Micro-bias and macro-evolution. Supplementary Material

August 1, 2008

Selection of parameter values

The study by Moreira et al. (*1*) demonstrated that populations comprised entirely of naive agents will efficiently—that is, rapidly and accurately—solve the density classification task under very broad conditions. In particular, they demonstrated:

- Infinitely large populations of agents in a one dimensinal circular lattice require only an infinitesimal number of random connections and an infinitesimal degree of noise in the communication between agents in order to be able to detect even extremely small majorities.
- There is a phase transition to an inefficient regime for a critical value of the noise amplitude.
- For fixed topology (that is circular networks with a certain probability of rewiring), the critical value of the noise amplitude is an increasing function of the average number k of neighbors of the agents.
- Finite populations require a finite fraction of random connections among agents in order to display efficient behavior. This fraction decreases with population size.

In the studies reported in our manuscript, we set our system to populations of 401 agents. For this size, (*1*) reported that for populations comprised entirely of naive agents the efficient regime is achieved, for a broad range of noise amplitudes, for a rewiring probability of $p \simeq 0.1$.

Rewiring probability

In this study, we are primarily interested in the effect of conservative and partisan strategies on the efficiency of the system. For this reason, we chose a set of system parmeters such that for which a population comprised entirely of naive agents would be efficient. First, we set the rewiring probability to 0.15. This way we assure that the system is connected as a small-world.

Magnitude of initial majority

In order to avoid finite-size effects, we determine the initial magnitude of the majority, that is the fraction of agents initially in the same state, above which perfect accuracy is achieved. Specifically, we define a parameter $c > 0.5$, which is the probability that an agent is initially in state "1." This implies that the majority state will always be "1."

To find the "best" value for c, we explore the accuracy with which a population of N agents completes the density classification task within $2N$ time steps as a function of c and for a variety of system sizes $N \geq 401$ (Fig. S1A). For these population sizes, a rewiring probability of 0.15 guaranties that the systems is connected as a small-world.

We want to select the combination $\{N, c\}$, such that c is as small as possible (so the problem is as hard as possible) while allowing the system to attain a high accuracy ($\geq 0.95\%$) with a low computational cost. Our results suggest that $N = 401$ and $c = 0.57$ fulfill our requirements [1].

[1]Note that while c sets the distribution of preferred states for well-intentioned agents, it does not for partisans. Partisans have a pre-assigned preferred state, thus they will immediately change to their preferred state. This effect is visible if one compares the results of Figs. 2C and S3. The results shown in Fig. 2C were obtained for systems in which the preferred states of partisan agents are equally distributed, while the results shown in Fig. S3 were obtained for systems in which the probability of a partisan

Definition of efficiency

To better quantify the effect of non-naive agents in the efficiency of the system, we introduce an alternative definition of efficiency to that considered in (*1*) (see main text for details). Figure S1B demonstrates that, despite the change in definition, the average efficiency we obtain for systems with naive agents using $c = 0.57$ is consistent with the efficiency measured using the definition in (*1*).

Robustness to change of parameter values

The results reported in the main manuscript show that the presence of conservatives also makes the system more robust to noise. To analyze whether conservatives make a system more robust to noise independently of k, we explore the efficiency for $k = 4$, $N = 401$, $p = 0.15$, $s = 2/5$, and $s = 3/5$ (Fig. S4). Despite the differences in the range of s, for $s = 2/5$ one can see the same trend since the presence of conservatives makes the system more robust to noise.

Estimation of the convergence time

To estimate the "typical" number of time steps it takes for a system to reach the steady state, we use the following method. First, we generate N_h time series $\varepsilon^i(t)$ for the same population parameters. For each time series, we start from different initial conditions and let the system evolve for T time steps (Fig. S5A). We then compute the average time series for the N_h histories $\overline{\varepsilon}(t) = \frac{1}{N_h}\sum_{i=1}^{N_h} \varepsilon^i(t)$. To estimate the convergence time we consider the "cumulative efficiency" at time t, that is, at each time step t we compute the average efficiency for the remaining time steps $\overline{\varepsilon_{av}(t)} = \frac{1}{T-t}\sum_{t'>t} \overline{\varepsilon(t')}$. The resulting

preferring state "1" is equal to $c = 0.57$. Our results show that having equally distributed preferred states or having slightly more partisans toward the majority position does not have a significant impact on the systems' ability to solve the density classification task.

