```
Remove["Global`*"]
```

Evaporative flux lab experiment: low load

Fixed Parameter and initial values that update

For in lab, set cloud=1, gs choked, psir=-.05

```
gs = 0.05(* conserved value based on Li-cor measurements, ψle varies *)
0.05

θvar = θair; (* θair *)
ψvar = ψr;  (* ψr *)

vsf = .69(*  first guess*)
0.69
```

Main Environmental variables

```
TI = 0 ; (* hypothetical temp inc C *)

θairbase = 273.15 + 28.31
301.46

θair = 273.15 + 28.31 + TI(* air temp *)
301.46

PARout = 50 (*PARo umol m-2 s-1*)
50

cloud = 1 (* fraction sky cloudy *)
1

windspeed = 3.86(* Logan airport mean for day m s-1*)
3.86

ψr = -.01(* stem potential from cov'd leaf in mpa *)
-0.01
```

Environmental parameters

```
R = 8.3145; (*gas constant Joules per mole per Kelvin*)

Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)

θsur = θair;
```


$$\chi a[relh\_, \theta\_] := relh \left(1.28\ \frac{R\ 298.15}{Patm}\ Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]\right)$$

$$cvbase = .437 \left(1.28\ \frac{298.15}{\theta airbase}\ Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta airbase}-\frac{1}{298.15}\right)\right]\right)$$

```
  (* wv conc conserved as temp inc *)
0.672255
```

$$cvcase = 1.28\ \frac{298.15}{\theta air}\ Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta air}-\frac{1}{298.15}\right)\right]$$

```
  (* wvc that would be satur at new temp *)
1.53834

RH = cvbase / cvcase (* new rh due to old wvc at new temp *)
0.437

χair = χa[RH, θair] (* new chi from new rh *)
0.0166338

αsr = 0.4(* absorbance over all solar PAR+NIR, 400 to 3000 nm, range .4 to .6 *)
0.4

r = 0.15 (* reflectance of surroundings to solar radiation Nobel *)
0.15

Assimilation = 19.7 × 10^-6(* mol m-2 s-1*)
0.0000197

AE = 479 000 * Assimilation (* 479 kJ per mole co2 fixed, J m-2 s-1  *)
```

$$FE = .01\ \alpha sr \left(\frac{1}{.45} * PARout * 10^{\wedge}(-6) * 2.35 * 10^{\wedge}5\right)(1 + r)$$

```
  (* J m-2 s-1 Energy lost to fluorescence, 3% of abs vis *)
9.4363

0.120111
```

$$SRsun = \alpha sr \left(\frac{1}{.45} * PARout * 10^{\wedge}(-6) * 2.35 * 10^{\wedge}5\right)(1 + r) - AE - FE$$

```
(* neglect refelctance *)
(* coeff J mol-1,
from Campbell intro env biophysics p 151: Total E incident is about 2x E in PAR,
then typical leaf abs is about half that amount,
so PAR plus NIR about equal to all E in PAR. OR 2.22 =1/.45 is factor
  mult PAR capmbell but also could subtract ps E at 479 kj per mol co2,
and then fluorescence too. ps here is 20 umol m-2 s-2 so ~10 watts*)
2.4547
```

Physical quantities

```
Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)
```
0.00317308

```
Pa = Patm * 10^-6 (* atm pressure in MPa*)
```
0.1013

$$c[\psi\_, \theta\_] := 1.28\ \frac{R\,298.15}{Patm}\ Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]$$
$$Exp\left[\frac{(\psi + Pa - Po)\ 18.07}{R\ \theta}\right]$$ (*mol fraction for psi in MPa*)

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```
0.00221782

0.000274585

```
λ = 44 000 - 43 (θvar - 298.15) (* Joules per Mol*)
```
43 857.7

$$Dv = \frac{Patm}{R\,\theta var}\ 2.13 * \left(\frac{\theta var}{273.15}\right)^{1.8} *$$
10^(-5) (* cDv for mol frac drving force out of leaf *)

0.00102805

```
kvh = .026;(* heat cond air J m-2 s-1 K-1*)
σ = 5.670373 * 10^(-8); (*stefan boltzmann J/m2/s/kelvin-4 *)
F = 1; (*view factor radiative from leaf*)
```

$$\epsilon ac = 1.72\left(\frac{Patm\ \chi a[RH, \theta air]}{1000\ \theta air}\right)^{1/7}$$
(*.32 rh logan airport,vapor pressure is in kpa, campbell p 162*)

0.819831

$$ealt = .553\left(\frac{Patm\ \chi a[RH, \theta air]}{100}\right)^{1/7}$$ (* Brutsaert 1975*)

0.827854

$$ealt2 = 1.24\left(\frac{Patm\ \chi a[RH, \theta air]}{100\ \theta air}\right)^{1/7}$$ (* Brutsaert 1975*)

0.821249

$$ebrunt = .51 + .066\left(\frac{Patm\ \chi a[RH, \theta air]}{100}\right)^{1/2}$$

0.780922

```
εa = (1 - .84 cloud) εac + .84 cloud (* campbell p 162*)
```
0.971173

Leaf parameters temp corrected

```
vcx = 1 + 0.024 TI (* xylem, see viscosity corr xls*)
```
1.

```
vct = 1 + .0812 TI (* tissue *)
```
1.

```
(*vcx=1/(1- 0.024 TI )(* xylem, for temp drop*)*)
(*vct=1/(1- .0812 TI )(* tissue for temp drop*)*)
εl = 0.96 (* emissivity leaf *)
```
0.96

```
αl = εl (* long wave abs leaf *)
```
0.96

```
charlength = .1 (* length normal to wind, m *)
```
0.1

$$\delta = 4\left(\frac{charlength}{windspeed}\right)^{1/2} * 10\hat{}(-3)$$ (* in m: cuvette leaf boundary layer*)

0.000643823

```
Al = .4(* .1263 oakair fraction, say 0.5 just mesophyll*)
Av = 1 - Al; (*Area fraction tissue in leaf from 2011 data 12.63%*)
```
0.4

```
AUl = .8;(* .1263 oakair fraction, say 0.5 just mesophyll*)
AUv = 1 - AUl; (*Area fraction tissue in leaf from 2011 data 12.63%*)
L = .72 × 131 × 10^(-6)(*-20 10^(-6)*)
```
0.00009432

```
LU = 1.28 * 131 × 10^(-6)
```
0.00016768

Check whole leaf air fraction

$$\frac{L\,Av + LU\,AUv}{L + LU}$$ (*vol weighted area fractions *)

0.344

```
klh = .28614;(* heat cond tissue*)

ktotal = vct 6.45 × 10^(-7);(* hyd cond leaf mol/m/s/mpa *)

kl = (ktotal - .1263 Dv Cvψ) / (1 - .1263)
  (* hyd cond tissue recon from on tree hydration (leaf at air temp)*)
```

6.97433×10^{-7}

```
H = .021; (* mol/m2/s/MPa from vein cutting, no steady state 1D scale factor *)

HA = vcx H vsf(* adj for viscosity (vcx) and 3D (vsf)*)
```

0.01449

Absorbed radiation

```
Q = 0.2 * SRsun / L (* w m-3 volumetric heat source,
   .2 is fraction of total abs solar rad abs in spongy *)
```

5205.05

```
QU = 0.8 * SRsun / LU
```

11711.4

```
Q L (* total solar in lower half of leaf*)
```

0.49094

```
QU LU
```

1.96376

```
Q L + QU LU - SRsun
```

0.

Description of fluxes and relations to env parameters

```
(*delta is from licor 1.42 mol/m2/s g_bw, d=Dv*C_a/g_bw, C_a=40.49 mol/m3*)
```

$$qsl = \frac{kvh}{\delta}\,(\theta le - \theta air);$$

$$qsu = \frac{kvh}{\delta}\,(\theta ue - \theta air);$$

$$qrl = \sigma\,\epsilon l\,\theta le^4 - F\,\alpha l\,\left(\sigma\,\theta sur^4\right);$$

$$qru = \sigma\,\epsilon l\,\theta ue^4 - F\,\alpha l\,\left(\sigma\,\epsilon a\,\theta air^4\right);$$

$$\chi[\psi\_,\theta\_] := 1.28\,\frac{R\,298.15}{Patm}\,Exp\left[-\frac{44\,000}{R}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]\,Exp\left[\frac{(\psi + Pa - Po)\,18.07}{R\,\theta}\right]$$

(*for psi in MPa*)

$\chi e = \chi[\psi le, \theta le];$

$$gbl = \frac{2.13 * \left(\frac{\theta air}{273.15}\right)^{1.8} * 10^{\wedge}(-5)\,Patm}{R\,\theta air}$$

(* c(tair)*Dv(Tair), boundary layer molar conductivity *)

0.00102805

gbl / δ

1.59679

$$Jtran = \left(\frac{1}{gs} + \frac{\delta}{gbl}\right)^{-1} (\chi e - \chi air);$$

$\psi o = \psi r - Jtran / HA$

$$-0.01 - 3.34589\left(-0.0166338 + 0.0313236\,e^{-5291.96\left(-0.00335402+\frac{1}{\theta le}\right)+\frac{2.17331\,(0.0981269+\psi le)}{\theta le}}\right)$$

