Supporting Text

A. The computer algorithm for calculating the offspring phenotypic distribution: The algorithm was developed to calculate phenotypic distribution of offspring from crossing any two autotetraploid genotypes at two linked loci. The algorithm first mimicked two cases of gametogenesis respectively, involving bivalent and quadrivalent pairing of homologous chromosomes of a given parental genotype. Then gamete genotypes generated from the two parents were paired into all possible offspring genotypes under each of these two pairing cases or mixture of them. For each of the three possible pairing types, the offspring genotypes were sorted according to the number of double reduction gametes if the gametogenesis involved quadrivalent chromosomal pairing and the number of recombinant gametes. These offspring genotypes were sorted again into phenotype groups by summing up the individuals that turned up to the same phenotype. In parallel of these sorting processes, double reduction and recombinant status for the individuals within the same phenotype groups were also updated and stored. These yielded three arrays of integers: $nc(m_1,1,0,0{:}4)$ for bivalent pairing, $nc(m_2,2,0{:}1,0{:}4)$ for mixed bivalent and quadrivalent pairing and $nc(m_3,3,0{:}2,0{:}4)$ for quadrivalent pairing respectively, where $m_i, i=1,2,3$ were the number of possible offspring phenotypes under the three pairing types. For the parental genotypes considered in the simulation study of this paper, values of $nc(m_3,3,0{:}2,0{:}4)$ were illustrated in Table 4. It indicated that there were $m_3=41$ possible offspring phenotypes from these parental lines under quadrivalent pairing, and probability of the first phenotype was calculated from $f_1=\sum_{k=0}^{2}[\sum_{l=0}^{4} nc(1,2,k,l)r^l(1-r)^{4-l}]\alpha^k(1-\alpha)^{2-k}/108^2 = 648(1-r)^4+\cdots$ $+672r^4(1-\alpha)^2/108^2=0.0499$ when α = 0.1 and r = 0.1. c_{ijkl} in equations (5)-(7) were calculated as $nc(i,j,k,l)/c$ with $c=12^2, 12\times108 \text{ or } 108^2$ for $j=1,2,3$ respectively.

B. The formulae needed for deriving equations (22)-(27): To derive the simplified forms of the second derivative equations (22)-(27), we used the following formulae:

$$\sum_{i=1}^{M} f_i = 1,\ \ E(n_i) = nf_i,\ \ E[\sum_{i=1}^{M} n_i \frac{\partial^2}{\partial x \partial y}(f_i)/f_i] = 0,\ \ E[\sum_{i=1}^{M} n_i \frac{\partial^2}{\partial x^2}(f_i)/f_i] = 0,\ x = \lambda,\ \alpha,\ r;$$

and

$$\sum_{i=1}^{M}\sum_{j=0}^{2} j\kappa_{ij} = \sum_{j=0}^{2} j\sum_{i=1}^{M}\kappa_{ij} = \sum_{j=0}^{2}(\sum_{i=1}^{M}\xi_{ij})\lambda^{j}(1-\lambda)^{2-j} = 2\lambda$$

$$\sum_{i=1}^{M}\sum_{j=0}^{4} j\psi_{ij} = 4r$$

$$\sum_{i=1}^{M}\sum_{j=1}^{2}\sum_{k=0}^{j} k\tau_{ijk} = \alpha\sum_{i=1}^{M} j\sum_{j=1}^{2}\sum_{k=0}^{j} k\tau_{ijk}$$