

Supplementary Material for Reduced rank regression via adaptive nuclear norm penalization

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1. ADDITIONAL SIMULATION RESULTS

In our paper, we have presented simulation results for the proposed adaptive nuclear norm penalized estimator $\hat{C}_S^{(\lambda w)}$ with $\gamma = 2$ and $\gamma = 0$. Here we show the results for $\gamma = 1$. The optimally tuned $\hat{C}_S^{(\lambda w)}$ with $\gamma = 2$ is also included for reference and ease of comparison.

In our paper, all the simulation results are for the optimally tuned reduced rank estimators. Here we show some results obtained from ten-fold cross validation, to demonstrate that they are similar to or slightly worse than those from the optimal tuning procedure.

Table 1. Additional simulation results using Model I. The estimation and prediction errors are reported along with their standard errors in parentheses.

b		$\hat{C}_S(2)$	$\hat{C}_S(1)$	$\hat{C}_S^{cv}(2)$	$\hat{C}_S^{cv}(1)$	\hat{C}_H^{cv}
$\rho = 0.9$						
0.05	Est	2.6 (0.4)	2.5 (0.4)	2.7 (0.5)	2.5 (0.4)	3.2 (0.7)
	Pred	9.9 (1.0)	9.7 (1.0)	10.2 (1.1)	9.9 (1.0)	12.7 (1.6)
	Rank	5.5, 0%	6.7, 0%	4.9, 0%	6.2, 0%	3.1, 0%
0.1	Est	4.5 (0.5)	4.3 (0.5)	4.6 (0.6)	4.3 (0.5)	5.7 (0.8)
	Pred	13.5 (1.2)	13.2 (1.1)	13.8 (1.2)	13.3 (1.1)	16.6 (1.6)
	Rank	8.1, 5%	9.2, 33%	7.6, 2%	8.8, 19%	5.1, 0%
0.3	Est	6.2 (0.7)	6.1 (0.7)	6.3 (0.7)	6.2 (0.7)	6.9 (0.8)
	Pred	15.1 (1.2)	15.9 (1.2)	16.1 (1.2)	15.9 (1.2)	17.1 (1.4)
	Rank	10.3, 64%	11.2, 11%	10.1, 74%	10.9, 27%	9.7, 68%
$\rho = 0.5$						
0.05	Est	0.9 (0.1)	0.9 (0.1)	0.9 (0.1)	0.9 (0.1)	1.2 (0.1)
	Pred	13.6 (1.2)	12.1 (1.1)	13.8 (1.3)	13.1 (1.2)	17.8 (2.0)
	Rank	8.0, 5%	9.2, 30%	7.6, 2%	8.9, 20%	5.6, 0%
0.1	Est	1.2 (0.1)	1.1 (0.1)	1.2 (0.1)	1.1 (0.1)	1.4 (0.2)
	Pred	15.7 (1.1)	15.4 (1.1)	15.9 (1.2)	15.5 (1.1)	17.8 (1.5)
	Rank	10.0, 58%	10.1, 24%	9.8, 56%	10.8, 34%	8.9, 25%
0.3	Est	1.2 (0.1)	1.2 (0.1)	1.2 (0.1)	1.2 (0.1)	1.3 (0.1)
	Pred	15.9 (1.2)	16.0 (1.2)	16.0 (1.2)	16.1 (1.2)	16.2 (1.2)
	Rank	10.2, 80%	11.0, 17%	10.1, 90%	10.6, 50%	10.0, 99%
$\rho = 0.1$						
0.05	Est	0.7 (0.1)	0.7 (0.1)	0.7 (0.1)	0.7 (0.1)	0.9 (0.1)
	Pred	14.1 (1.2)	13.5 (1.1)	14.4 (1.3)	13.7 (1.1)	18.2 (1.8)
	Rank	8.5, 12%	9.6, 40%	7.1, 6%	9.2, 29%	6.2, 0%
0.1	Est	0.8 (0.1)	0.8 (0.1)	0.8 (0.1)	0.8 (0.1)	0.9 (0.1)
	Pred	15.9 (1.2)	15.6 (1.2)	16.0 (1.2)	15.7 (1.2)	17.6 (1.4)
	Rank	10.2, 64%	11.1, 15%	9.9, 65%	10.9, 31%	9.3, 40%
0.3	Est	0.8 (0.1)	0.8 (0.1)	0.8 (0.1)	0.8 (0.1)	0.8 (0.1)
	Pred	15.1 (1.2)	16.0 (1.2)	16.1 (1.2)	16.1 (1.2)	16.2 (1.2)
	Rank	10.2, 83%	10.9, 17%	10.1, 88%	10.7, 48%	10.0, 99%
Time		0.15	0.15	0.18	0.18	0.02