First iteration

$$\Pi\psi = 1 + \frac{Al\,klh + Av\,kvh}{\lambda\,Av\,Dv\,Cv\theta} + \frac{Al\,klh + Av\,kvh}{\lambda\,Al\,kl}\,\frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

4.48372

$$\Pi\theta = 1 + \frac{\lambda\,Al\,kl}{Al\,klh + Av\,kvh}\,\frac{Av\,Dv\,Cv\theta}{Al\,kl + Av\,Dv\,Cv\psi}$$

(* greater than one favors latent over conduction *)

1.28705

$$\Pi U\psi = 1 + \frac{AUl\,klh + AUv\,kvh}{\lambda\,AUv\,Dv\,Cv\theta} + \frac{AUl\,klh + AUv\,kvh}{\lambda\,AUl\,kl}\,\frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

13.8905

$$\Pi U\theta = 1 + \frac{\lambda\,AUl\,kl}{AUl\,klh + AUv\,kvh}\,\frac{AUv\,Dv\,Cv\theta}{AUl\,kl + AUv\,Dv\,Cv\psi}$$

(* greater than one favors latent over conduction *)

1.07758

Global energy conservation

```
SRsun == qsu + qru + qrl + qsl + λ Jtran ;
```

Solution lower thermal field

$$\theta l[x\_] := \theta o + \left(-\frac{x^2}{2\,L^2} + \frac{x}{L}\right)\frac{Q\,L^2}{\Pi\theta\ (Al\ klh + Av\ kvh)} + \left(-\frac{(qsl + qrl)\ L}{\Pi\theta\ (Al\ klh + Av\ kvh)} - \frac{\lambda\ Al\ kl\ Jtran\ L}{\Pi\theta\ (Al\ klh + Av\ kvh)}\ \frac{1}{(Al\ kl + Av\ Dv\ Cv\psi)}\right)\frac{x}{L}$$

Solution upper thermal field

$$\theta U[x\_] := \theta o + \left(-\frac{x^2}{2\,LU^2} + \frac{x}{LU}\right)\frac{QU\,LU^2}{\Pi U\theta\ (AUl\ klh + AUv\ kvh)} - \left(\frac{(qsu + qru)\ LU}{\Pi U\theta\ (AUl\ klh + AUv\ kvh)}\right)\frac{x}{LU}$$

Solution upper potential field

$$\psi U[x\_] := \psi o + \left(\frac{x^2}{2\,LU^2} - \frac{x}{LU}\right)\frac{QU\,LU^2}{\Pi U\psi\ AUl\ kl\ \lambda} + \left(\frac{(qsu + qru)\ LU}{\Pi U\psi\ AUl\ kl\ \lambda}\right)\frac{x}{LU}$$

Solution lower potential field

$$\psi l[x\_] := \psi o + \left(\frac{x^2}{2\,L^2} - \frac{x}{L}\right)\frac{Q\,L^2}{\Pi\psi\ Al\ kl\ \lambda} + \left(\frac{(qsl + qrl)\ L}{\Pi\psi\ Al\ kl\ \lambda} - \frac{Jtran\ L}{\Pi\psi\ Al\ kl\ \lambda}\ \frac{Al\ klh + Av\ kvh}{Av\ Dv\ Cv\theta}\right)\frac{x}{L}$$

Solve system

sol =

$$\text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\ Jtran,\ \frac{\psi l[L]}{\psi le} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\}, \text{PrecisionGoal} \to 4\right]$$

{θo → 300.88, θue → 300.89, θle → 300.88, ψle → -0.283297}

update seed values

$$Biotout = \frac{H\ L\ (\psi o - \psi le)}{(Al\ kl + Av\ Dv\ Cv\psi)\ (\psi o - \psi le) + Av\ Dv\ Cv\theta\ (\theta o - \theta le)}\ /.\ sol$$

4.43585


```
θvar = θo /. sol
ψvar = ψo /. sol
```

300.88

-0.0773024

```
vsf = (.151 Biotout + 1.03) ^ (-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.5883

Second iteration

```
Cvθ = ∂θ c[ψvar, θ] /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar] /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00215133

0.000265834

$$\Pi\psi = 1 + \frac{Al\ klh + Av\ kvh}{\lambda\ Av\ Dv\ Cv\theta} + \frac{Al\ klh + Av\ kvh}{\lambda\ Al\ kl}\ \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

4.54815

$$\Pi\theta = 1 + \frac{\lambda\ Al\ kl}{Al\ klh + Av\ kvh}\ \frac{Av\ Dv\ Cv\theta}{Al\ kl + Av\ Dv\ Cv\psi}$$ (* greater than one favors latent over conduction *)

1.28184

$$\Pi U\psi = 1 + \frac{AUl\ klh + AUv\ kvh}{\lambda\ AUv\ Dv\ Cv\theta} + \frac{AUl\ klh + AUv\ kvh}{\lambda\ AUl\ kl}\ \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

14.2499

$$\Pi U\theta = 1 + \frac{\lambda\ AUl\ kl}{AUl\ klh + AUv\ kvh}\ \frac{AUv\ Dv\ Cv\theta}{AUl\ kl + AUv\ Dv\ Cv\psi}$$ (* greater than one favors latent over conduction *)

1.07547

sol =

$$\text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\ Jtran,\ \frac{\psi l[L]}{\psi le} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\}, \text{PrecisionGoal} \to 4\right]$$

{θo → 300.88, θue → 300.89, θle → 300.88, ψle → -0.285234}

update seed values

$$\text{Biotout} = \frac{\text{H L } (\psi\text{o} - \psi\text{le})}{(\text{Al kl} + \text{Av Dv Cv}\psi)\ (\psi\text{o} - \psi\text{le}) + \text{Av Dv Cv}\theta\ (\theta\text{o} - \theta\text{le})}\ \text{/. sol}$$

4.47777

```
sf = (.151 Biotout + 1.03) ^ (-1)
```

0.586117

```
θvar = θo /. sol
ψvar = ψo /. sol
```

300.88

-0.0773001

```
vsf = (.151 Biotout + 1.03) ^ (-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.586117

Third iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00215134

0.000265835

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$

(* greater than one favors conduction over latent *)

4.54814

$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$

(* greater than one favors latent over conduction *)

1.28184

$$\Pi\text{U}\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$

(* greater than one favors conduction over latent *)

14.2499

$$\Pi\text{U}\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\ \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$$

(* greater than one favors latent over conduction *)

1.07547

$$\text{sol} = \text{FindRoot}\left[\left\{\frac{\theta\text{U[LU]}}{\theta\text{ue}} == 1,\ \frac{\theta\text{l[L]}}{\theta\text{le}} == 1,\ \text{SRsun} == \text{qsu} + \text{qru} + \text{qrl} + \text{qsl} + \lambda\ \text{Jtran},\ \frac{\psi\text{l[L]}}{\psi\text{le}} == 1\right\},\ \{\{\theta\text{o}, \theta\text{air}\}, \{\theta\text{ue}, \theta\text{air}\}, \{\theta\text{le}, \theta\text{air}\}, \{\psi\text{le}, \psi\text{r}\}\},\ \text{PrecisionGoal} \to 4\right]$$

{θo → 300.88, θue → 300.89, θle → 300.88, ψle → -0.285234}

update seed values

$$\text{Biotout} = \frac{\text{H L } (\psi\text{o} - \psi\text{le})}{(\text{Al kl} + \text{Av Dv Cv}\psi)\ (\psi\text{o} - \psi\text{le}) + \text{Av Dv Cv}\theta\ (\theta\text{o} - \theta\text{le})}\ \text{/. sol}$$

4.47776

```
sf = (.151 Biotout + 1.03) ^ (-1)
```

0.586118

```
θvar = θo /. sol
ψvar = ψo /. sol
```

300.88

-0.0773001

```
vsf = (.151 Biotout + 1.03) ^ (-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.586118

Fourth iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00215134

0.000265835

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$

(* greater than one favors conduction over latent *)

4.54814

$$\Pi\theta = 1 + \frac{\lambda\ \mathrm{Al\ kl}}{\mathrm{Al\ klh + Av\ kvh}}\ \frac{\mathrm{Av\ Dv\ Cv}\theta}{\mathrm{Al\ kl + Av\ Dv\ Cv}\psi}$$

(* greater than one favors latent over conduction *)

1.28184

$$\Pi U\psi = 1 + \frac{\mathrm{AUl\ klh + AUv\ kvh}}{\lambda\ \mathrm{AUv\ Dv\ Cv}\theta} + \frac{\mathrm{AUl\ klh + AUv\ kvh}}{\lambda\ \mathrm{AUl\ kl}}\ \frac{\mathrm{Cv}\psi}{\mathrm{Cv}\theta}$$

(* greater than one favors conduction over latent *)

14.2499

$$\Pi U\theta = 1 + \frac{\lambda\ \mathrm{AUl\ kl}}{\mathrm{AUl\ klh + AUv\ kvh}}\ \frac{\mathrm{AUv\ Dv\ Cv}\theta}{\mathrm{AUl\ kl + AUv\ Dv\ Cv}\psi}$$

