The influence of social norms on the dynamics of vaccinating behaviour for paediatric infectious diseases

Tamer Oraby[1,*], Vivek Thampi[1], Chris T. Bauch[1,2]

1 Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario, Canada

2 Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

*Currently: Department of Mathematical Sciences, University of Cincinnati, Cincinnati, Ohio, USA.

E-mail: tamer.oraby@uc.edu

Supplementary material

S1: Model derivations

Transmission model

A susceptible-infected-removed (SIR) model with fixed population size and frequency-dependent transmission term is postulated to depict the childhood disease dynamics. Let S, I and R be the proportions of the susceptible, infected and recovered individuals in the population. The SIR model is given by

$$\left.\begin{aligned} \frac{dS}{dt} &= \mu\,(1-x) - \beta\,S\,I - \mu\,S \\ \frac{dI}{dt} &= \beta\,S\,I - (\mu+\gamma)\,I \\ \frac{dR}{dt} &= \mu\,x + \gamma\,I - \mu\,R \end{aligned}\right\} \tag{1}$$

where μ is the birth (death) rate, β is the disease transmission rate, γ is the recovery rate and x is the proportion of vaccinated newborn (assumed to be equal to vaccinators by birth uniformity in the population). Since $S+I+R=1$, then the third equation of $\frac{dR}{dt}$ follows from the first two equations in (1) and so it is removed.

Behaviour model

Imitation dynamics description Parents adopt one of two strategies: vaccination V or no-vaccination N. Let $p_V := x$ and $p_N := 1-p_V$ be the proportion of vaccinators and non-vaccinators in the population. People sample one another or seek sources of information (people are also sources of information) at a rate of κ. Thus, a parent following strategy Θ ($\Theta = V, N$) samples a strategy Θ' (in the form of adopter

of or promoter for Θ') with rate $\kappa\, p_{\Theta'}$. Then that individual compares between the payoffs of the current strategy $\pi(\Theta)$ and the other strategy $\pi(\Theta')$. The parent, afterwards, switches to the new strategy only if $\pi(\Theta') > \pi(\Theta)$ according to a probability given as $c'\,(\pi(\Theta') - \pi(\Theta))_+$; where c' is a proportionality constant and $z_+ = z$ if $z \in \mathbb{R}_+$ and zero otherwise. Accordingly, the larger is the difference between the two payoffs, the larger is the probability of switching strategies.

The imitation dynamics can be described by

$$\frac{dx}{dt} = \kappa\, x\,(1-x)\;\left(c'\,(\pi(V) - \pi(N))_+ - c'\,(\pi(N) - \pi(V))_+\right) \tag{2}$$

where, again, $c'\,(\pi(V) - \pi(N))_+$ is the probability that a non-vaccinator switches to a vaccinator (inflow) and $c'\,(\pi(N) - \pi(V))_+$ is the probability that a vaccinator switches to a non-vaccinator (outflow). Thus, equation (2) can be rewritten as

$$\frac{dx}{dt} = \kappa\, c'\, x\,(1-x)\;(\pi(V) - \pi(N)) \tag{3}$$

Payoff function Each of the two groups adopting one of the strategies imposes, uniformly, a social group pressure δ_0 on its members. Hence, a parent adopting strategy Θ experiences an average group pressure of $\delta_0\, p_\Theta$. The average group pressure is comprised of the injunctive norm component δ_0 and the descriptive norm component p_Θ. See also the introduction and [1].

Therefore, the payoff of any strategy Θ is defined by the gain of full health due to applying that strategy, in one's own perspective, and the pressure (reward) imposed by the group adopting strategy Θ minus the perceived risks the strategy imposes. That is, the payoff of vaccination is given by

$$e_v = -r_v + \delta_0\, p_V$$

where r_v is the perceived risk of vaccination's side effects which we posit, based on [2], to be of the form $r_v(t) = m + m(\sigma - 1)(1 - (t - t_s)/D)\,\mathbf{1}_{[t_s, t_s + D]}(t)$; where m is the perceived magnitude of risk before the year t_s, σ is the relative risk at the scare, D is the number of years of decay for the risk perception (representing memory loss effect or vividness of the information), and $\mathbf{1}_{[t_s, t_s+D]}(t)$ is the Heaviside function that is equal to one if $t_s \le t \le t_s + D$ and zero otherwise. The payoff of no-vaccination

