```
Remove["Global`*"]
```

Oak-like condensing leaf model (Base Case)

Changing klh, area fractions, and kl to do Berry like simulation

Fixed Parameter and initial values that update

```
(*ψle=-2.498(* conserved value based on numerical exposed leaf sol,
gs varies *)*)

ψrinit = -1.29;

θvar = θair; (* θair *)
ψvar = ψrinit;  (* ψr *)

vsf = .69(*  first guess*)
0.69
```

Main Environmental variables

```
rhbase = .85
0.85

QI = 0
0

TI = 0 ; (* hypothetical temp inc C *)

θair = 273.15 + 28.31 + TI(* new air temp *)
301.46

θairbase = 273.15 + 28.31 (* air temp *)
301.46
```



```
PARout = 1700 (*PARo umol m-2 s-1*)
1700

cloud = 0 (* fraction sky cloudy *)
0

windspeed = 3.86(* Logan airport mean for day m s-1, 1 m/s is 2.37 mph *)
3.86
```

Soil dependance of ψr

```
Krt = .0056 / (1.29 + -.2);
(*root plus trunk K calc for base case assuming soil -2 bars, isothermal *)

ψg = -.15 ;(* stem potential from cov'd leaf in mpa *)

ψs = -.2 ;(* stem potential from cov'd leaf in mpa *)

Krt = .0056 / (1.29 + ψs + ψg);
(*root plus trunk K calc for base case assuming soil -2 bars, isothermal *)

ψr = ψg + ψs - Jtran / Krt
-0.35 - 167.857 Jtran
```

Other Environmental parameters

```
R = 8.3145; (*gas constant Joules per mole per Kelvin*)

Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)

θsur = θair;
```

$$\chi a[relh\_, \theta\_] := relh \left(1.28 \frac{R\ 298.15}{Patm} Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right]\right)$$

$$cvbase = rhbase \left(1.28 \frac{298.15}{\theta airbase} Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta airbase} - \frac{1}{298.15}\right)\right]\right)$$

```
(* wv conc at init temp conserved as temp inc *)
1.30759
```

$$cvcase = 1.28 \frac{298.15}{\theta air} Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta air} - \frac{1}{298.15}\right)\right]$$

```
(* wvc that would be satur at new temp *)
1.53834

RH = cvbase / cvcase(* new chi from new rh *)
0.85
```

```
χair = χa[RH, θair] (* From Licor matching external conditions *)
```
0.032354

```
αsr = 0.4(* absorbance over all solar PAR+NIR, 400 to 3000 nm, range .4 to .6 *)
```
0.4

```
r = 0.15 (* reflectance of surroundings to solar radiation Nobel *)
```
0.15

```
Assimilation = 19.7 × 10^-6(* mol m-2 s-1*)
```
0.0000197

```
AE = 479 000 * Assimilation (* 479 kJ per mole co2 fixed, J m-2 s-1  *)
```
$$FE = .01\,\alpha sr\left(\frac{1}{.45} * PARout * 10^{\wedge}(-6) * 2.35 * 10^{\wedge}5\right)(1 + r)$$
```
  (* J m-2 s-1 Energy lost to fluorescence, 3% of abs vis *)
```
9.4363

4.08378

$$SRsun = \alpha sr\left(\frac{1}{.45} * PARout * 10^{\wedge}(-6) * 2.35 * 10^{\wedge}5\right)(1 + r) - AE - FE + QI$$
```
(* neglect reflectance *)
(* coeff J mol-1,
from Campbell intro env biophysics p 151: Total E incident is about 2x E in PAR,
then typical leaf abs is about half that amount,
so PAR plus NIR about equal to all E in PAR. OR 2.22 =1/.45 is factor
  mult PAR capmbell but also could subtract ps E at 479 kj per mol co2,
and then fluorescence too. ps here is 20 umol m-2 s-2 so ~10 watts*)
```
394.858

```
N[479 * 20 * 10^-3]
```
9.58

Physical quantities

```
Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)
```
0.00317308

```
Pa = Patm * 10^-6 (* atm pressure in MPa*)
```
0.1013

$$c[\psi\_, \theta\_] := 1.28\,\frac{R\,298.15}{Patm}\,Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right]$$
$$Exp\left[\frac{(\psi + Pa - Po)\ 18.07}{R\ \theta}\right]\ (*mol\ fraction\ for\ psi\ in\ MPa*)$$


```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```
0.00219861

0.000272063

```
λ = 44 000 - 43 (θvar - 298.15) (* Joules per Mol*)
```
43 857.7

$$Dv = \frac{Patm}{R\,\theta var}\,2.13 * \left(\frac{\theta var}{273.15}\right)^{1.8} *$$
```
  10^(-5)(* cDv for mol frac drving force out of leaf *)
```
0.00102805