curve (Fig. S5B) shows how the system reaches a constant average efficiency. We find that this curve can be fit to a stretched exponential

$$f(x) = A(1 - Be^{-(\frac{t}{\tau})^{\beta}}) \tag{1}$$

We estimate the convergence time t^* as the time at which the average efficiency is within 5% of the asymptotic efficiency $\overline{\varepsilon_\infty(t)}$

$$t^* = \tau(-\ln\frac{0.05}{B})^{\beta}. \tag{2}$$

As shown in Fig. 3A in the main text, the time needed for a system to reach the steady state increases with the fraction of conservatives f_c. The effect of conservatives on the convergence time can be explained in terms of resistance to a driving force. The driving force that pushes the system toward consensus is provided by three parameters: c, p and η. Because $c > 0.5$, the system is driven toward state "1." Conservatives holding state "-1," however, oppose the driving force by resisting the influence of their naive neighbors. Therefore, the larger the bias strength and the larger the fraction of conservatives, the longer it takes the system to overcome that resistance.

The same idea explains why the system is more robust to noise if there are conservatives present. The stronger the bias strength, the stronger the resistance to noise and, as a consequence, to the loss of efficiency. Therefore, increasing the fraction of conservatives also helps the system to increase its efficiency. In particular, for the cases in which the noise is high enough to prevent the system from being efficient, increasing the fraction of conservatives will make the system transition from an inefficient to an inefficient regime. The fraction of conservatives needed to enter in the efficient regime depends on the bias strength. Systems comprising conservatives with a larger bias strength need less conservatives to efficiently solve the density classification task than systems with a lower bias strength.

References

1. A. A. Moreira, A. Mathur, D. Diermeier, L. A. N. Amaral, *Proc. Natl. Acad. Sci. USA* **101**, 12083 (2004).

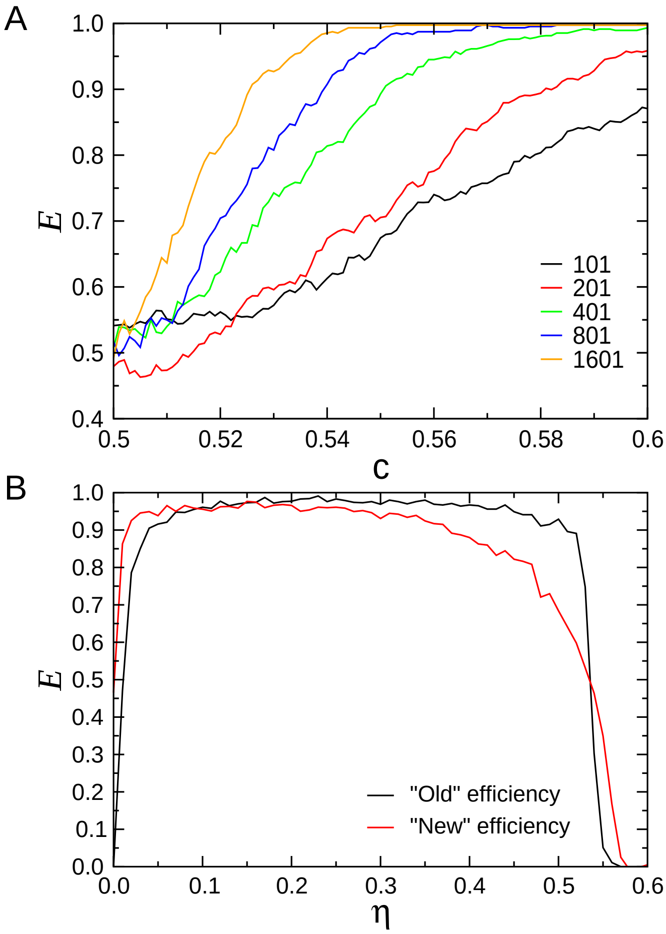

Figure S1: Effect of the initial distribution of states on the system's efficiency—(**A**) Efficiency attained by a system of N naive agents after $2N$ time steps as a function of c, the probability that an agent's state at $t = 0$ is "1." We consider systems of sizes $N = 101, 201, 401, 801$ and 1601 with a noise intensity $\eta = 0.2$. (**B**) Comparison between the efficiency obtained for a system of $N = 401$ agents following: (i) the method in (*1*) for $c = 0.5$, and (ii) the definition of efficiency we use for $c = 0.57$.