is given by

$$e_n = -r_i + \delta_0\, p_N$$

where r_i is the perceived risk of infection. The perceived risk of infection r_i is given by $c\, I$ where I is the actual prevalence given via equation (1) and the parameter c is the multiplicative product of three quantities: the proportionality constant of the probability of catching the disease, the reporting probability (assumed to be constant over time), and the cost/consequence of infection. Let $\kappa' = \kappa\, c'\, c$, $\delta = \dfrac{\delta_0}{c}$ and

$$\omega(t) := \frac{1}{c}\, r_v(t) = m' + m'(\sigma - 1)(1 - (t - t_s)/D)\, \mathbf{1}_{[t_s, t_s + D]}(t) \tag{4}$$

Therefore, equation (3) becomes

$$\frac{dx}{dt} = \kappa'\, x\, (1 - x)\, \left(-\omega(t) + I + \delta\, (2x - 1)\right) \tag{5}$$

In the main part of the paper, we will drop the prime in the parameters' notation for brevity and as they will be irrelevant to the results.

S2: Pertussis vaccine coverage and incidence in the UK

Table 1. Pertussis vaccine coverage and disease incidence in the UK in the period 1967-2010.

Year	Coverage	Incidence	Year	Coverage	Incidence
1967	0.78	0.065702282	1989	0.84	0.02205659
1968	0.78	0.034970569	1990	0.88	0.029764297
1969	0.78	0.011656856	1991	0.92	0.011260422
1970	0.78	0.033910855	1992	0.93	0.004929348
1971	0.78	0.033910855	1993	0.93	0.008456968
1972	0.78	0.004238857	1994	0.94	0.008670274
1973	0.77	0.005298571	1995	0.94	0.004300184
1974	0.59	0.032071056	1996	0.92	0.004882743
1975	0.38	0.0195433	1997	0.91	0.006576646
1976	0.39	0.010022205	1998	0.92	0
1977	0.31	0.033073277	1999	0.91	0
1978	0.35	0.129286446	2000	0.91	0.001552296
1979	0.41	0.05913101	2001	0.91	0.001880322
1980	0.46	0.041091041	2002	0.91	0.001889284
1981	0.53	0.038468628	2003	0.92	0.000933887
1982	0.59	0.127137732	2004	0.91	0.000302931
1983	0.65	0.038698067	2005	0.92	0.000641711
1984	0.65	0.011505993	2006	0.92	0.00085681
1985	0.67	0.043457128	2007	0.92	0.002084666
1986	0.73	0.071590259	2008	0.93	0.00184268
1987	0.75	0.0300977	2009	0.94	0.001516447
1988	0.78	0.009810298	2010	0.95	0.00092851

S3: Model extension

This extension owes to the fact that there is another group that do not vaccinate at all–we call them all-time nonvaccinators–due to religious or alternative medicine reasons [3, 4, 5]. Let q be the proportion of all-time nonvaccintors such that $q + p_V + p_N = 1$ and $0 < q < 1$ is constant. Then the coupled incidence-behaviour model (equation (4) in main text) will be given by

$$\left.\begin{aligned} \frac{dS}{dt} &= \mu\,(1-x) - \beta\,S\,I - \mu\,S \\ \frac{dI}{dt} &= \beta\,S\,I - (\mu+\gamma)\,I \\ \frac{dx}{dt} &= \kappa\,x\,(1-x-q)\;(-\omega(t) + I + \delta\,(2\,x-1)) \end{aligned}\right\} \tag{6}$$

where $x = p_V$; under the assumption that all-time nonvaccinators are also invoking a pressure. The equilibrium points of (equation (4) in main text) and (6) are the same except for the first one $\mathcal{E}_1$; instead we have $\mathcal{E}_1' = (\frac{1}{R_0}, \frac{\mu}{\mu+\gamma}\,(q - \frac{1}{R_0}), 1-q)$ as another equilibrium point for the model (6). Hence, we can have a below full vaccine coverage as an equilibrium if $R_0 > \dfrac{1}{q}$ which is stable if and only if

$$m < \delta\,(1-2q) + \frac{\mu}{\mu+\gamma}(q - \frac{1}{R_0}) \tag{7}$$

The model can possess from one to two bistability regions depending on the proportion of all-time non-vaccinators q, Figure FS3 (c). $\mathcal{E}_1'$ can be the only stable fixed point with less then 100% vaccine coverage rate and extremely reduced incidence, Figure FS3 (a) and (b).