```
kvh = .026;(* heat cond air J m-2 s-1 K-1*)
σ = 5.670373 * 10^(-8); (*stefan boltzmann J/m2/s/kelvin-4 *)
F = 1; (*view factor radiative from leaf*)
```

$$\epsilon ac = 1.72\left(\frac{Patm\,\chi a[RH, \theta air]}{1000\,\theta air}\right)^{1/7}$$
```
  (*.32 rh logan airport,vapor pressure is in kpa, campbell p 162*)
```
0.901573

$$ealt = .553\left(\frac{Patm\,\chi a[RH, \theta air]}{100}\right)^{1/7}\ (*\ Brutsaert\ 1975*)$$
0.910396

$$ealt2 = 1.24\left(\frac{Patm\,\chi a[RH, \theta air]}{100\,\theta air}\right)^{1/7}\ (*\ Brutsaert\ 1975*)$$
0.903132

$$ebrunt = .51 + .066\left(\frac{Patm\,\chi a[RH, \theta air]}{100}\right)^{1/2}$$
0.887844

```
ϵa = (1 - .84 cloud) ϵac + .84 cloud (* campbell p 162*)
```
0.901573

Leaf parameters temp corrected

```
vcx = 1 + 0.024 TI (* xylem, see viscosity corr xls*)
```
1.

```
vct = 1 + .0812 TI (**)
1.

ϵl = 0.96 (* emissivity leaf *)
0.96

αl = ϵl (* long wave abs leaf *)
0.96

charlength = .1 (* length normal to wind, m *)
0.1
```

$\delta = 4\left(\frac{\text{charlength}}{\text{windspeed}}\right)^{1/2} * 10\hat{}(-3)$ (* in m: cuvette leaf boundary layer*)

```
0.000643823

Al = .15(* .1263 oakair fraction, say 0.5 just mesophyll*)
Av = 1 - Al; (*Area fraction tissue in leaf from 2011 data 12.63%*)
0.15

AUl = .8;(* .1263 oakair fraction, say 0.5 just mesophyll*)
AUv = 1 - AUl; (*Area fraction tissue in leaf from 2011 data 12.63%*)

L = .72 × 131 × 10^(-6) (*-20 10^(-6)*)
0.00009432

LU = 1.28 * 131 × 10^(-6)
0.00016768
```

Check whole leaf air fraction

$\frac{\text{L Av + LU AUv}}{\text{L + LU}}$ (*vol weighted area fractions *)

```
0.434

klh = .28614;(* heat cond tissue*)

ktotal = vct 6.45 × 10^(-7);(* hyd cond leaf mol/m/s/mpa *)

kl = (ktotal - .1263 Dv Cvψ) / (1 - .1263)
  (* hyd cond tissue recon from on tree hydration (leaf at air temp)*)
```

6.97808×10^{-7}

```
H = .021; (* mol/m2/s/MPa from vein cutting, no steady state 1D scale factor *)

HA = vcx H vsf(* adj for viscosity (vcx) and 3D (vsf)*)
0.01449
```


Absorbed radiation

```
Q = 0.2 * SRsun / L (* w m-3 volumetric heat source,
   .2 is fraction of total abs solar rad abs in spongy *)
837272.

QU = 0.8 * SRsun / LU
```

1.88386×10^{6}

```
Q L (* total solar in lower half of leaf*)
78.9715

QU LU
315.886

Q L + QU LU - SRsun
0.
```

Description of fluxes and relations to env parameters

```
(*delta is from licor 1.42 mol/m2/s g_bw, d=Dv*C_a/g_bw, C_a=40.49 mol/m3*)
```

$qsl = \frac{kvh}{\delta} (\theta le - \theta air);$

$qsu = \frac{kvh}{\delta} (\theta ue - \theta air);$

$qrl = \sigma\, \epsilon l\, \theta le^4 - F\, \alpha l\, (\sigma\, \theta sur^4);$

$qru = \sigma\, \epsilon l\, \theta ue^4 - F\, \alpha l\, (\sigma\, \epsilon a\, \theta air^4);$

$\chi[\psi\_, \theta\_] := 1.28\, \frac{R\, 298.15}{Patm}\, Exp\left[-\frac{44000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right] Exp\left[\frac{(\psi + Pa - Po)\ 18.07}{R\, \theta}\right]$

(*for psi in MPa*)

$\chi e = \chi[\psi le, \theta le];$

$gbl = \frac{2.13 * \left(\frac{\theta air}{273.15}\right)^{1.8} * 10\hat{}(-5)\ Patm}{R\, \theta air}$