(* greater than one favors latent over conduction *)

1.07547

sol =

$$\mathrm{FindRoot}\left[\left\{\frac{\theta\mathrm{U[LU]}}{\theta\mathrm{ue}} == 1,\ \frac{\theta\mathrm{l[L]}}{\theta\mathrm{le}} == 1,\ \mathrm{SRsun} == \mathrm{qsu + qru + qrl + qsl} + \lambda\,\mathrm{Jtran},\ \frac{\psi\mathrm{l[L]}}{\psi\mathrm{le}} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\},\ \mathrm{PrecisionGoal} \to 4\right]$$

{θo → 300.88, θue → 300.89, θle → 300.88, ψle → -0.285234}

$$\mathrm{Biotout} = \frac{\mathrm{H\ L}\ (\psi o - \psi le)}{(\mathrm{Al\ kl + Av\ Dv\ Cv}\psi)\ (\psi o - \psi le) + \mathrm{Av\ Dv\ Cv}\theta\ (\theta o - \theta le)}\ \text{/. sol}$$

4.47776

```
sf = (.151 Biotout + 1.03)^(-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.586118

check convergence

θvar - θo /. sol

-2.72848×10^{-12}

ψvar - ψo /. sol

-6.51701×10^{-14}

sf - vsf

-6.09072×10^{-11}

Results

ψue = ψU[LU] /. sol (* Water potential upper epidermis *)

-0.0842432

Temps = {θair, θsur, θue, θo, θle} /. sol

{301.46, 301.46, 300.89, 300.88, 300.88}

Potentials = {ψr, ψo, ψue, ψle} /. sol

{-0.01, -0.0773001, -0.0842432, -0.285234}

Fluxes

qsu /. sol (*Sensible flux upper*)

-23.0251

qru /. sol (*Radiative flux upper*)

9.5684

qsl /. sol (*Sensible flux lower*)

-23.4096

qrl /. sol (*Radiative flux lower*)

-3.44799

λ Jtran /. sol

42.769

Jtran /. sol

0.000975178

Leaf water potential and apparent conductance

$$\mathrm{LowerAvgPotential} = \frac{1}{\mathrm{L}} \int_0^{\mathrm{L}} \psi\mathrm{l}[x]\, dx\ \text{/. sol}$$

-0.181336

$$\mathrm{UpperAvgPotential} = \frac{1}{\mathrm{LU}} \int_0^{\mathrm{LU}} \psi\mathrm{U}[x]\, dx\ \text{/. sol}$$

-0.0808504

$$\mathrm{LWP} = \frac{\mathrm{L\ LowerAvgPotential + LU\ UpperAvgPotential}}{\mathrm{L + LU}}$$

-0.117025

Naive view of gradients

$$\text{Kleaf} = \frac{\text{Jtran}}{\psi\text{r} - \text{LWP}} \text{ /. sol (* Apparent conductance, mol m-2 s-1 MPa-1*)}$$

0.00911166

Informed view of gradients

$$\text{nondnumericpsiavg} = .721 - \frac{.219}{\text{Biotout}}$$

0.672092

$$\text{ScaledPsiAverageLeaf} = \text{nondnumericpsiavg} * (\psi\text{r} - \psi\text{le}) + \psi\text{le} \text{ /. sol}$$

(* cf 0.65 for oak (scaled k est on tree and leaf paper): given true psie and psir as defined in numeric simulation gives average potential in domain. Justified if 1D steady sol is scaled to 3d as it is here *)

-0.100252

$$\text{kleaf} = \text{Jtran} / (\psi\text{r} - \text{ScaledPsiAverageLeaf}) \text{ /. sol}$$

0.0108051

Evaporation distribution

$$\text{ProportionPeristomatal} = \frac{-\text{Al kl } \partial_x \psi\text{l}[x] \text{ /. } x \to \text{L}}{\text{Jtran}} \text{ /. sol}$$

0.629405

$$\text{ProportionEvaporationLower} = \frac{\text{Al kl } \psi\text{l}''[x] \text{ L}}{\text{Jtran}} \text{ /. sol}$$

0.00252386

$$\text{EvapOriginUpper} = \frac{\text{QU LU} - (\text{qsu} + \text{qru})}{\Pi\text{U}\psi\ \lambda\ \text{Jtran}} \text{ /. sol}$$

0.0253021

$$\text{VaporFluxintoLower} = \frac{1}{\text{Jtran}} (-\text{Av Dv Cv}\psi\ (\partial_x \psi\text{l}[x] \text{ /. } x \to 0) - \text{Av Dv Cv}\theta\ (\partial_x \theta\text{l}[x] \text{ /. } x \to 0)) \text{ /. sol}$$

0.368071

$$\text{PerivascularEvap} = \text{VaporFluxintoLower} - \text{EvapOriginUpper}$$

0.342769

ProportionPeristomatal + PerivascularEvap + EvapOriginUpper + ProportionEvaporationLower (* Check=1 *)

1.

ApparentLength = (ProportionPeristomatal + 0.5 ProportionEvaporationLower) (* Dist from evap site to vasc *)

0.630667

Analysis

Analyze proportion peristomatal at Lower Epidermis

$$\text{ProportionPeristomatal} = \frac{-\text{Al kl } \partial_x \psi\text{l}[x] \text{ /. } x \to \text{L}}{\text{Jtran}} \text{ /. sol}$$

0.629405

$$\text{PropPeri} = \frac{-(\text{qsl} + \text{qrl})}{\Pi\psi\ \text{Jtran}\ \lambda} + \left(1 + \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1} \text{ /. sol}$$

0.629405

$$\frac{-(\text{qsl} + \text{qrl})}{\Pi\psi\ \text{Jtran}\ \lambda} \text{ /. sol}$$

0.138072

$$\left(1 + \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1} \text{ /. sol}$$

0.491333

$$\text{ProportionInternalVapor} = \frac{1}{\text{Jtran}} (-\text{Av Dv Cv}\psi\ (\partial_x \psi\text{l}[x] \text{ /. } x \to \text{L}) - \text{Av Dv Cv}\theta\ (\partial_x \theta\text{l}[x] \text{ /. } x \to \text{L})) \text{ /. sol}$$

0.370595

ProportionPeristomatal + ProportionInternalVapor

1.

Condensation on lower epidermis if following is larger than one:

$$\frac{(\text{qsl} + \text{qrl})}{\text{Jtran}\ \lambda} \frac{\lambda\ \text{Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} \text{ /. sol}$$

-0.281014

Origin of vapor internally transported

$$\text{VaporFluxintoLower} = \frac{-\text{Av Dv Cv}\psi\ (\partial_x \psi\text{l}[x] \text{ /. } x \to 0) - \text{Av Dv Cv}\theta\ (\partial_x \theta\text{l}[x] \text{ /. } x \to 0)}{\text{Jtran}} \text{ /. sol}$$

0.368071

$$\text{ProportionEvaporationLower} = \frac{\text{Al kl } \psi\text{l}''[\text{x}]\ \text{L}}{\text{Jtran}}\ /.\ \text{sol}$$

0.00252386

$$\frac{\text{Q L}}{\Pi\psi\ \lambda\ \text{Jtran}}\ /.\ \text{sol}\ (*\text{integrating evap 2nd derv over L }*)$$

0.00252386

ProportionPeristomatal + VaporFluxintoLower + ProportionEvaporationLower (* Check=1 *)

1.

Analyze evaporation in upper part of leaf

$$\text{LiquidFluxintoUpper} = \frac{-\text{AUl kl } \partial_x \psi\text{U}[\text{x}]\ /.\ \text{x} \to 0}{\text{Jtran}}\ /.\ \text{sol}$$

0.0253021

$$\text{EvapOriginUpper} = \frac{\text{QU LU} - (\text{qsu} + \text{qru})}{\Pi\text{U}\psi\ \lambda\ \text{Jtran}}\ /.\ \text{sol}$$

0.0253021

VaporFluxintoUpper =

$$\frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[\text{x}]\ /.\ \text{x} \to 0) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[\text{x}]\ /.\ \text{x} \to 0)}{\text{Jtran}}\ /.\ \text{sol}$$

-0.0253021

CondensationUpperEpidermis =

$$\frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[\text{x}]\ /.\ \text{x} \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[\text{x}]\ /.\ \text{x} \to \text{LU})}{1}\ /.\ \text{sol}$$

-0.0000215319

$$\text{FluxFromUpperEpidermis} = \frac{-\text{AUl kl } \partial_x \psi\text{U}[\text{x}]\ /.\ \text{x} \to \text{LU}}{1}\ /.\ \text{sol}$$

0.0000215319

$$\frac{\lambda}{\text{QU LU}}\ \frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[\text{x}]\ /.\ \text{x} \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[\text{x}]\ /.\ \text{x} \to \text{LU})}{1}\ /.\ \text{sol}$$

-0.480883

$$\frac{\lambda}{\text{qsu} + \text{qru}}\ \frac{-\text{AUv Dv Cv}\psi\ (\partial_x \psi\text{U}[\text{x}]\ /.\ \text{x} \to \text{LU}) - \text{AUv Dv Cv}\theta\ (\partial_x \theta\text{U}[\text{x}]\ /.\ \text{x} \to \text{LU})}{1}\ /.\ \text{sol}$$

0.070176

qru /. sol

9.5684

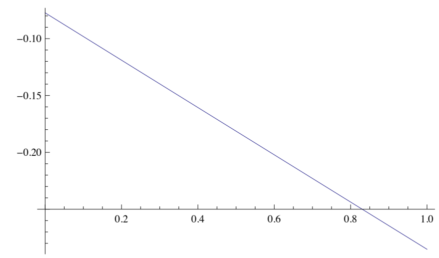
Plots

Lower Potential

psiL = ψl[x] /. sol /. x → L X

$-0.0773001 - 0.207518\ \text{X} + 0.00083213\ (-1.\ \text{X} + 0.5\ \text{X}^2)$

Plot[psiL, {X, 0, 1}]

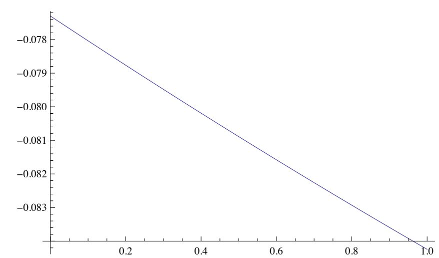


Upper Potential

psiU = ψU[x] /. sol /. x → LU X

$-0.0773001 - 0.006471\ \text{X} + 0.000944323\ (-1.\ \text{X} + 0.5\ \text{X}^2)$

Plot[psiU, {X, 0, 1}]

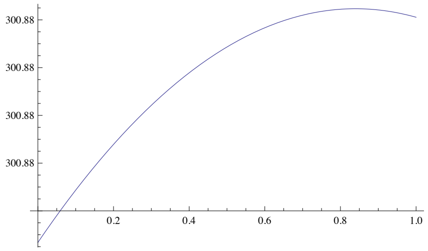


Lower Thermal

thetaL = θl[x] /. sol /. x → L X

$300.88 - 0.0000445442\ \text{X} + 0.000277759\ (1.\ \text{X} - 0.5\ \text{X}^2)$

