*)
```
10.5052

```
qru /. sol (*Radiative flux upper*)
```
1.67573

```
qsl /. sol (*Sensible flux lower*)
```
8.97971

```
qrl /. sol (*Radiative flux lower*)
```
1.43208

```
latentlow = λ Jtran /. sol
```
74.0602

```
latentup = λ UJtran /. sol
```
63.347

```
totallatent = latentlow + latentup
```
137.407

```
tranlow = Jtran /. sol (* target at Q160 is .003 *)
```
0.00168319

```
tranup = UJtran /. sol
```
0.00143971

```
totaltran = tranlow + tranup
```
0.00312289

Matching Q160

```
totaltran (* expect about .0031 *)
```
0.00312289

```
blMFdrop = totaltran / 2.84
```
0.00109961

```
χsurf = χair + blMFdrop (* app mol frac at leaf surface *)
```
0.0272548

```
appgradientLS = χe - χsurf /. sol (* app MF grad at leaf surf exp .0044 *)
```
0.00445978

$$\text{appgst} = \left(\frac{(\chi e - \chi air)}{\text{totaltran}} - \frac{1}{2.84}\right)^{-1} \text{ /. sol(* 0.7 *)}$$

0.700234

Evaporation distribution percent of totals

$$\text{Liquidtolower} = \frac{-\text{Al kl } \partial_x \psi l[x] \text{ /. } x \to L}{\text{Jtran + UJtran}} \text{ /. sol}$$

0.41302

$$\text{Liquidtoupper} = \frac{-\text{AUl kl } \partial_x \psi U[x] \text{ /. } x \to LU}{\text{Jtran + UJtran}} \text{ /. sol}$$

0.350052

$$\text{VaportoLower} = \frac{1}{\text{Jtran + UJtran}} (-\text{Av Dv Cv}\psi \; (\partial_x \psi l[x] \text{ /. } x \to L) - \text{Av Dv Cv}\theta \; (\partial_x \theta l[x] \text{ /. } x \to L)) \text{ /. sol}$$

0.125963

$$\text{VaportoUpper} = \frac{1}{\text{Jtran + UJtran}} (-\text{AUv Dv Cv}\psi \; (\partial_x \psi U[x] \text{ /. } x \to LU) - \text{AUv Dv Cv}\theta \; (\partial_x \theta U[x] \text{ /. } x \to LU)) \text{ /. sol}$$

0.110964

```
(Liquidtolower + Liquidtoupper + VaportoLower + VaportoUpper)
```
1.

Condensation on lower epidermis if following is larger than one:

$$\frac{(\text{qsl + qrl})}{\text{Jtran } \lambda} \frac{\lambda \text{ Av Dv Cv}\theta}{\text{Al klh + Av kvh}} \text{ /. sol}$$

0.0242022

Condensation on upper epidermis if following is larger than one:

$$\frac{(\text{qsu + qru})}{\text{UJtran } \lambda} \frac{\lambda \text{ AUv Dv Cv}\theta}{\text{AUl klh + AUv kvh}} \text{ /. sol}$$

0.0331031

Leaf water potential and apparent conductance

$$\text{LowerAvgPotential} = \frac{1}{L} \int_0^L \psi l[x] \, dx \text{ /. sol}$$

-0.3257

```
UpperAvgPotential = 1/LU ∫_0^LU ψU[x] ⅆx /. sol
```
-0.330382

```
LWP = (L LowerAvgPotential + LU UpperAvgPotential)/(L + LU)
```
-0.328041

Naive view of gradients

```
Kleaf = Jtran/(ψr - LWP) /. sol (* Apparent conductance, mol m-2 s-1 MPa-1*)
```
0.0131457

Analysis

Analyze proportion peristomatal at Lower Epidermis

```
ProportionPeristomatal = (-Al kl ∂x ψl[x] /. x → L)/Jtran /. sol
```
0.766295

```
PropPeri = -(qsl + qrl)/(Πψ Jtran λ) + (1 + λ Av Dv Cvθ/(Al klh + Av kvh) + Av Dv Cvψ/(Al kl))^-1 /. sol
```
0.766295

```
-(qsl + qrl)/(Πψ Jtran λ) /. sol
```
-0.019006

```
(1 + λ Av Dv Cvθ/(Al klh + Av kvh) + Av Dv Cvψ/(Al kl))^-1 /. sol
```
0.785301

Evaporation in lower

```
ProportionEvaporationLower = (Al kl ψl''[x] L)/Jtran /. sol
```
0.0292068

```
Q L/(Πψ λ Jtran) /. sol (*integrating evap 2nd derv over L *)
```
0.0292068

Analyze evaporation in upper part of leaf