```
 (* c(tair)*Dv(Tair), boundary layer molar conductivity *)
0.00102805

gbl / δ
1.59679
```

$Jtran = \left(\frac{1}{gs} + \frac{\delta}{gbl}\right)^{-1} (\chi e - \chi air);$

$$\psi o = \psi r - \mathrm{Jtran} / \mathrm{HA}$$

$$-0.35 - \frac{1}{0.626256 + \frac{1}{gs}} 236.87 \left(-0.032354 + 0.0313236\, e^{-5291.96 \left(-0.00335402 + \frac{1}{\theta le}\right) + \frac{2.17331\,(0.0981269 + \psi le)}{\theta le}}\right)$$

First iteration

$$\Pi\psi = 1 + \frac{\mathrm{Al\ klh + Av\ kvh}}{\lambda\ \mathrm{Av\ Dv\ Cv}\theta} + \frac{\mathrm{Al\ klh + Av\ kvh}}{\lambda\ \mathrm{Al\ kl}} \frac{\mathrm{Cv}\psi}{\mathrm{Cv}\theta}$$ (* greater than one favors conduction over latent *)

3.52435

$$\Pi\theta = 1 + \frac{\lambda\ \mathrm{Al\ kl}}{\mathrm{Al\ klh + Av\ kvh}} \frac{\mathrm{Av\ Dv\ Cv}\theta}{\mathrm{Al\ kl + Av\ Dv\ Cv}\psi}$$ (* greater than one favors latent over conduction *)

1.39614

$$\Pi U\psi = 1 + \frac{\mathrm{AUl\ klh + AUv\ kvh}}{\lambda\ \mathrm{AUv\ Dv\ Cv}\theta} + \frac{\mathrm{AUl\ klh + AUv\ kvh}}{\lambda\ \mathrm{AUl\ kl}} \frac{\mathrm{Cv}\psi}{\mathrm{Cv}\theta}$$ (* greater than one favors conduction over latent *)

13.9915

$$\Pi U\theta = 1 + \frac{\lambda\ \mathrm{AUl\ kl}}{\mathrm{AUl\ klh + AUv\ kvh}} \frac{\mathrm{AUv\ Dv\ Cv}\theta}{\mathrm{AUl\ kl + AUv\ Dv\ Cv}\psi}$$ (* greater than one favors latent over conduction *)

1.07697

Stomatal conductance

gmax = 0.36

0.36

stomata[z_] = gmax (1 - Exp[(-5.2 - z) / 1.5])

$$0.36 \left(1 - e^{0.666667\,(-5.2 - z)}\right)$$

Global energy conservation

SRsun == qsu + qru + qrl + qsl + λ Jtran ;

Solution lower thermal field

$$\theta l[x\_] := \theta o + \left(-\frac{x^2}{2 L^2} + \frac{x}{L}\right) \frac{Q L^2}{\Pi\theta\ (\mathrm{Al\ klh + Av\ kvh})} + \left(-\frac{(\mathrm{qsl + qrl})\ L}{\Pi\theta\ (\mathrm{Al\ klh + Av\ kvh})} - \frac{\lambda\ \mathrm{Al\ kl\ Jtran}\ L}{\Pi\theta\ (\mathrm{Al\ klh + Av\ kvh})} \frac{1}{(\mathrm{Al\ kl + Av\ Dv\ Cv}\psi)}\right) \frac{x}{L}$$

Solution upper thermal field

$$\theta U[x\_] := \theta o + \left(-\frac{x^2}{2\ LU^2} + \frac{x}{LU}\right) \frac{QU\ LU^2}{\Pi U\theta\ (\mathrm{AUl\ klh + AUv\ kvh})} - \left(\frac{(\mathrm{qsu + qru})\ LU}{\Pi U\theta\ (\mathrm{AUl\ klh + AUv\ kvh})}\right) \frac{x}{LU}$$

Solution upper potential field

$$\psi U[x\_] := \psi o + \left(\frac{x^2}{2\ LU^2} - \frac{x}{LU}\right) \frac{QU\ LU^2}{\Pi U\psi\ \mathrm{AUl\ kl}\ \lambda} + \left(\frac{(\mathrm{qsu + qru})\ LU}{\Pi U\psi\ \mathrm{AUl\ kl}\ \lambda}\right) \frac{x}{LU}$$

Solution lower potential field

$$\psi l[x\_] := \psi o + \left(\frac{x^2}{2 L^2} - \frac{x}{L}\right) \frac{Q L^2}{\Pi\psi\ \mathrm{Al\ kl}\ \lambda} + \left(\frac{(\mathrm{qsl + qrl})\ L}{\Pi\psi\ \mathrm{Al\ kl}\ \lambda} - \frac{\mathrm{Jtran}\ L}{\Pi\psi\ \mathrm{Al\ kl}\ \lambda} \frac{\mathrm{Al\ klh + Av\ kvh}}{\mathrm{Av\ Dv\ Cv}\theta}\right) \frac{x}{L}$$

Solve system

$$\mathrm{sol} = \mathrm{FindRoot}\Big[\Big\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ \mathrm{SRsun} == \mathrm{qsu + qru + qrl + qsl} + \lambda\ \mathrm{Jtran},\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{\mathrm{stomata}[\psi le]}{gs} == 1\Big\},$$
$$\{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi rinit\}, \{gs, 0.3\}\}, \mathrm{PrecisionGoal} \to 4\Big]$$

{θo → 303.883, θue → 303.884, θle → 303.771, ψle → -1.24945, gs → 0.334148}

update seed values

$$\mathrm{Biotout} = \frac{H\ L\ (\psi o - \psi le)}{(\mathrm{Al\ kl + Av\ Dv\ Cv}\psi)\ (\psi o - \psi le) + \mathrm{Av\ Dv\ Cv}\theta\ (\theta o - \theta le)}\ /.\ \mathrm{sol}$$

1.35971

θvar = θo /. sol
ψvar = ψo /. sol

303.883

-1.05637