```
Plot[thetaL, {X, 0, 1}]
```

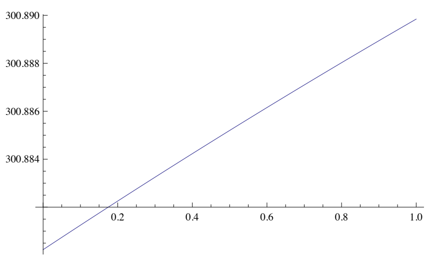


Upper Thermal

```
thetaU = θU[x] /. sol /. x → LU X
```

$300.88 + 0.00896185\,X + 0.00130782\,(1.\,X - 0.5\,X^2)$

```
Plot[thetaU, {X, 0, 1}]
```

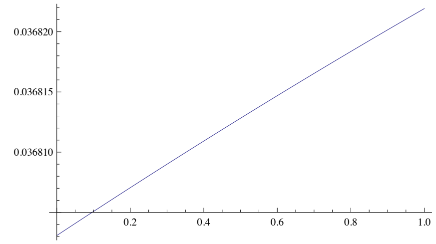


Upper Vapor Concentration

```
cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;
```



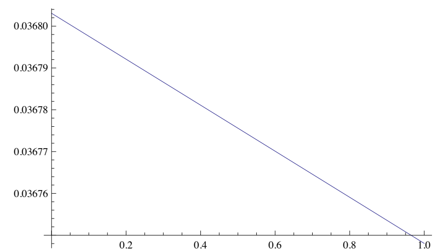
```
Plot[cU, {X, 0, 1}]
```



Lower Vapor Concentration

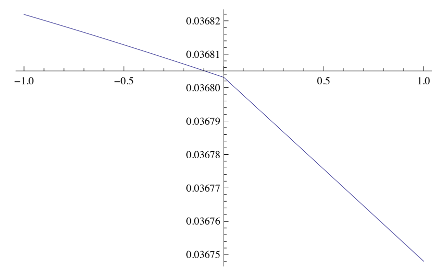
```
cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;
```

```
Plot[cL, {X, 0, 1}]
```



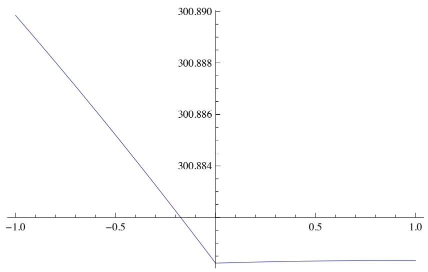
Whole leaf vapor

```
Show[Plot[cU /. X → -z, {z, 0, -1}], Plot[cL, {X, 0, 1}], PlotRange → All]
```



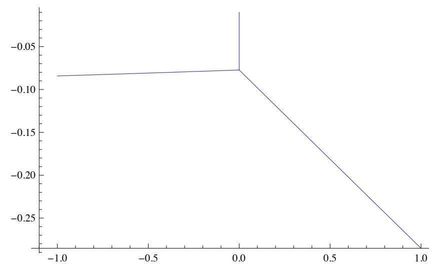
Whole leaf temperature

```
Show[Plot[thetaU /. X → -z, {z, 0, -1}], Plot[thetaL, {X, 0, 1}], PlotRange → All]
```



Whole leaf potential

```
Show[Plot[psiU /. X → -z, {z, 0, -1}], Plot[psiL, {X, 0, 1}],
 ListPlot[{{0, ψr}, {0, ψo /. sol}}, Joined → True],
 AxesOrigin → {-1.1, ψle /. sol}, PlotRange → All]
```

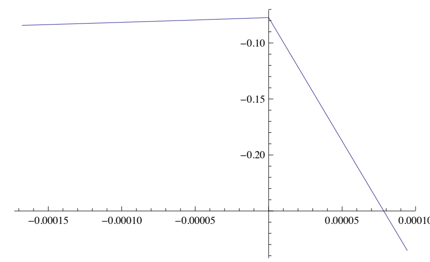


Export plot data

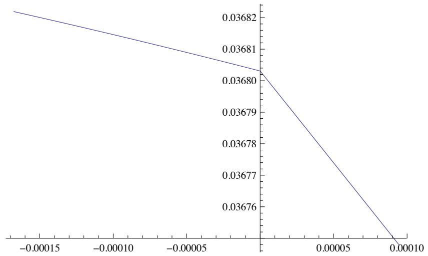
```
position = Range[-1, 1, .01];

outpos = Table[0, {Length[position]}];
```



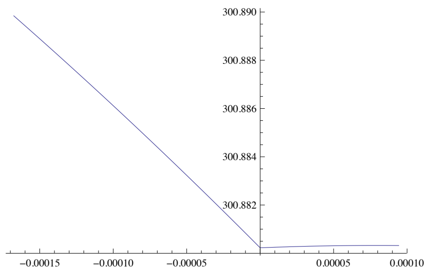
```
potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]

opdata = Transpose[{outpos, potential}];

otdata = Transpose[{outpos, temperature}];

ocdata = Transpose[{outpos, vapor}];

ListPlot[opdata, Joined → True]
```



```
ListPlot[ocdata, Joined → True]
```




```
ListPlot[otdata, Joined → True]
```




```
(*SetDirectory[ToFileName[NotebookDirectory[]]]*)
/Users/Tony/Dropbox/Manuscripts/Active
  manuscripts/vapor and liquid/mol fraction/Potometer

(*outfile="low_airy_"*)
low_airy_

(*Export[outfile<>"potential.xls",opdata]*)
low_airy_potential.xls

(*Export[outfile<>"vapor.xls",ocdata]*)
low_airy_vapor.xls

(*Export[outfile<>"temperature.xls",otdata]*)
low_airy_temperature.xls
```


Evaporative flux experiment: high load

```
Remove["Global`*"]
```

Remove::rmnsm : There are no symbols matching "Global`*". ≫

Fixed Parameter and initial values that update

For in lab, set cloud=1, gs choked, psir=-.05

```
gs = 0.1(* conserved value based on Li-cor measurements, ψle varies *)
0.1

θvar = θair; (* θair *)
ψvar = ψr;  (* ψr *)

vsf = .69(*  first guess*)
0.69
```

Main Environmental variables

```
TI = 0 ; (* hypothetical temp inc C *)

θairbase = 273.15 + 24.31 (* WITH TEMP COMPTO MATCH LOW LOAD EP T*)
297.46

θair = 273.15 + 24.31 + TI(* air temp WITH TMP COMP*)
297.46

PARout = 2000 (*PARo umol m-2 s-1*)
2000

cloud = 1 (* fraction sky cloudy *)
1

windspeed = 3.86(* Logan airport mean for day m s-1*)
3.86

ψr = -.01 ;(* stem potential from cov'd leaf in mpa *)
```

Environmental parameters

```
R = 8.3145; (*gas constant Joules per mole per Kelvin*)

Patm = 1.013 × 10^5 ;(*atm pressure in Pa*)