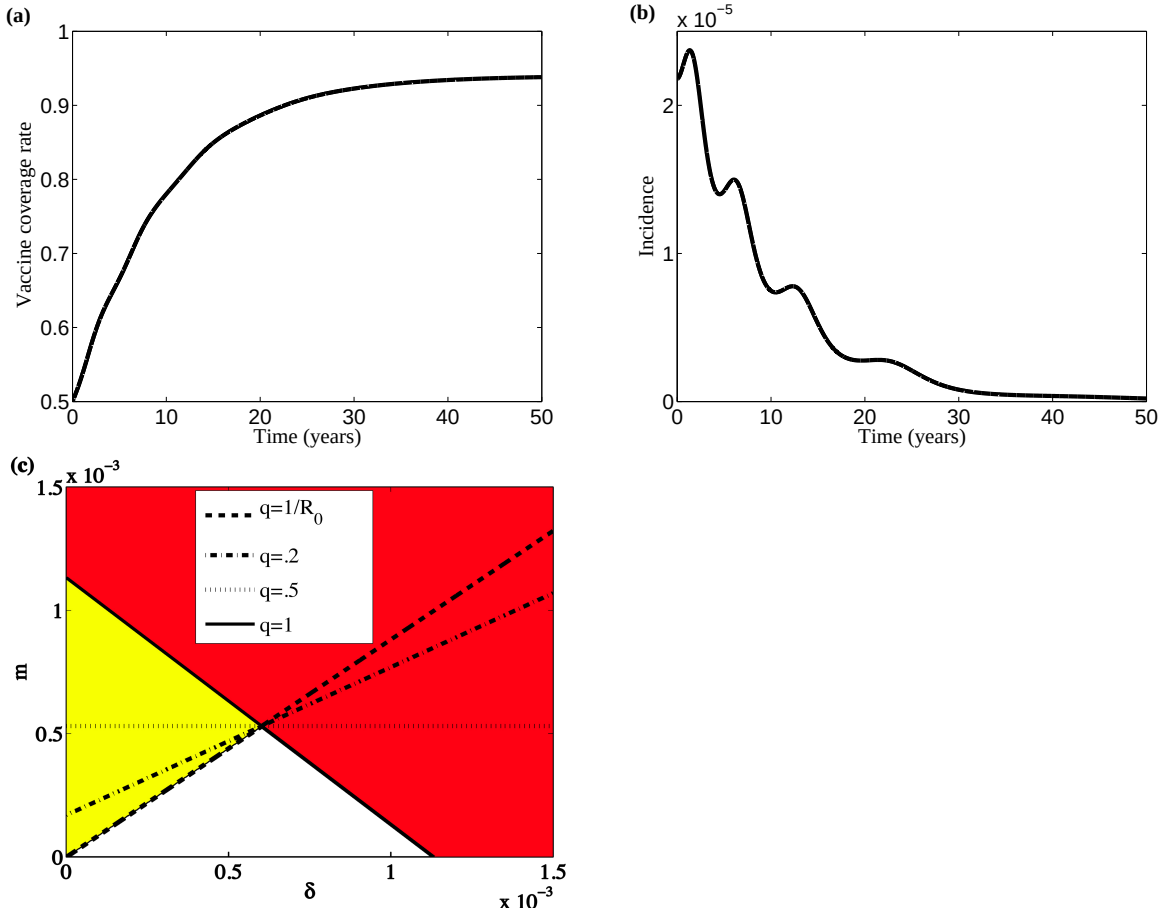

Figure FS3. Simulation of vaccination rate and incidence (panels (a) and (b)) without vaccine scare when proportion of all-time nonvaccinators $q = .06$ for $(\delta, m) = (.0005, .00025)$. (c) $\delta - m$ parameter diagram for stability of the equilibrium points for the model (6). $\mathcal{E}_1'$ is stable below each line corresponding to the values of $q = 1/R_0, .2, .5$ and 1. The equilibrium point $\mathcal{E}_4$ is stable in the red colored region. $\mathcal{E}_5$ is stable in the yellow colored region.