```
vsf = (.151 Biotout + 1.03)^(-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
0.809509
```

Second iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
0.00249289
0.000310989
```

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

3.44751

$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

1.40858

$$\Pi U\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

12.6072

$$\Pi U\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\ \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

1.08615

$$\text{sol} = \text{FindRoot}\Big[\Big\{\frac{\theta U[\text{LU}]}{\theta \text{ue}} == 1,\ \frac{\theta l[L]}{\theta \text{le}} == 1,\ \text{SRsun} == \text{qsu} + \text{qru} + \text{qrl} + \text{qsl} + \lambda\ \text{Jtran},\ \frac{\psi l[L]}{\psi \text{le}} == 1,\ \frac{\text{stomata}[\psi \text{le}]}{\text{gs}} == 1\Big\},$$
$$\{\{\theta o, \theta \text{air}\}, \{\theta \text{ue}, \theta \text{air}\}, \{\theta \text{le}, \theta \text{air}\}, \{\psi \text{le}, \psi \text{rinit}\}, \{\text{gs}, 0.3\}\}, \text{PrecisionGoal} \to 4\Big]$$

```
{θo → 303.876, θue → 303.877, θle → 303.768, ψle → -1.18808, gs → 0.335184}
```


update seed values

$$\text{Biotout} = \frac{\text{H L } (\psi o - \psi \text{le})}{(\text{Al kl} + \text{Av Dv Cv}\psi)\ (\psi o - \psi \text{le}) + \text{Av Dv Cv}\theta\ (\theta o - \theta \text{le})}\ /.\ \text{sol}$$

```
0.906002
sf = (.151 Biotout + 1.03)^(-1)
0.85704
θvar = θo /. sol
ψvar = ψo /. sol
303.876
-1.05895
vsf = (.151 Biotout + 1.03)^(-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
0.85704
```

Third iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
0.00249189
0.000310855
```

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

3.44774

$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$
(* greater than one favors latent over conduction *)

1.40854

$$\Pi U\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$
(* greater than one favors conduction over latent *)

12.6113

$$\Pi U\theta = 1 + \frac{\lambda\, AUl\, kl}{AUl\, klh + AUv\, kvh}\; \frac{AUv\, Dv\, Cv\theta}{AUl\, kl + AUv\, Dv\, Cv\psi}$$
(* greater than one favors latent over conduction *)

1.08612

$$sol = FindRoot\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\, Jtran,\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{stomata[\psi le]}{gs} == 1\right\}, \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi rinit\}, \{gs, 0.3\}\}, PrecisionGoal \to 4\right]$$

$\{\theta o \to 303.876, \theta ue \to 303.877, \theta le \to 303.768, \psi le \to -1.18827, gs \to 0.335181\}$

update seed values

$$Biotout = \frac{H\, L\, (\psi o - \psi le)}{(Al\, kl + Av\, Dv\, Cv\psi)\, (\psi o - \psi le) + Av\, Dv\, Cv\theta\, (\theta o - \theta le)} \ /.\ sol$$

0.907401

```
sf = (.151 Biotout + 1.03) ^ (-1)
```

0.856885

```
θvar = θo /. sol
ψvar = ψo /. sol
```

303.876

-1.05894

```
vsf = (.151 Biotout + 1.03) ^ (-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.856885