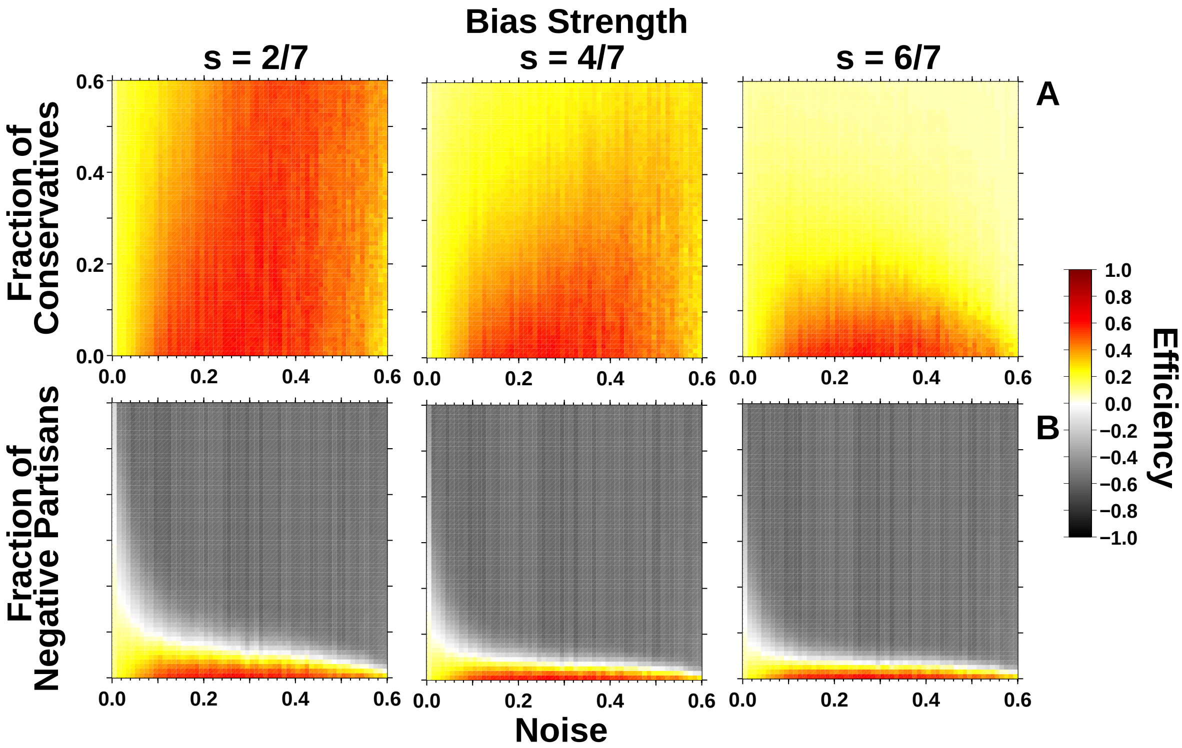

Figure S2: Efficiency of the system when $c = 0.52$—Efficiency of the system at solving the density classification versus noise intensity η and the fraction of (**A**) conservative agents and (**B**) partisans holding the minority position. Panels A and B are directly comparable with Figs. 2A and 2B in the main text, respectively. Note that, in both cases, there is an overall reduction in efficiency.

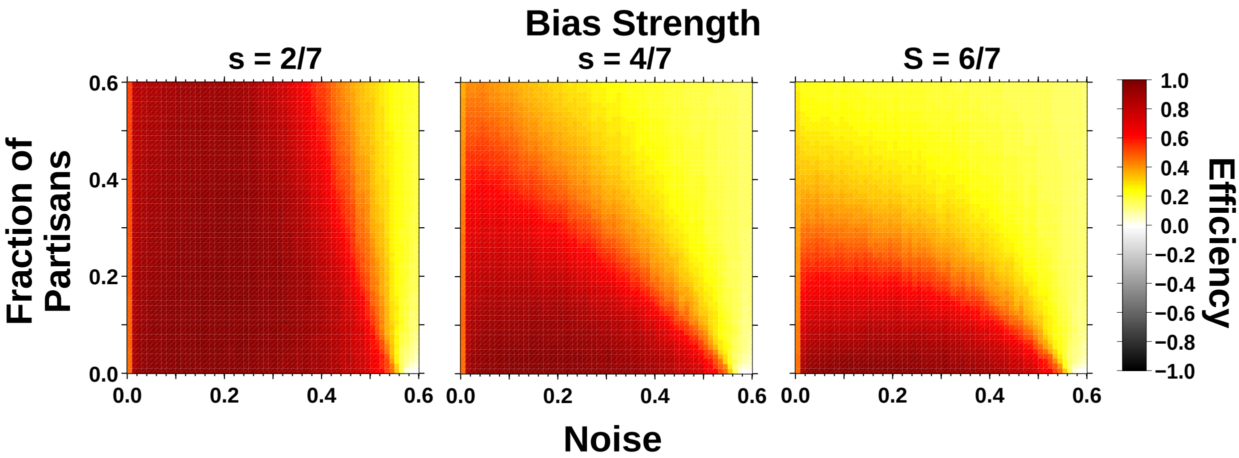

Figure S3: Distribution of preferred states for partisan agents—Efficiency of a system in solving the density classification task as a function of noise intensity and the fraction of partisans for three bias strengths, $s = 2/7$, $4/7$, and $6/7$. In here, we consider that the probability of a partisan agent preferring state 1 is 0.57.