θsur = θair;
```

$$\chi a[\text{relh\_}, \theta\_] := \text{relh}\left(1.28\ \frac{\text{R }298.15}{\text{Patm}}\ \text{Exp}\left[-\frac{44\,000}{\text{R}}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]\right)$$

$$\text{cvbase} = .437\left(1.28\ \frac{298.15}{\theta\text{airbase}}\ \text{Exp}\left[-\frac{44\,000}{\text{R}}\left(\frac{1}{\theta\text{airbase}}-\frac{1}{298.15}\right)\right]\right)$$

(* wv conc conserved as temp inc *)

0.538043

$$\text{cvcase} = 1.28\ \frac{298.15}{\theta\text{air}}\ \text{Exp}\left[-\frac{44\,000}{\text{R}}\left(\frac{1}{\theta\text{air}}-\frac{1}{298.15}\right)\right]$$

(* wvc that would be satur at new temp *)

1.23122

RH = cvbase / cvcase (* new rh due to old wvc at new temp *)

0.437

χair = χa[RH, θair] (* new chi from new rh *)

0.0131363

αsr = 0.4(* absorbance over all solar PAR+NIR, 400 to 3000 nm, range .4 to .6 *)

0.4

r = 0.15 (* reflectance of surroundings to solar radiation Nobel *)

0.15

Assimilation = 19.7 × 10^-6(* mol m-2 s-1*)

0.0000197

AE = 479 000 * Assimilation (* 479 kJ per mole co2 fixed, J m-2 s-1 *)

$$\text{FE} = .01\,\alpha\text{sr}\left(\frac{1}{.45} * \text{PARout} * 10\wedge(-6) * 2.35 * 10\wedge 5\right)(1+\text{r})$$

(* J m-2 s-1 Energy lost to fluorescence, 3% of abs vis *)

9.4363

4.80444

$$\text{SRsun} = \alpha\text{sr}\left(\frac{1}{.45} * \text{PARout} * 10\wedge(-6) * 2.35 * 10\wedge 5\right)(1+\text{r}) - \text{AE} - \text{FE}$$

(* neglect refelctance *)
(* coeff J mol-1,
from Campbell intro env biophysics p 151: Total E incident is about 2x E in PAR,
then typical leaf abs is about half that amount,
so PAR plus NIR about equal to all E in PAR. OR 2.22 =1/.45 is factor
mult PAR capmbell but also could subtract ps E at 479 kj per mol co2,
and then fluorescence too. ps here is 20 umol m-2 s-2 so ~10 watts*)

466.204

Physical quantities

Po = 1.28 R 298.15 * 10^(-6) (* ref vapor pressure MPa *)

0.00317308

Pa = Patm * 10^-6 (* atm pressure in MPa*)

0.1013

$$\text{c}[\psi\_, \theta\_] := 1.28\ \frac{\text{R }298.15}{\text{Patm}}\ \text{Exp}\left[-\frac{44\,000}{\text{R}}\left(\frac{1}{\theta}-\frac{1}{298.15}\right)\right]\ \text{Exp}\left[\frac{(\psi+\text{Pa}-\text{Po})\ 18.07}{\text{R}\ \theta}\right]$$

(*mol fraction for psi in MPa*)

Cvθ = ∂θ c[ψvar, θ] /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar] /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)

0.00179893

0.000219768

λ = 44 000 - 43 (θvar - 298.15) (* Joules per Mol*)

44 029.7

$$\text{Dv} = \frac{\text{Patm}}{\text{R}\ \theta\text{var}}\ 2.13 * \left(\frac{\theta\text{var}}{273.15}\right)^{1.8} * 10\wedge(-5)$$

(* cDv for mol frac drving force out of leaf *)

0.00101712

kvh = .026;(* heat cond air J m-2 s-1 K-1*)

σ = 5.670373 * 10^(-8); (*stefan boltzmann J/m2/s/kelvin-4 *)

F = 1; (*view factor radiative from leaf*)

$$\epsilon\text{ac} = 1.72\left(\frac{\text{Patm}\ \chi\text{a}[\text{RH}, \theta\text{air}]}{1000\ \theta\text{air}}\right)^{1/7}$$

(*.32 rh logan airport,vapor pressure is in kpa, campbell p 162*)

0.794159

$$\text{ealt} = .553\left(\frac{\text{Patm}\ \chi\text{a}[\text{RH}, \theta\text{air}]}{100}\right)^{1/7}$$ **(* Brutsaert 1975*)**

0.800402

$$\text{ealt2} = 1.24\left(\frac{\text{Patm}\ \chi\text{a}[\text{RH}, \theta\text{air}]}{100\ \theta\text{air}}\right)^{1/7}$$ **(* Brutsaert 1975*)**

0.795532

ebrunt = .51 + .066 $\left(\frac{\text{Patm }\chi\text{a[RH, }\theta\text{air]}}{100}\right)^{1/2}$

0.75076

εa = (1 - .84 cloud) εac + .84 cloud (* campbell p 162*)

0.967065

Leaf parameters temp corrected

vcx = 1 + 0.024 TI (* xylem, see viscosity corr xls*)

1.

vct = 1 + .0812 TI (* tissue *)

1.

(*vcx=1/(1- 0.024 TI)(* xylem, for temp drop*)*)

(*vct=1/(1- .0812 TI)(* tissue for temp drop*)*)

εl = 0.96 (* emissivity leaf *)

0.96

αl = εl (* long wave abs leaf *)

0.96

charlength = .1 (* length normal to wind, m *)

0.1

δ = 4 $\left(\frac{\text{charlength}}{\text{windspeed}}\right)^{1/2}$ * 10^(-3) (* in m: cuvette leaf boundary layer*)

0.000643823

Al = .4(* .1263 oakair fraction, say 0.5 just mesophyll*)
Av = 1 - Al; (*Area fraction tissue in leaf from 2011 data 12.63%*)

0.4

AUl = .8;(* .1263 oakair fraction, say 0.5 just mesophyll*)
AUv = 1 - AUl; (*Area fraction tissue in leaf from 2011 data 12.63%*)

L = .72 × 131 × 10^(-6) (*-20 10^(-6)*)

0.00009432

LU = 1.28 * 131 × 10^(-6)

0.00016768

Check whole leaf air fraction

$\frac{\text{L Av + LU AUv}}{\text{L + LU}}$ (*vol weighted area fractions *)

0.344

klh = .28614;(* heat cond tissue*)

ktotal = vct 6.45 × 10^(-7);(* hyd cond leaf mol/m/s/mpa *)

kl = (ktotal - .1263 Dv Cvψ) / (1 - .1263)
  (* hyd cond tissue recon from on tree hydration (leaf at air temp)*)

7.05927×10^{-7}

H = .021; (* mol/m2/s/MPa from vein cutting, no steady state 1D scale factor *)

HA = vcx H vsf(* adj for viscosity (vcx) and 3D (vsf)*)

0.01449

Absorbed radiation

Q = 0.2 * SRsun / L (* w m-3 volumetric heat source,
  .2 is fraction of total abs solar rad abs in spongy *)

988 557.

QU = 0.8 * SRsun / LU

2.22425×10^{6}

Q L (* total solar in lower half of leaf*)

93.2407

QU LU

372.963

Q L + QU LU - SRsun

0.

Description of fluxes and relations to env parameters

(*delta is from licor 1.42 mol/m2/s g_bw, d=Dv*C_a/g_bw, C_a=40.49 mol/m3*)

qsl = $\frac{\text{kvh}}{\delta}$ (θle - θair);

qsu = $\frac{\text{kvh}}{\delta}$ (θue - θair);

qrl = $\sigma\,\epsilon l\,\theta le^4 - F\,\alpha l\,(\sigma\,\theta sur^4)$;

qru = $\sigma\,\epsilon l\,\theta ue^4 - F\,\alpha l\,(\sigma\,\epsilon a\,\theta air^4)$;

$$\chi[\psi\_, \theta\_] := 1.28 \frac{R\, 298.15}{Patm} \text{Exp}\left[-\frac{44\,000}{R}\left(\frac{1}{\theta} - \frac{1}{298.15}\right)\right] \text{Exp}\left[\frac{(\psi + Pa - Po)\, 18.07}{R\, \theta}\right]$$

(*for psi in MPa*)

$\chi e = \chi[\psi le, \theta le];$

$$gbl = \frac{2.13 * \left(\frac{\theta air}{273.15}\right)^{1.8} * 10\text{^}(-5)\, Patm}{R\, \theta air}$$

(* c(tair)*Dv(Tair), boundary layer molar conductivity *)

0.00101712

gbl / δ

1.57982

$$Jtran = \left(\frac{1}{gs} + \frac{\delta}{gbl}\right)^{-1} (\chi e - \chi air);$$

$\psi o = \psi r - Jtran / HA$

$$-0.01 - 6.49047 \left(-0.0131363 + 0.0313236\, e^{-5291.96\left(-0.00335402 + \frac{1}{\theta le}\right) + \frac{2.17331\,(0.0981269 + \psi le)}{\theta le}}\right)$$

First iteration

$$\Pi\psi = 1 + \frac{Al\, klh + Av\, kvh}{\lambda\, Av\, Dv\, Cv\theta} + \frac{Al\, klh + Av\, kvh}{\lambda\, Al\, kl} \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

4.96854

$$\Pi\theta = 1 + \frac{\lambda\, Al\, kl}{Al\, klh + Av\, kvh} \frac{Av\, Dv\, Cv\theta}{Al\, kl + Av\, Dv\, Cv\psi}$$

(* greater than one favors latent over conduction *)

1.25198

$$\Pi U\psi = 1 + \frac{AUl\, klh + AUv\, kvh}{\lambda\, AUv\, Dv\, Cv\theta} + \frac{AUl\, klh + AUv\, kvh}{\lambda\, AUl\, kl} \frac{Cv\psi}{Cv\theta}$$ (* greater than one favors conduction over latent *)

16.68

$$\Pi U\theta = 1 + \frac{\lambda\, AUl\, kl}{AUl\, klh + AUv\, kvh} \frac{AUv\, Dv\, Cv\theta}{AUl\, kl + AUv\, Dv\, Cv\psi}$$

(* greater than one favors latent over conduction *)

1.06378

Global energy conservation

$SRsun == qsu + qru + qrl + qsl + \lambda\, Jtran;$

Solution lower thermal field

$$\theta l[x\_] := \theta o + \left(-\frac{x^2}{2\, L^2} + \frac{x}{L}\right) \frac{Q\, L^2}{\Pi\theta\, (Al\, klh + Av\, kvh)} + \left(-\frac{(qsl + qrl)\, L}{\Pi\theta\, (Al\, klh + Av\, kvh)} - \frac{\lambda\, Al\, kl\, Jtran\, L}{\Pi\theta\, (Al\, klh + Av\, kvh)} \frac{1}{(Al\, kl + Av\, Dv\, Cv\psi)}\right) \frac{x}{L}$$

Solution upper thermal field

$$\theta U[x\_] := \theta o + \left(-\frac{x^2}{2\, LU^2} + \frac{x}{LU}\right) \frac{QU\, LU^2}{\Pi U\theta\, (AUl\, klh + AUv\, kvh)} - \left(\frac{(qsu + qru)\, LU}{\Pi U\theta\, (AUl\, klh + AUv\, kvh)}\right) \frac{x}{LU}$$

Solution upper potential field

$$\psi U[x\_] := \psi o + \left(\frac{x^2}{2\, LU^2} - \frac{x}{LU}\right) \frac{QU\, LU^2}{\Pi U\psi\, AUl\, kl\, \lambda} + \left(\frac{(qsu + qru)\, LU}{\Pi U\psi\, AUl\, kl\, \lambda}\right) \frac{x}{LU}$$

Solution lower potential field

$$\psi l[x\_] := \psi o + \left(\frac{x^2}{2\, L^2} - \frac{x}{L}\right) \frac{Q\, L^2}{\Pi\psi\, Al\, kl\, \lambda} + \left(\frac{(qsl + qrl)\, L}{\Pi\psi\, Al\, kl\, \lambda} - \frac{Jtran\, L}{\Pi\psi\, Al\, kl\, \lambda} \frac{Al\, klh + Av\, kvh}{Av\, Dv\, Cv\theta}\right) \frac{x}{L}$$

Solve system

$$sol = \text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1, \frac{\theta l[L]}{\theta le} == 1, SRsun == qsu + qru + qrl + qsl + \lambda\, Jtran, \frac{\psi l[L]}{\psi le} == 1\right\}, \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\}, \text{PrecisionGoal} \to 4\right]$$

$\{\theta o \to 301.321, \theta ue \to 301.317, \theta le \to 301.208, \psi le \to -0.387611\}$

update seed values

$$Biotout = \frac{H\, L\, (\psi o - \psi le)}{(Al\, kl + Av\, Dv\, Cv\psi)\, (\psi o - \psi le) + Av\, Dv\, Cv\theta\, (\theta o - \theta le)} \;/.\; sol$$

2.02135