A further extension to model (6) is to incorporate p $(0 \leq p < 1)$ the proportion of parents who vaccinate all the time. Under the assumption that the all-time vaccinators are also invoking a pressure δ, the third equation in the model becomes

$$\frac{dx}{dt} = \kappa\,(x-p)\,(1-x-q)\;(-\omega(t) + I + \delta\,(2\,x-1)) \tag{8}$$

where $x = p + p_V$. The equilibrium points of the new model are the same as of (6) except $\mathcal{E}_2$ and $\mathcal{E}_4$ which become $\mathcal{E}_2' = (1-p, 0, p)$ and $\mathcal{E}_4' = (\frac{1}{R_0}, \frac{\mu}{\mu+\gamma}\,(1-p-\frac{1}{R_0}), p)$, respectively. The endemic equilibrium

point $\mathcal{E}_4'$ exists if and only if $R_0 > \dfrac{1}{1-p}$ and is stable when

$$m > (2p-1)\,\delta + \frac{\mu}{\mu+\gamma}(1-p-\frac{1}{R_0}) \tag{9}$$

The disease-free equilibrium $\mathcal{E}_2'$ is stable if $R_0 < \dfrac{1}{1-p}$ and $m > (2p-1)\,\delta$. This time a large proportion of all-time vaccinators insures the first condition for many childhood diseases while small pressure is needed for the second condition so as to insure disease eradication stability.

S4: Parameter estimates

Table 2 shows the estimates of the parameters in the main model (equation (4) in main text) with and without group pressure. Apparently, there is no significant difference between estimates.

Table 2. Parameters' estimated values for the model without and with pressure.

Parameters	**Without pressure in the period 1971-1988**	**Without pressure in the period 1967-2010**	**With pressure in the period 1967-2010**
κ	1.48	2.49	1.69
m	8.41×10^{-5}	4.21×10^{-5}	8.43×10^{-5}
σ	27	33.59	26.1
D (years)	5.85	9.1	5.89
δ	–	–	1.95×10^{-4}
RSS[1]	.0098	.0748	.0487
AIC_c	-73.17	-144.14	-160.32

[1]RSS: residuals sum of squares.

S5: The equilibrium point $\mathcal{E}_5$

The last equilibrium $\mathcal{E}_5$ exists in two regions depending on whether δ is less or greater than $\frac{1}{2}\frac{\mu}{\mu+\gamma}$. The two regions are

$$\mathcal{R}_1 = \left\{ (\delta, m) \in \mathbb{R}^2_+ : \delta < \frac{1}{2}\frac{\mu}{\mu+\gamma} \text{ and } \delta\,(1-\frac{2}{R_0}) < m < -\delta + \frac{\mu}{\mu+\gamma}\,(1-\frac{1}{R_0}) \right\}$$

and

$$\mathcal{R}_2 = \left\{ (\delta, m) \in \mathbb{R}^2_+ : \delta > \frac{1}{2}\frac{\mu}{\mu+\gamma} \text{ and } \delta\,(1-\frac{2}{R_0}) > m > -\delta + \frac{\mu}{\mu+\gamma}\,(1-\frac{1}{R_0}) \right\}$$

Using the necessary and sufficient condition of the Routh-Hurwitz criterion, the disease endemic equilibrium $\mathcal{E}_5$ is stable if and only if $a_i > 0$ for $i = 1, 2, 3$ and $a_1\, a_2 - a_3 > 0$ where

$$\begin{aligned} a_1 &= \mu + \beta\, I_5 - 2\kappa\delta\, x_5(1-x_5) \\ a_2 &= \beta\, I_5(\mu+\gamma) - 2\kappa\delta\, x_5(1-x_5)\,(\mu + \beta\, I_5) \\ a_3 &= \kappa\, x_5\,(1-x_5)\,\beta\, I_5(\mu - 2\delta(\mu+\gamma)) \end{aligned} \tag{10}$$

Clearly, the condition $a_3 > 0$ can be only valid in the region $\mathcal{R}_1$ which is the dark grey region in Figure 2. We have found in the numerical solutions below that $\mathcal{E}_5$ is stable in the region $\mathcal{R}_1$.