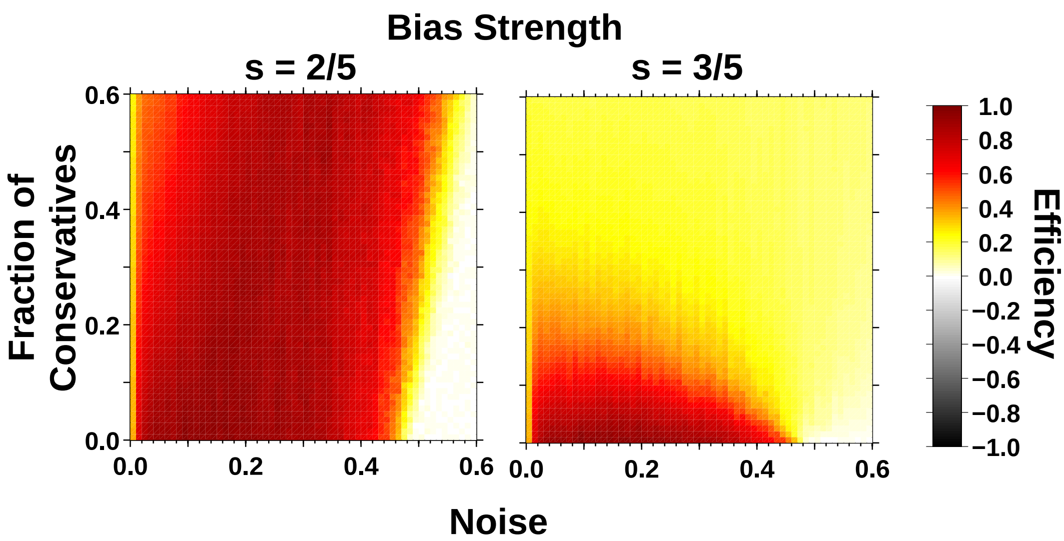

Figure S4: Effect of the agents' degree k—Efficiency of the system in solving the density classification task as a function of the fraction of conservatives and of the noise intensity considering $k = 4$ instead of $k = 6$. Note that the change in k means a change in the range of s. Despite such factor, the overall trend is similar to the results shown in Fig. 2A in the main text, so that conservatives will increase the robustness of the system regardless of k.

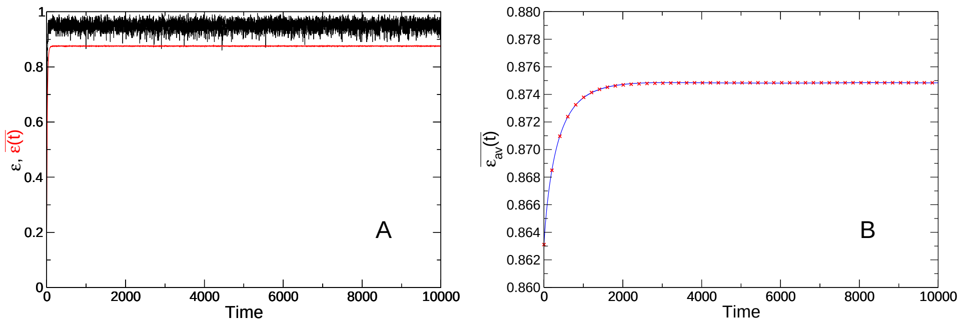

Figure S5: Estimation of the convergence time t^*—(**A**) Efficiency versus time for a system with $N = 401$, $\eta = 0.4$, $p = 0.15$, $f_c = 0.4$, and $s = 4/7$. We show for a single time history the efficiency at each time step and the the average time series $\overline{\varepsilon(t)}$ for 5000 histories. (**B**) Average cumulative efficiency $\overline{\varepsilon_{av}(t)} = \sum_{t'>t} \overline{\varepsilon(t')} \frac{1}{T-t}$ versus time. Note that with this transformation one can measure the time needed for the system to reach the stationary state. The solid line shows the fit to the curve using a stretched exponential.