Fourth iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00249189

0.000310856

$$\Pi\psi = 1 + \frac{Al\, klh + Av\, kvh}{\lambda\, Av\, Dv\, Cv\theta} + \frac{Al\, klh + Av\, kvh}{\lambda\, Al\, kl}\, \frac{Cv\psi}{Cv\theta}$$
(* greater than one favors conduction over latent *)

3.44774

$$\Pi\theta = 1 + \frac{\lambda\, Al\, kl}{Al\, klh + Av\, kvh}\; \frac{Av\, Dv\, Cv\theta}{Al\, kl + Av\, Dv\, Cv\psi}$$
(* greater than one favors latent over conduction *)

1.40854

$$\Pi U\psi = 1 + \frac{AUl\, klh + AUv\, kvh}{\lambda\, AUv\, Dv\, Cv\theta} + \frac{AUl\, klh + AUv\, kvh}{\lambda\, AUl\, kl}\, \frac{Cv\psi}{Cv\theta}$$
(* greater than one favors conduction over latent *)

12.6113

$$\Pi U\theta = 1 + \frac{\lambda\, AUl\, kl}{AUl\, klh + AUv\, kvh}\; \frac{AUv\, Dv\, Cv\theta}{AUl\, kl + AUv\, Dv\, Cv\psi}$$
(* greater than one favors latent over conduction *)

1.08612

$$sol = FindRoot\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\, Jtran,\ \frac{\psi l[L]}{\psi le} == 1,\ \frac{stomata[\psi le]}{gs} == 1\right\}, \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi rinit\}, \{gs, 0.3\}\}, PrecisionGoal \to 4\right]$$

$\{\theta o \to 303.876, \theta ue \to 303.877, \theta le \to 303.768, \psi le \to -1.18827, gs \to 0.335181\}$

$$Biotout = \frac{H\, L\, (\psi o - \psi le)}{(Al\, kl + Av\, Dv\, Cv\psi)\, (\psi o - \psi le) + Av\, Dv\, Cv\theta\, (\theta o - \theta le)} \ /.\ sol$$

0.907396

```
sf = (.151 Biotout + 1.03) ^ (-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.856886

check convergence

```
θvar - θo /. sol
```

7.18443×10^{-8}

```
ψvar - ψo /. sol
```

2.43227×10^{-8}

```
sf - vsf
```

4.76636×10^{-7}

Results

```
MolFracs = {(χe - χair), χe, χair} /. sol
```
{0.0108037, 0.0431577, 0.032354}

```
gs /. sol
```
0.335181

```
ψue = ψU[LU] /. sol (* Water potential upper epidermis *)
```
-1.05976

```
Temps = {θair, θsur, θue, θo, θle, θle - θair} /. sol
```
{301.46, 301.46, 303.877, 303.876, 303.768, 2.30848}

```
Potentials = {ψr, ψo, ψue, ψle} /. sol
```
{-0.852389, -1.05894, -1.05976, -1.18827}

Fluxes

```
qsu /. sol (*Sensible flux upper*)
```
97.5979

```
qru /. sol (*Radiative flux upper*)
```
58.8411

```
qsl /. sol (*Sensible flux lower*)
```
93.225

```
qrl /. sol (*Radiative flux lower*)
```
13.9297

```
λ Jtran /. sol
```
131.264

```
Jtran /. sol
```
0.00299295

Leaf water potential and apparent conductance

$$\text{LowerAvgPotential} = \frac{1}{L}\int_0^L \psi l[x]\, dx \text{ /. sol}$$

-1.16282

$$\text{UpperAvgPotential} = \frac{1}{LU}\int_0^{LU} \psi U[x]\, dx \text{ /. sol}$$

-1.07365

$$\text{LWP} = \frac{L\ \text{LowerAvgPotential} + LU\ \text{UpperAvgPotential}}{L + LU}$$

-1.10575

Naive view of gradients

$$\text{Kleaf} = \frac{\text{Jtran}}{\psi r - \text{LWP}} \text{ /. sol (* Apparent conductance, mol m-2 s-1 MPa-1*)}$$

0.011813

Informed view of gradients

$$\text{nondnumericpsiavg} = .721 - \frac{.219}{\text{Biotout}}$$

0.47965

```
ScaledPsiAverageLeaf = nondnumericpsiavg * (ψr - ψle ) + ψle /. sol
  (* cf 0.65 for oak (scaled k est on tree and leaf paper): given true psie
    and psir as defined in numeric simulation gives average potential in
    domain. Justified if 1D steady sol is scaled to 3d as it is here *)
```
-1.02716