```
θvar = θo /. sol
ψvar = ψo /. sol
```

301.321

-0.167684

```
vsf = (.151 Biotout + 1.03)^(-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.748938

Second iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00219962

0.000272189

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$ (* greater than one favors conduction over latent *)

4.49491

$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$ (* greater than one favors latent over conduction *)

1.28613

$$\Pi\text{U}\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$ (* greater than one favors conduction over latent *)

14.0481

$$\Pi\text{U}\theta = 1 + \frac{\lambda\ \text{AUl kl}}{\text{AUl klh} + \text{AUv kvh}}\ \frac{\text{AUv Dv Cv}\theta}{\text{AUl kl} + \text{AUv Dv Cv}\psi}$$ (* greater than one favors latent over conduction *)

1.07664

sol =
$$\text{FindRoot}\left[\left\{\frac{\theta\text{U[LU]}}{\theta\text{ue}} == 1,\ \frac{\theta\text{l[L]}}{\theta\text{le}} == 1,\ \text{SRsun} == \text{qsu} + \text{qru} + \text{qrl} + \text{qsl} + \lambda\ \text{Jtran},\ \frac{\psi\text{l[L]}}{\psi\text{le}} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\},\ \text{PrecisionGoal} \to 4\right]$$

{θo → 301.317, θue → 301.314, θle → 301.21, ψle → -0.327861}

update seed values

$$\text{Biotout} = \frac{\text{H L}\ (\psi o - \psi le)}{(\text{Al kl} + \text{Av Dv Cv}\psi)\ (\psi o - \psi le) + \text{Av Dv Cv}\theta\ (\theta o - \theta le)}\ /.\ \text{sol}$$

1.46977

```
sf = (.151 Biotout + 1.03)^(-1)
```

0.798764

```
θvar = θo /. sol
ψvar = ψo /. sol
```

301.317

-0.167815

```
vsf = (.151 Biotout + 1.03)^(-1) (* vasc scale factor
   steady state based on geometry of veis and eff biot number*)
```

0.798764

Third iteration

```
Cvθ = ∂θ c[ψvar, θ]  /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂ψ c[ψ, θvar]  /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)
```

0.00219918

0.000272132

$$\Pi\psi = 1 + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Av Dv Cv}\theta} + \frac{\text{Al klh} + \text{Av kvh}}{\lambda\ \text{Al kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$ (* greater than one favors conduction over latent *)

4.49533

$$\Pi\theta = 1 + \frac{\lambda\ \text{Al kl}}{\text{Al klh} + \text{Av kvh}}\ \frac{\text{Av Dv Cv}\theta}{\text{Al kl} + \text{Av Dv Cv}\psi}$$ (* greater than one favors latent over conduction *)

1.2861

$$\Pi\text{U}\psi = 1 + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUv Dv Cv}\theta} + \frac{\text{AUl klh} + \text{AUv kvh}}{\lambda\ \text{AUl kl}}\ \frac{\text{Cv}\psi}{\text{Cv}\theta}$$ (* greater than one favors conduction over latent *)

14.0504

$$\Pi U\theta = 1 + \frac{\lambda\, AUl\, kl}{AUl\, klh + AUv\, kvh}\,\frac{AUv\, Dv\, Cv\theta}{AUl\, kl + AUv\, Dv\, Cv\psi}$$
(* greater than one favors latent over conduction *)

1.07663

sol =

$$\text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\, Jtran,\ \frac{\psi l[L]}{\psi le} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\}, \text{PrecisionGoal} \to 4\right]$$

$\{\theta o \to 301.317,\ \theta ue \to 301.314,\ \theta le \to 301.21,\ \psi le \to -0.32792\}$

update seed values

$$Biotout = \frac{H\, L\, (\psi o - \psi le)}{(Al\, kl + Av\, Dv\, Cv\psi)\,(\psi o - \psi le) + Av\, Dv\, Cv\theta\,(\theta o - \theta le)}\ /.\ sol$$

1.47031

sf = (.151 Biotout + 1.03) ^ (-1)

0.798712

θvar = θo /. sol
ψvar = ψo /. sol

301.317

-0.167815

vsf = (.151 Biotout + 1.03) ^ (-1) (* vasc scale factor steady state based on geometry of veis and eff biot number*)

0.798712

Fourth iteration

Cvθ = ∂_θ c[ψvar, θ] /. θ → θvar (* linearization of dχ/dT 1/k *)
Cvψ = ∂_ψ c[ψ, θvar] /. ψ → ψvar (* linearization of dχ/dpsi 1/mpa *)

0.00219919

0.000272132

$$\Pi\psi = 1 + \frac{Al\, klh + Av\, kvh}{\lambda\, Av\, Dv\, Cv\theta} + \frac{Al\, klh + Av\, kvh}{\lambda\, Al\, kl}\,\frac{Cv\psi}{Cv\theta}$$
(* greater than one favors conduction over latent *)

4.49533

$$\Pi\theta = 1 + \frac{\lambda\, Al\, kl}{Al\, klh + Av\, kvh}\,\frac{Av\, Dv\, Cv\theta}{Al\, kl + Av\, Dv\, Cv\psi}$$
(* greater than one favors latent over conduction *)

1.2861

$$\Pi U\psi = 1 + \frac{AUl\, klh + AUv\, kvh}{\lambda\, AUv\, Dv\, Cv\theta} + \frac{AUl\, klh + AUv\, kvh}{\lambda\, AUl\, kl}\,\frac{Cv\psi}{Cv\theta}$$
(* greater than one favors conduction over latent *)