```
(-AUl kl ∂x ψU[x] /. x → LU)/UJtran /. sol
```
0.759305

```
1/UJtran (-AUv Dv Cvψ (∂x ψU[x] /. x → LU) - AUv Dv Cvθ (∂x θU[x] /. x → LU)) /. sol
```
0.240695

```
VaportoUpperEpidermis =
  1/UJtran (-AUv Dv Cvψ (∂x ψU[x] /. x → LU) - AUv Dv Cvθ (∂x θU[x] /. x → LU)) /. sol
```
0.240695

```
LiquidtoUpperEpidermis = (-AUl kl ∂x ψU[x] /. x → LU)/UJtran /. sol
```
0.759305

```
λ/(QU LU) 1/1 (-AUv Dv Cvψ (∂x ψU[x] /. x → LU) - AUv Dv Cvθ (∂x θU[x] /. x → LU)) /. sol
```
0.105884

```
λ/(qsu + qru) 1/1 (-AUv Dv Cvψ (∂x ψU[x] /. x → LU) - AUv Dv Cvθ (∂x θU[x] /. x → LU)) /. sol
```
1.25174

```
qru /. sol
```
1.67573

Perivascular vapor

```
VaporFluxintoLower =
  1/Jtran (-Av Dv Cvψ (∂x ψl[x] /. x → 0) - Av Dv Cvθ (∂x θl[x] /. x → 0)) /. sol
```
0.204498

```
PerivascularEvap = VaporFluxintoLower - EvapOriginUpper
```
0.204498 - EvapOriginUpper

```
ProportionPeristomatal + PerivascularEvap +
  EvapOriginUpper + ProportionEvaporationLower (* Check=1 *)
```
1.

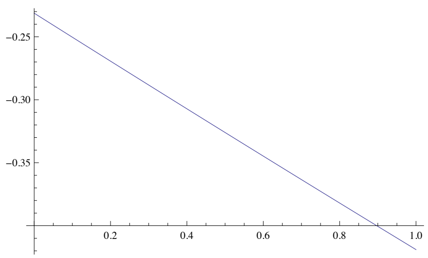
Plots

Lower Potential

```
psiL = ψl[x] /. sol /. x → L X
```

$$-0.231229 - 0.18426\, X + 0.00702293 \left(-X + \frac{X^2}{2}\right)$$

```
Plot[psiL, {X, 0, 1}]
```

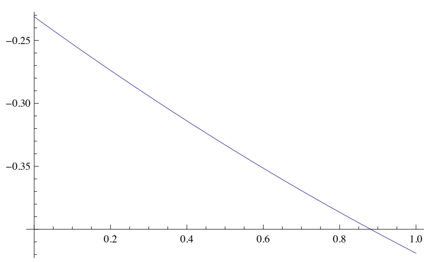


Upper Potential

```
psiU = ψU[x] /. sol /. x → LU X
```

$$-0.231229 - 0.156168\, X + 0.0632064 \left(-X + \frac{X^2}{2}\right)$$

```
Plot[psiU, {X, 0, 1}]
```

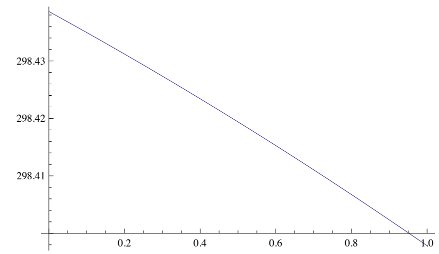


Lower Thermal

```
thetaL = θl[x] /. sol /. x → L X
```

$$298.439 - 0.0454423\, X + 0.00936193 \left(X - \frac{X^2}{2}\right)$$

```
Plot[thetaL, {X, 0, 1}]
```

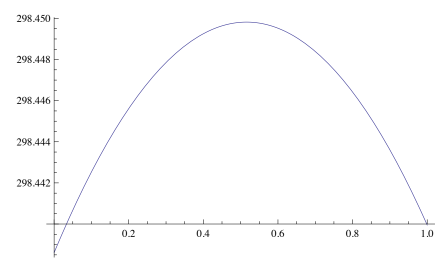


Upper Thermal

```
thetaU = θU[x] /. sol /. x → LU X
```

$$298.439 - 0.0407853\, X + 0.0842574 \left(X - \frac{X^2}{2}\right)$$

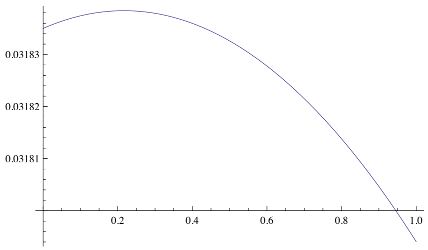
```
Plot[thetaU, {X, 0, 1}]
```



Upper Vapor Concentration

