Appendix to "Automated Factor Slice Sampling" published in the Journal of Computational and Graphical Statistics

In this appendix, we compare the performance of the standard and factor univariate slice sampling algorithms applied to a non-linear, "banana-shaped" distribution (cf. Rosenbrock, 1960; Wraith et al., 2009).

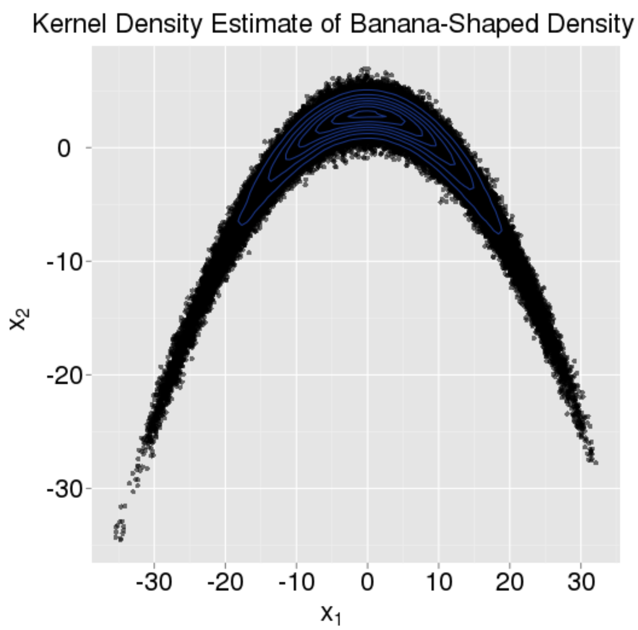

Figure 1: Kernel density estimate of Banana-shaped Distribution

Example 1 (Banana-shaped Distribution). *Following the work of Wraith et al. (2009), we construct a p-dimensional "banana-shaped" distribution from a p-dimensional multivariate normal distribution with mean zero and covariance $\Sigma = \mathrm{diag}(\sigma_1^2, 1, 1, \ldots, 1)$. However, the distribution is twisted by altering the second coordinate x_2 to be $x_2 + b\left(x_1^2 - \sigma_1^2\right)$. With the remaining coordinates unchanged, and noting that the Jacobian of this twist being 1, the banana-shaped distribution is given as:*

$$\left(x_1, x_2 + b\left(x_1^2 - \sigma_1^2\right), x_3, \ldots, x_p\right) \sim \mathcal{N}_p\left(\mathbf{0}, \Sigma\right)$$

We choose $p = 4$, $\sigma_1^2 = 100$, and $b = 0.03$.

For reference, we include the kernel density estimate of the first two components of the above distribution in Figure 1. We proceed, as suggested in Section 2, by varying the length of the tuning phase as well as the tuning parameters values between the standard and factor slice samplers to ensure that both samplers function at peak performance. As before, the ES/sec times reflect the total run time (including time spent in the tuning phase) so that a fair comparison can be made. The univariate slice sampler was run for a shorter tuning phase of $10,000$ samples during which the interval widths were tuned to ensure optimal performance (see Algorithm 5 in Section 3.2). The univariate factor slice sampler ran for a much longer tuning phase of $120,000$ samples during which the interval widths were also tuned to ensure optimal performance using Algorithm 5 of Section 3.2. Algorithm 3 was used to tune the factor slice sampler for iteration number $10,000$ through $110,000$. After the tuning phase, both samplers drew $500,000$ samples for the ESS and ES/sec computations listed below in Table 1.

Table 1: Comparison of Standard and Factor Slice Samplers on Non-Linear, Banana-shaped distribution (Example 1)

Algorithm	x_1		x_2		x_3		x_4	
	ESS	ES/sec	ESS	ES/sec	ESS	ES/sec	ESS	ES/sec
Univariate Slice Sampler	12298	5097	9036	3745	500000	207213	500000	207213
Univariate Factor Slice Sampler	12846	3014	9534	2237	500000	117304	500000	117304

In comparing the performance of the samplers, we see that the factor slice sampler provides a minimal improvement in the ESS of the first two components. However, the prolonged tuning phase and more expensive updates yielded a total run time of 4.26 seconds, in comparison with the shorter, 2.41 seconds, of the standard slice sampler. Hence, the standard slice sampler posts an ES/sec roughly 1.77 times better than that of the factor slice sampler.

This model accurately depicts the worst-case for the factor slice sampler. During the tuning phase, it estimated an orthogonal basis, $\mathbf{\Gamma}$, very close to a four dimension identity matrix. In practice, one could easily detect a lack of correlation in the posterior distribution of the parameters of interest (as the estimated orthogonal basis approaches an identity) and then switch to the standard slice sampler. This would've dropped the total run time from 4.26 seconds to roughly 3.21 seconds. Still, even in the worst case of strong non-linear dependence with no linear dependence, the factor slice sampler does make a small improvement in ESS, while suffering only a marginal impact on total run time.

References

Rosenbrock, H. H. (1960). An Automatic Method for Finding the Greatest or Least Value of a Function. *The Computer Journal*, 3(3):175–184.

Wraith, D., Kilbinger, M., Benabed, K., Cappé, O., Cardoso, J.-F. m. c., Fort, G., Prunet, S., and Robert, C. P. (2009). Estimation of cosmological parameters using adaptive importance sampling. *Phys. Rev. D*, 80:023507.