14.0504

$$\Pi U\theta = 1 + \frac{\lambda\, AUl\, kl}{AUl\, klh + AUv\, kvh}\,\frac{AUv\, Dv\, Cv\theta}{AUl\, kl + AUv\, Dv\, Cv\psi}$$
(* greater than one favors latent over conduction *)

1.07663

sol =

$$\text{FindRoot}\left[\left\{\frac{\theta U[LU]}{\theta ue} == 1,\ \frac{\theta l[L]}{\theta le} == 1,\ SRsun == qsu + qru + qrl + qsl + \lambda\, Jtran,\ \frac{\psi l[L]}{\psi le} == 1\right\},\ \{\{\theta o, \theta air\}, \{\theta ue, \theta air\}, \{\theta le, \theta air\}, \{\psi le, \psi r\}\}, \text{PrecisionGoal} \to 4\right]$$

$\{\theta o \to 301.317,\ \theta ue \to 301.314,\ \theta le \to 301.21,\ \psi le \to -0.32792\}$

$$Biotout = \frac{H\, L\, (\psi o - \psi le)}{(Al\, kl + Av\, Dv\, Cv\psi)\,(\psi o - \psi le) + Av\, Dv\, Cv\theta\,(\theta o - \theta le)}\ /.\ sol$$

1.47031

sf = (.151 Biotout + 1.03) ^ (-1) (* vasc scale factor steady state based on geometry of veis and eff biot number*)

0.798712

check convergence

θvar - θo /. sol

3.68965×10^{-9}

ψvar - ψo /. sol

1.26746×10^{-10}

sf - vsf

5.13642×10^{-8}

Results

```
ψue = ψU[LU] /. sol (* Water potential upper epidermis *)
```
-0.165083

```
Temps = {θair, θsur, θue, θo, θle} /. sol
```
{297.46, 297.46, 301.314, 301.317, 301.21}

```
Potentials = {ψr, ψo, ψue, ψle} /. sol
```
{-0.01, -0.167815, -0.165083, -0.32792}

Fluxes

```
qsu /. sol (*Sensible flux upper*)
```
155.62

```
qru /. sol (*Radiative flux upper*)
```
36.5536

```
qsl /. sol (*Sensible flux lower*)
```
151.444

```
qrl /. sol (*Radiative flux lower*)
```
21.9017

```
λ Jtran /. sol
```
100.684

```
Jtran /. sol
```
0.00228674

Leaf water potential and apparent conductance

$$\text{LowerAvgPotential} = \frac{1}{L}\int_0^L \psi l[x]\, dx \text{ /. sol}$$
-0.26098

$$\text{UpperAvgPotential} = \frac{1}{LU}\int_0^{LU} \psi U[x]\, dx \text{ /. sol}$$
-0.181366

$$\text{LWP} = \frac{L\,\text{LowerAvgPotential} + LU\,\text{UpperAvgPotential}}{L + LU}$$
-0.210027

Naive view of gradients

$$\text{Kleaf} = \frac{\text{Jtran}}{\psi r - \text{LWP}} \text{ /. sol (* Apparent conductance, mol m-2 s-1 MPa-1*)}$$
0.0114321

Informed view of gradients

$$\text{nondnumericpsiavg} = .721 - \frac{.219}{\text{Biotout}}$$
0.572051

```
ScaledPsiAverageLeaf = nondnumericpsiavg * (ψr - ψle) + ψle /. sol
  (* cf 0.65 for oak (scaled k est on tree and leaf paper): given true psie
    and psir as defined in numeric simulation gives average potential in
    domain. Justified if 1D steady sol is scaled to 3d as it is here *)
```
-0.146053

```
kleaf = Jtran / (ψr - ScaledPsiAverageLeaf) /. sol
```
0.0168077

Evaporation distribution

$$\text{ProportionPeristomatal} = \frac{-\text{Al kl } \partial_x \psi l[x] \text{ /. } x \to L}{\text{Jtran}} \text{ /. sol}$$
0.106603

$$\text{ProportionEvaporationLower} = \frac{\text{Al kl } \psi l''[x]\, L}{\text{Jtran}} \text{ /. sol}$$
0.206007

$$\text{EvapOriginUpper} = \frac{QU\, LU - (\text{qsu} + \text{qru})}{\Pi U\psi\, \lambda\, \text{Jtran}} \text{ /. sol}$$
0.127797

$$\text{VaporFluxintoLower} = \frac{1}{\text{Jtran}}(-\text{Av Dv Cv}\psi\,(\partial_x \psi l[x] \text{ /. } x \to 0) - \text{Av Dv Cv}\theta\,(\partial_x \theta l[x] \text{ /. } x \to 0)) \text{ /. sol}$$
0.68739

```
PerivascularEvap = VaporFluxintoLower - EvapOriginUpper
```
0.559593

```
ProportionPeristomatal + PerivascularEvap +
 EvapOriginUpper + ProportionEvaporationLower (* Check=1 *)
```
1.

```
ApparentLength = (ProportionPeristomatal + 0.5 ProportionEvaporationLower)
  (* Dist from evap site to vasc *)
```

0.209606

Analysis

Analyze proportion peristomatal at Lower Epidermis

$$\text{ProportionPeristomatal} = \frac{-\text{Al kl } \partial_x \psi l[x] \text{ /. } x \to L}{\text{Jtran}} \text{ /. sol}$$

0.106603

$$\text{PropPeri} = \frac{-(\text{qsl} + \text{qrl})}{\Pi\psi \text{ Jtran } \lambda} + \left(1 + \frac{\lambda \text{ Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1} \text{ /. sol}$$

0.106603

$$\frac{-(\text{qsl} + \text{qrl})}{\Pi\psi \text{ Jtran } \lambda} \text{ /. sol}$$

-0.382992

$$\left(1 + \frac{\lambda \text{ Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} + \frac{\text{Av Dv Cv}\psi}{\text{Al kl}}\right)^{-1} \text{ /. sol}$$

0.489595

$$\text{ProportionInternalVapor} = \frac{1}{\text{Jtran}}(-\text{Av Dv Cv}\psi \text{ } (\partial_x \psi l[x] \text{ /. } x \to L) - \text{Av Dv Cv}\theta \text{ } (\partial_x \theta l[x] \text{ /. } x \to L)) \text{ /. sol}$$

0.893397

```
ProportionPeristomatal + ProportionInternalVapor
```

1.

Condensation on lower epidermis if following is larger than one:

$$\frac{(\text{qsl} + \text{qrl})}{\text{Jtran } \lambda} \frac{\lambda \text{ Av Dv Cv}\theta}{\text{Al klh} + \text{Av kvh}} \text{ /. sol}$$

0.782263

Origin of vapor internally transported

$$\text{VaporFluxintoLower} = \frac{1}{\text{Jtran}}(-\text{Av Dv Cv}\psi \text{ } (\partial_x \psi l[x] \text{ /. } x \to 0) - \text{Av Dv Cv}\theta \text{ } (\partial_x \theta l[x] \text{ /. } x \to 0)) \text{ /. sol}$$

0.68739

$$\text{ProportionEvaporationLower} = \frac{\text{Al kl } \psi l''[x] \text{ L}}{\text{Jtran}} \text{ /. sol}$$

0.206007

$$\frac{\text{Q L}}{\Pi\psi \text{ } \lambda \text{ Jtran}} \text{ /. sol (*integrating evap 2nd derv over L *)}$$

0.206007

```
ProportionPeristomatal + VaporFluxintoLower +
  ProportionEvaporationLower (* Check=1 *)
```

1.

Analyze evaporation in upper part of leaf

$$\text{LiquidFluxintoUpper} = \frac{-\text{AUl kl } \partial_x \psi U[x] \text{ /. } x \to 0}{\text{Jtran}} \text{ /. sol}$$

0.127797

$$\text{EvapOriginUpper} = \frac{\text{QU LU} - (\text{qsu} + \text{qru})}{\Pi U\psi \text{ } \lambda \text{ Jtran}} \text{ /. sol}$$

0.127797

$$\text{VaporFluxintoUpper} = \frac{1}{\text{Jtran}}(-\text{AUv Dv Cv}\psi \text{ } (\partial_x \psi U[x] \text{ /. } x \to 0) - \text{AUv Dv Cv}\theta \text{ } (\partial_x \theta U[x] \text{ /. } x \to 0)) \text{ /. sol}$$

-0.127797

$$\text{CondensationUpperEpidermis} = \frac{1}{1}(-\text{AUv Dv Cv}\psi \text{ } (\partial_x \psi U[x] \text{ /. } x \to LU) - \text{AUv Dv Cv}\theta \text{ } (\partial_x \theta U[x] \text{ /. } x \to LU)) \text{ /. sol}$$

0.000310642

$$\text{FluxFromUpperEpidermis} = \frac{-\text{AUl kl } \partial_x \psi U[x] \text{ /. } x \to LU}{1} \text{ /. sol}$$