```
kleaf = Jtran / (ψr - ScaledPsiAverageLeaf) /. sol
```
0.0171248

Evaporation distribution

$$\text{ProportionPeristomatal} = \frac{-\text{Al kl } \partial_x \psi l[x] \text{ /. } x \to L}{\text{Jtran}} \text{ /. sol}$$

-0.0392977

$$\text{ProportionEvaporationLower} = \frac{\text{Al kl } \psi l''[x]\ L}{\text{Jtran}} \text{ /. sol}$$

0.174498

$$\text{EvapOriginUpper} = \frac{\text{QU LU} - (\text{qsu} + \text{qru})}{\Pi U\psi\ \lambda\ \text{Jtran}} \text{ /. sol}$$

0.0963186

$$\text{VaporFluxintoLower} = \frac{1}{\text{Jtran}}\left(-\text{Av Dv Cv}\psi\ (\partial_x \psi l[x]\ /.\ x \to 0) - \text{Av Dv Cv}\theta\ (\partial_x \theta l[x]\ /.\ x \to 0)\right)\ /.\ \text{sol}$$

0.8648

```
PerivascularEvap = VaporFluxintoLower - EvapOriginUpper
```

0.768481

```
ProportionPeristomatal + PerivascularEvap +
 EvapOriginUpper + ProportionEvaporationLower (* Check=1 *)
```

1.

```
ApparentLength = (ProportionPeristomatal + 0.5 ProportionEvaporationLower)
 (* Dist from evap site to vasc *)
```

0.0479514

Analysis

Analyze proportion peristomatal at Lower Epidermis

$$\text{ProportionPeristomatal} = \frac{-\text{Al kl}\ \partial_x \psi l[x]\ /.\ x \to L}{\text{Jtran}}\ /.\ \text{sol}$$

-0.0392977

$$\text{PropPeri} = \frac{-(\text{qsl} + \text{qrl})}{\Pi\psi\ \text{Jtran}\ \lambda} + \left(1 + \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1}\ /.\ \text{sol}$$

-0.0392977

$$\frac{-(\text{qsl} + \text{qrl})}{\Pi\psi\ \text{Jtran}\ \lambda}\ /.\ \text{sol}$$

-0.236772

$$\left(1 + \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1}\ /.\ \text{sol}$$

0.197475

Condensation on lower epidermis if following is larger than one:

$$\frac{(\text{qsl} + \text{qrl})}{\text{Jtran}\ \lambda}\ \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}}\ /.\ \text{sol}$$

1.199

Total internal vapor arriving at lower epidermis (non-peristomatal)

$$\text{ProportionInternalVapor} = \frac{1}{\text{Jtran}}\left(-\text{Av Dv Cv}\psi\ (\partial_x \psi l[x]\ /.\ x \to L) - \text{Av Dv Cv}\theta\ (\partial_x \theta l[x]\ /.\ x \to L)\right)\ /.\ \text{sol}$$

1.0393

```
ProportionPeristomatal + ProportionInternalVapor
```

1.

Evaporation in lower

$$\text{ProportionEvaporationLower} = \frac{\text{Al kl}\ \psi l''[x]\ L}{\text{Jtran}}\ /.\ \text{sol}$$

0.174498

$$\frac{Q\ L}{\Pi\psi\ \lambda\ \text{Jtran}}\ /.\ \text{sol}\ (*\text{integrating evap 2nd derv over L }*)$$

0.174498

Analyze evaporation in upper part of leaf

$$\text{LiquidFluxintoUpper} = \frac{-\text{AUl kl}\ \partial_x \psi U[x]\ /.\ x \to 0}{\text{Jtran}}\ /.\ \text{sol}$$

0.0963186

$$\text{EvapOriginUpper} = \frac{\text{QU LU} - (\text{qsu} + \text{qru})}{\Pi U\psi\ \lambda\ \text{Jtran}}\ /.\ \text{sol}$$

0.0963186

$$\text{VaporFluxintoUpper} = \frac{1}{\text{Jtran}}\left(-\text{AUv Dv Cv}\psi\ (\partial_x \psi U[x]\ /.\ x \to 0) - \text{AUv Dv Cv}\theta\ (\partial_x \theta U[x]\ /.\ x \to 0)\right)\ /.\ \text{sol}$$

-0.0963186

$$\text{CondensationUpperEpidermis} = \frac{1}{1}\left(-\text{AUv Dv Cv}\psi\ (\partial_x \psi U[x]\ /.\ x \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta U[x]\ /.\ x \to \text{LU})\right)\ /.\ \text{sol}$$

0.000282838

$$\text{FluxFromUpperEpidermis} = \frac{-\text{AUl kl}\ \partial_x \psi U[x]\ /.\ x \to \text{LU}}{1}\ /.\ \text{sol}$$

-0.000282838

$$\frac{\lambda}{QU\,LU}\,\frac{1}{1}\left(-AUv\,Dv\,Cv\psi\,(\partial_x\,\psi U[x]\ /.\ x\to LU) - AUv\,Dv\,Cv\theta\,(\partial_x\,\theta U[x]\ /.\ x\to LU)\right)\ /.\ sol$$

0.0392693

$$\frac{\lambda}{qsu+qru}\,\frac{1}{1}\left(-AUv\,Dv\,Cv\psi\,(\partial_x\,\psi U[x]\ /.\ x\to LU) - AUv\,Dv\,Cv\theta\,(\partial_x\,\theta U[x]\ /.\ x\to LU)\right)\ /.\ sol$$

0.0792937