The reason that simulation show large oscillations where $\mathcal{E}_5$ should be stable is the very small value of the real eigenvalue of the Jacobian matrix of the system (6) at the fixed point $\mathcal{E}_5$ (Figure FS5.1). Choosing other pairs like $(\delta, m) = (1.5 \times 10^{-5}, .00067)$ would result in a better numerical simulation of the vaccine coverage convergence to $\mathcal{E}_5$ (Figure FS5.2) even if it is very slow.

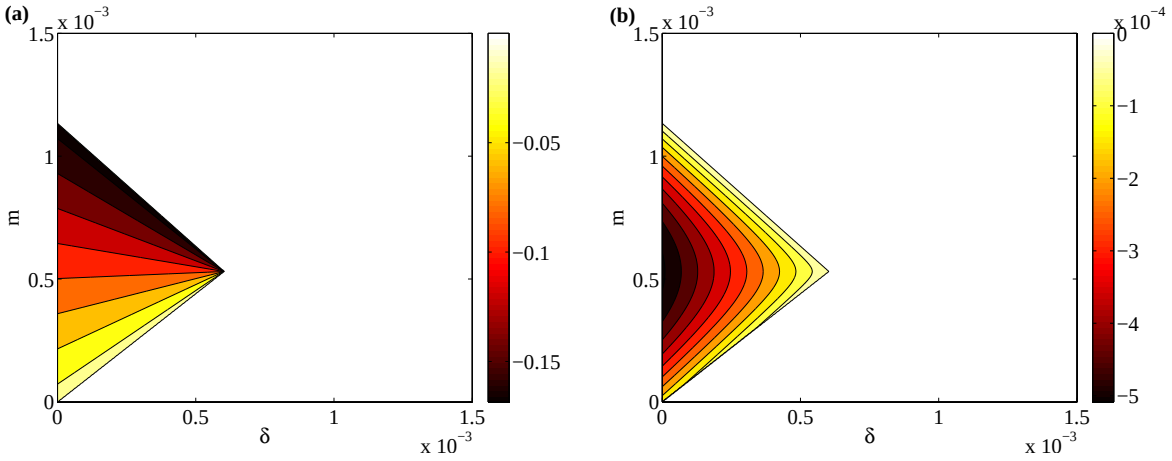

Figure FS5.1. (a) The real part of the two conjugate complex eigenvalues of the Jacobian matrix of the system (equation (4) in main text) at the fixed point $\mathcal{E}_5$ shown in its stability region. (b) The third eigenvalue of the Jacobian matrix of the system (equation (4) in main text) at the fixed point $\mathcal{E}_5$ shown in its stability region.