-0.000310642

$$\frac{\lambda}{\text{QU LU}} \frac{1}{1}(-\text{AUv Dv Cv}\psi \text{ } (\partial_x \psi U[x] \text{ /. } x \to LU) - \text{AUv Dv Cv}\theta \text{ } (\partial_x \theta U[x] \text{ /. } x \to LU)) \text{ /. sol}$$

0.0366724

$$\frac{\lambda}{\text{qsu} + \text{qru}} \frac{1}{1}(-\text{AUv Dv Cv}\psi \text{ } (\partial_x \psi U[x] \text{ /. } x \to LU) - \text{AUv Dv Cv}\theta \text{ } (\partial_x \theta U[x] \text{ /. } x \to LU)) \text{ /. sol}$$

0.0711722

```
qru /. sol
```

36.5536

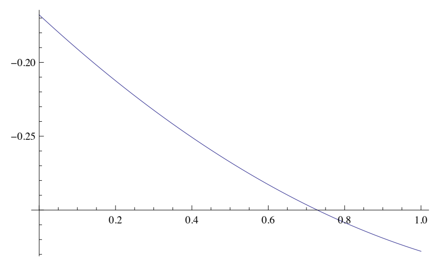
Plots

Lower Potential

```
psiL = ψl[x] /. sol /. x → L X
```

$-0.167815 - 0.081427\, X + 0.157356\, \left(-1.\, X + 0.5\, X^2\right)$

```
Plot[psiL, {X, 0, 1}]
```

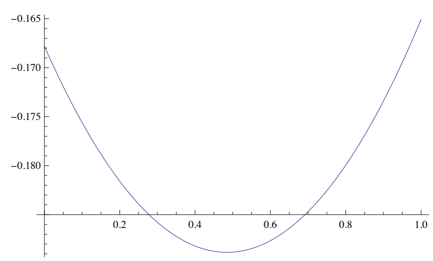


Upper Potential

```
psiU = ψU[x] /. sol /. x → LU X
```

$-0.167815 + 0.0922341\, X + 0.179004\, \left(-1.\, X + 0.5\, X^2\right)$

```
Plot[psiU, {X, 0, 1}]
```

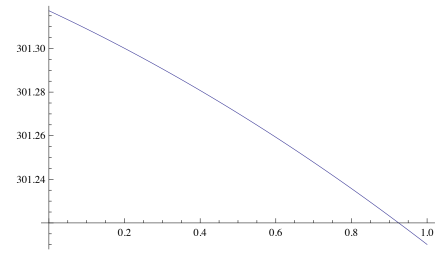


Lower Thermal

```
thetaL = θl[x] /. sol /. x → L X
```

$301.317 - 0.133499\, X + 0.0525782\, \left(1.\, X - 0.5\, X^2\right)$


```
Plot[thetaL, {X, 0, 1}]
```

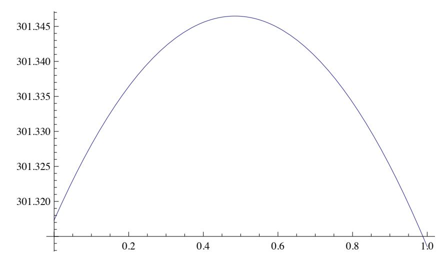


Upper Thermal

```
thetaU = θU[x] /. sol /. x → LU X
```

$301.317 - 0.127846\, X + 0.248118\, \left(1.\, X - 0.5\, X^2\right)$

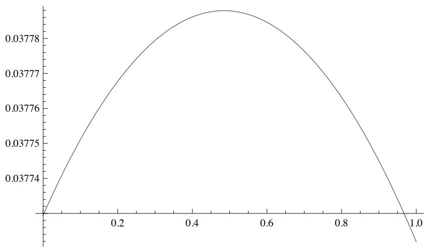
```
Plot[thetaU, {X, 0, 1}]
```



Upper Vapor Concentration

```
cU = c[ψ, θ] /. ψ → psiU /. θ → thetaU;
```

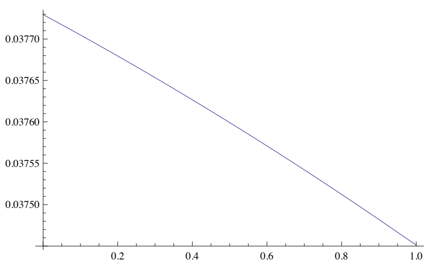
```
Plot[cU, {X, 0, 1}]
```



Lower Vapor Concentration

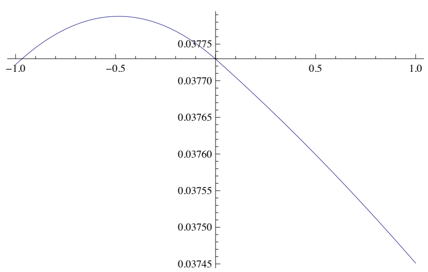
```
cL = c[ψ, θ] /. ψ → psiL /. θ → thetaL;
```

```
Plot[cL, {X, 0, 1}]
```



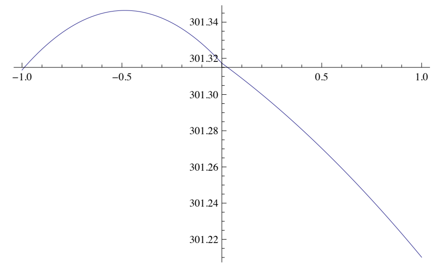
Whole leaf vapor

```
Show[Plot[cU /. X → -z, {z, 0, -1}], Plot[cL, {X, 0, 1}], PlotRange → All]
```



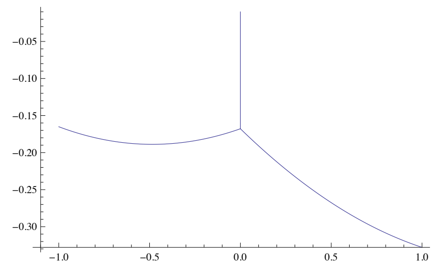
Whole leaf temperature

```
Show[Plot[thetaU /. X → -z, {z, 0, -1}], Plot[thetaL, {X, 0, 1}], PlotRange → All]
```



Whole leaf potential

```
Show[Plot[psiU /. X → -z, {z, 0, -1}], Plot[psiL, {X, 0, 1}],
 ListPlot[{{0, ψr}, {0, ψo /. sol}}, Joined → True],
 AxesOrigin → {-1.1, ψle /. sol}, PlotRange → All]
```

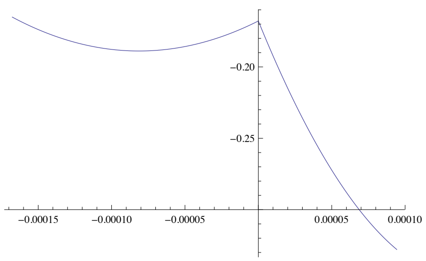


Export plot data

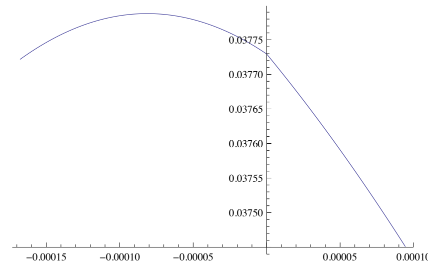
```
position = Range[-1, 1, .01];
```

```
outpos = Table[0, {Length[position]}];
```

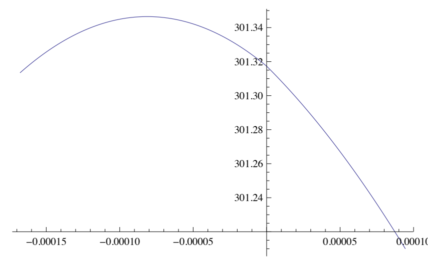
```
potential = Table[0, {Length[position]}];
vapor = Table[0, {Length[position]}];
temperature = Table[0, {Length[position]}];

Do[
 If[ NonNegative[position[[i]]], potential[[i]] = psiL /. X → position[[i]],
  potential[[i]] = psiU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  vapor[[i]] = cL /. X → position[[i]], vapor[[i]] = cU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]], temperature[[i]] = thetaL /. X → position[[i]],
  temperature[[i]] = thetaU /. X → -position[[i]]],
 {i, Length[position]}]
Do[
 If[ NonNegative[position[[i]]],
  outpos[[i]] = L * position[[i]], outpos[[i]] = LU * position[[i]]],
 {i, Length[position]}]

opdata = Transpose[{outpos, potential}];

otdata = Transpose[{outpos, temperature}];

ocdata = Transpose[{outpos, vapor}];

ListPlot[opdata, Joined → True]
```




```
ListPlot[ocdata, Joined → True]
```




```
ListPlot[otdata, Joined → True]
```




```
(*SetDirectory[ToFileName[NotebookDirectory[]]]*)

/Users/Tony/Dropbox/Manuscripts/Active
  manuscripts/vapor and liquid/mol fraction/Potometer

(*outfile="highTC_airy_"*)

highTC_airy_

(*Export[outfile<>"potential.xls",opdata]*)

highTC_airy_potential.xls

(*Export[outfile<>"vapor.xls",ocdata]*)

highTC_airy_vapor.xls

(*Export[outfile<>"temperature.xls",otdata]*)

highTC_airy_temperature.xls
```