```
qru /. sol
```

58.8411

Perivascular vapor

$$VaporFluxintoLower = \frac{1}{Jtran}\left(-Av\,Dv\,Cv\psi\,(\partial_x\,\psi l[x]\ /.\ x\to 0) - Av\,Dv\,Cv\theta\,(\partial_x\,\theta l[x]\ /.\ x\to 0)\right)\ /.\ sol$$

0.8648

```
PerivascularEvap = VaporFluxintoLower - EvapOriginUpper
```

0.768481

```
ProportionPeristomatal + PerivascularEvap +
 EvapOriginUpper + ProportionEvaporationLower (* Check=1 *)
```

1.

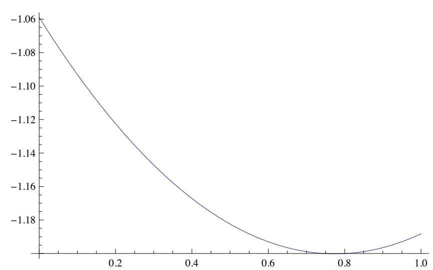
Plots

Lower Potential

```
psiL = ψl[x] /. sol /. x → L X
```

$-1.05894 + 0.105985\,X + 0.470617\,(-1.\,X + 0.5\,X^2)$

```
Plot[psiL, {X, 0, 1}]
```

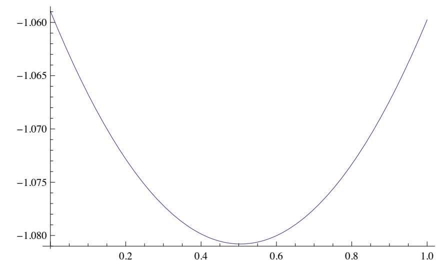


Upper Potential

```
psiU = ψU[x] /. sol /. x → LU X
```

$-1.05894 + 0.084956\,X + 0.171546\,(-1.\,X + 0.5\,X^2)$

```
Plot[psiU, {X, 0, 1}]
```

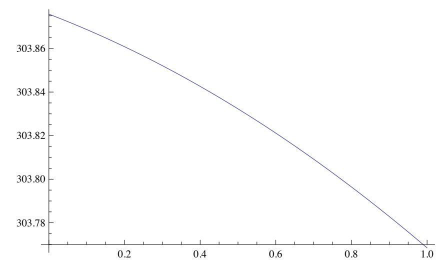


Lower Thermal

```
thetaL = θl[x] /. sol /. x → L X
```

$303.876 - 0.147957\,X + 0.0813301\,(1.\,X - 0.5\,X^2)$

```
Plot[thetaL, {X, 0, 1}]
```

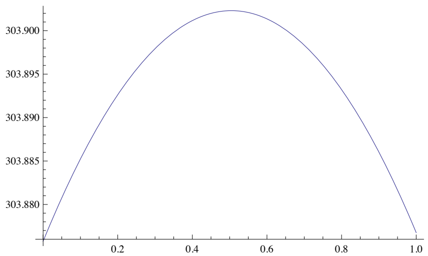


Upper Thermal

```
thetaU = θU[x] /. sol /. x → LU X
```

$303.876 - 0.103163\,X + 0.20831\,(1.\,X - 0.5\,X^2)$

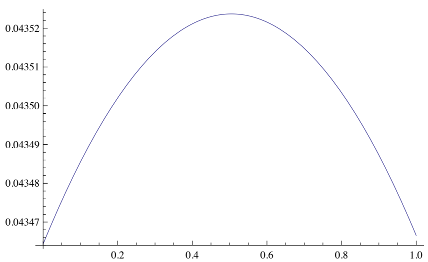
```
Plot[thetaU, {X, 0, 1}]
```



Upper Vapor Concentration