```
cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;
```

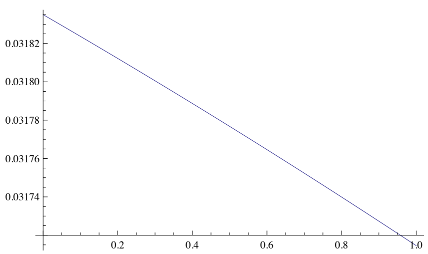
```
Plot[cU, {X, 0, 1}]
```



Lower Vapor Concentration

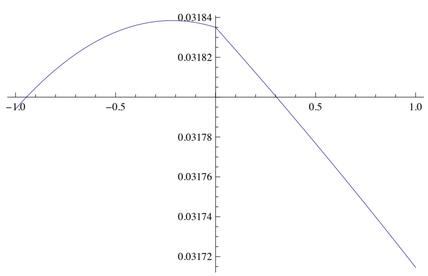
```
cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;
```

```
Plot[cL, {X, 0, 1}]
```



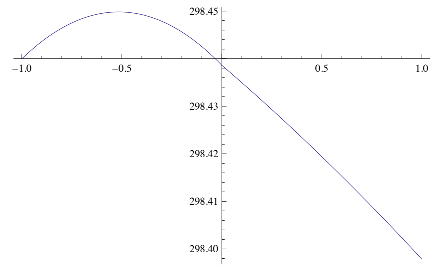
Whole leaf vapor

```
Show[Plot[cU /. X → -z, {z, 0, -1}], Plot[cL, {X, 0, 1}], PlotRange → All]
```



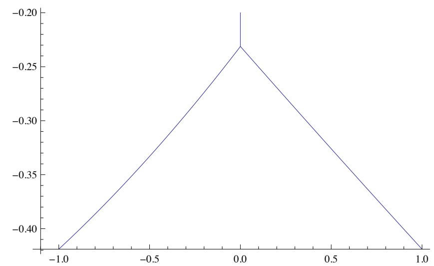
Whole leaf temperature

```
Show[Plot[thetaU /. X → -z, {z, 0, -1}], Plot[thetaL, {X, 0, 1}], PlotRange → All]
```



Whole leaf potential

```
Show[Plot[psiU /. X → -z, {z, 0, -1}], Plot[psiL, {X, 0, 1}],
 ListPlot[{{0, ψr}, {0, ψo /. sol}}, Joined → True],
 AxesOrigin → {-1.1, ψle /. sol}, PlotRange → All]
```

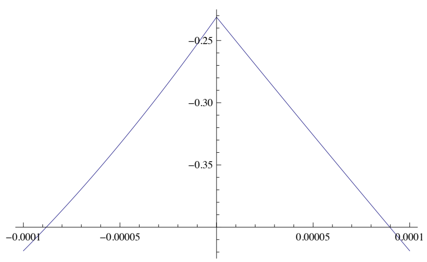


Export plot data

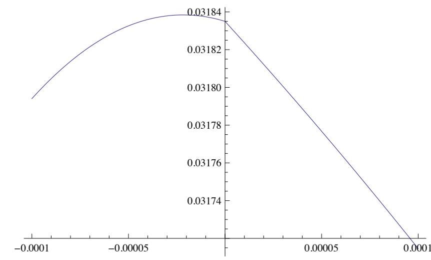
```
position = Range[-1, 1, .01];
```

```
outpos = Table[0, {Length[position]}];
```

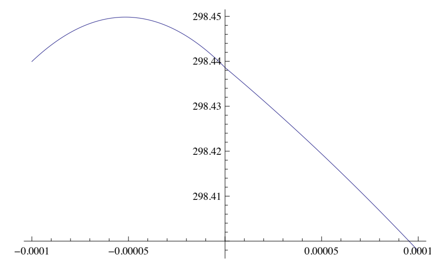
```
potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]

opdata = Transpose[{outpos, potential}];

otdata = Transpose[{outpos, temperature}];

ocdata = Transpose[{outpos, vapor}];

ListPlot[opdata, Joined → True]
```




```
ListPlot[ocdata, Joined → True]
```




```
ListPlot[otdata, Joined → True]
```




```
(*SetDirectory[ToFileName[NotebookDirectory[]]]*)

(*outfile="oak_numeric_"*)

(*Export[outfile<>"potential.xls",opdata]*)

(*Export[outfile<>"vapor.xls",ocdata]*)

(*Export[outfile<>"temperature.xls",otdata]*)
```