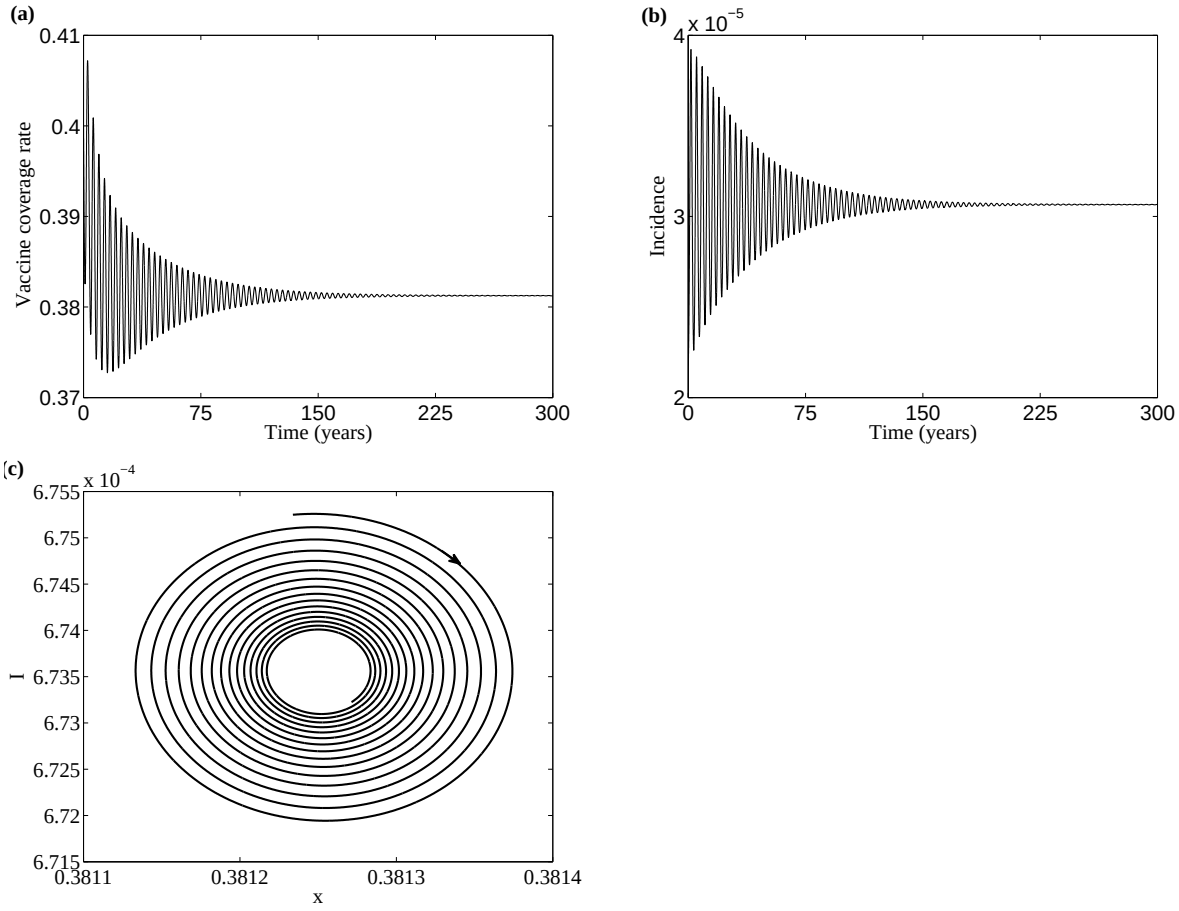

Figure FS5.2. Simulation of vaccine coverage rate and incidence without vaccine scare for $(\delta, m) = (1.5 \times 10^{-5}, .00067)$ (panels (a) and (b)). (c) Phase diagram for the last few years of the simulation showing that the convergence is slow.

References

[1] Lapinski, M. K. & Rimal, R. N. 2005 An explication of social norms. Commun. Theor. **15**, 127–147.

[2] Bauch, C. T. & Bhattacharyya, S. 2012 Evolutionary game theory and social learning can determine how vaccine scares unfold. PLoS Comput. Biol. **8**, e1002452.

[3] Smith, P. J., Humiston, S. G., Marcuse, E. K., Zhao, Z., Dorell, C. G., Howes, C. & Hibbs, B. 2011 Parental delay or refusal of vaccine doses, childhood vaccination coverage at 24 months of age, and the health belief model. Public Health Rep. **126 Suppl 2**, 135–146.

[4] Mills, E., Jadad, A. R., Ross, C. & Wilson, K. 2005 Systematic review of qualitative studies exploring parental beliefs and attitudes toward childhood vaccination identifies common barriers to vaccination. J. Clin. Epidemiol. **58**, 1081–1088.

[5] Downs, J. S., de Bruin, W. B. & Fischhoff, B. 2008 Parents vaccination comprehension and decisions. Vaccine **26**, 1595–1607.