```
cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;
```

```
Plot[cU, {X, 0, 1}]
```

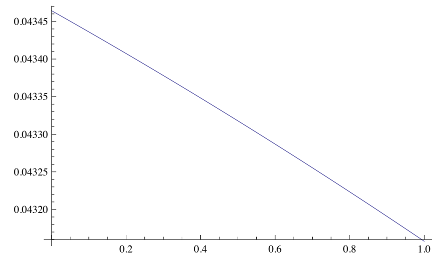


Lower Vapor Concentration

```
cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;
```

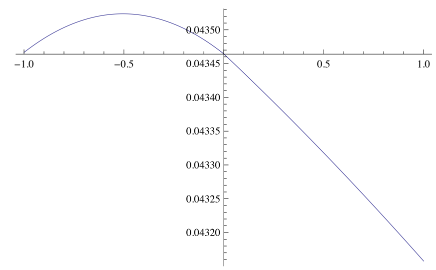


```
Plot[cL, {X, 0, 1}]
```



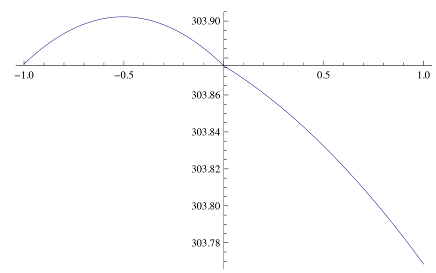
Whole leaf vapor

```
Show[Plot[cU /. X → -z, {z, 0, -1}], Plot[cL, {X, 0, 1}], PlotRange → All]
```



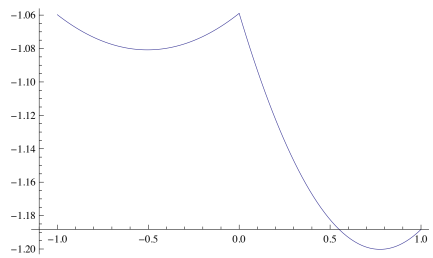
Whole leaf temperature

```
Show[Plot[thetaU /. X → -z, {z, 0, -1}], Plot[thetaL, {X, 0, 1}], PlotRange → All]
```



Whole leaf potential

```
Show[Plot[psiU /. X → -z, {z, 0, -1}], Plot[psiL, {X, 0, 1}],
 ListPlot[{{0, ψr}, {0, ψo /. sol}}, Joined → True],
 AxesOrigin → {-1.1, ψle /. sol}, PlotRange → All]
```

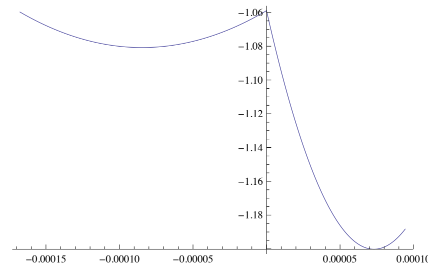


Export plot data

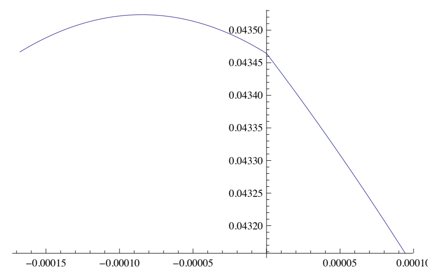
```
position = Range[-1, 1, .01];


outpos = Table[0, {Length[position]}];

potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]

opdata = Transpose[{outpos, potential}];

otdata = Transpose[{outpos, temperature}];

ocdata = Transpose[{outpos, vapor}];
```



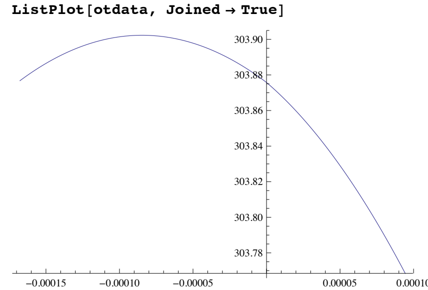
```
ListPlot[opdata, Joined → True]
```




```
ListPlot[ocdata, Joined → True]
```




```
ListPlot[otdata, Joined → True]
```




```
SetDirectory[ToFileName[NotebookDirectory[]]]

/Users/Tony/Dropbox/Manuscripts/Active manuscripts/vapor and liquid/mol
  fraction/stomatal g function of wp/Hypothetical 15 80 AF oak gs
```

```
outfile = "sim_Q0_"
sim_Q0_
Export[outfile <> "potential.xls", opdata]
sim_Q0_potential.xls
Export[outfile <> "vapor.xls", ocdata]
sim_Q0_vapor.xls
Export[outfile <> "temperature.xls", otdata]
sim_Q0_temperature.xls
```