Flow in bulk solution:

The Navier Stokes equations of incompressible flows in 3D, non-porous systems contains 3 momentum equations and 1 incompressibility constraint. Here, ρ is the fluid density (kg·m^{-3}); u is the flow velocity (L·min^{-1}); P is the pressure (Pa); f is the given force (kg·m·t^{-2}); σ is the stress kg·m^{-1}·t^{-2}.

$$\rho\left(\frac{\partial u}{\partial t}+u\cdot\nabla u-f\right)-\nabla\cdot\sigma=0$$
$$\nabla\cdot u=0$$

In order to define initial conditions, stress must be defined in terms of pressure and fluid viscosity.

$$\sigma=-pI+2\eta\varepsilon$$

Here, η is the dynamic viscosity (kg·m^{-1}s^{-1}). I is the identity matrix. The strain-rate tensor, ε, can be expanded into:

$$\varepsilon=\frac{1}{2}(\nabla u+(\nabla u)^T)$$

Substituting and simplifying:

$$\nabla\cdot\sigma=\nabla\cdot(-PI+\eta(\nabla u+(\nabla u)^T))$$
$$\nabla\cdot\sigma=\nabla\cdot(-PI+\eta(\nabla\cdot(\nabla u)+\nabla\cdot(\nabla u)^T)$$
$$\nabla\cdot\sigma=\nabla\cdot(-PI+\eta\big(\nabla\cdot(\nabla u)+\nabla(\nabla\cdot u)\big)$$

Since $\nabla\cdot u=0$ due to the incompressibility constraint:

$$\nabla\cdot\sigma=-\nabla P+\eta\nabla^2u$$

Therefore, assuming that f is 0 for laminar flow:

$$\rho\left(\frac{\partial u}{\partial t}+u\cdot\nabla u\right)+\nabla P-\eta\nabla^2u=0$$

Since the time-independent Navier Stokes is solved in order to reduce the complexity of the system, $\frac{\partial u}{\partial t}$ drops out:

$$\rho(u\cdot\nabla u)+\nabla P-\eta\nabla^2u=0$$

Flow in porous beads:

For flow in porous medium, we use Brinkman's equation,[24] a modification of Navier Stokes to take into account permeability and porosity of the porous medium. We assume f is zero. Here, k is the permeability (m^2); and ε_p is the porosity (dimensionless).

$$\rho\left(\frac{\partial u}{\partial t}+u\cdot\nabla u-f\right)-\nabla\cdot\sigma=0$$
$$\frac{\rho}{\varepsilon_p}\left(\frac{\partial u}{\partial t}+u\cdot\nabla u\right)-\nabla\cdot\sigma=0$$
$$\frac{\rho}{\varepsilon_p}\left(\frac{\partial u}{\partial t}\right)+\frac{\eta}{k}u-f-\nabla\cdot\sigma=0$$

In a similar fashion, we expand stress out into its pressure and viscosity terms.

$$\nabla\cdot\sigma=\nabla\cdot(-PI+\frac{\eta}{\varepsilon_p}(\nabla u+(\nabla u)^T))$$

After substituting and simplifying, we end up with:

$$\frac{\rho}{\varepsilon_p}\left(\frac{\partial u}{\partial t}\right)+\frac{\eta}{k}u+\nabla P-\frac{\eta}{\varepsilon_p}\nabla^2u=0$$