$\hat{C}_S(\gamma)$: optimally tuned adaptive nuclear norm penalized estimator with weight parameter γ ; $\hat{C}_S^{cv}(\gamma)$: adaptive nuclear norm penalized estimator with weight parameter γ tuned by cross validation; \hat{C}_H^{cv} , rank penalized estimator tuned by cross validation; Est, estimation error; Pred, prediction error; Rank, average of estimated rank and percentage of correct rank identification; Time, average computation time in seconds per simulation run.

Table 2. Additional simulation results using Model II. The layout of the table is the same as in Table 1.

b		$\hat{C}_S(2)$	$\hat{C}_S(1)$	$\hat{C}_S^{cv}(2)$	$\hat{C}_S^{cv}(1)$	\hat{C}_H^{cv}
$\rho = 0.9$						
0.05	Est	1.2 (0.2)	1.2 (0.2)	1.2 (0.2)	1.2 (0.2)	1.2 (0.2)
	Pred	29.1 (3.5)	28.9 (3.4)	29.6 (3.5)	29.4 (3.4)	32.3 (4.6)
	Rank	5.2, 71%	5.7, 36%	4.10, 73%	5.4, 56%	4.6, 64%
0.1	Est	4.5 (0.7)	4.5 (0.7)	4.5 (0.7)	4.5 (0.7)	4.5 (0.7)
	Pred	30.0 (3.6)	30.1 (3.7)	30.4 (3.8)	30.5 (3.8)	31.0 (3.8)
	Rank	5.2, 79%	5.6, 43%	5.1, 87%	5.5, 60%	4.1, 97%
0.3	Est	40.6 (5.9)	40.6 (5.9)	40.6 (5.9)	40.6 (5.9)	40.6 (5.9)
	Pred	30.1 (3.5)	30.2 (3.5)	30.3 (3.6)	30.4 (3.6)	30.2 (3.6)
	Rank	5.2, 82%	5.5, 56%	5.2, 87%	5.3, 75%	5.0, 100%
$\rho = 0.5$						
0.05	Est	1.1 (0.2)	1.1 (0.2)	1.1 (0.2)	1.1 (0.2)	1.1 (0.2)
	Pred	29.1 (3.6)	29.1 (3.6)	29.6 (3.8)	29.4 (3.7)	31.6 (4.2)
	Rank	5.2, 77%	5.7, 38%	5.1, 79%	5.4, 63%	4.8, 78%
0.1	Est	4.5 (0.6)	4.5 (0.6)	4.5 (0.6)	4.5 (0.6)	4.5 (0.6)
	Pred	29.6 (3.3)	29.7 (3.4)	29.9 (3.6)	30.0 (3.5)	30.3 (3.6)
	Rank	5.2, 81%	5.6, 42%	5.1, 90%	5.4, 70%	4.1, 98%
0.3	Est	40.1 (6.0)	40.1 (6.0)	40.1 (6.0)	40.1 (6.0)	40.1 (6.0)
	Pred	30.2 (3.1)	30.3 (3.1)	30.4 (3.1)	30.5 (3.1)	30.3 (3.1)
	Rank	5.2, 78%	5.5, 57%	5.1, 89%	5.2, 81%	5.0, 100%
$\rho = 0.1$						
0.05	Est	1.1 (0.2)	1.1 (0.2)	1.1 (0.2)	1.1 (0.2)	1.1 (0.2)
	Pred	29.3 (3.5)	29.2 (3.5)	29.7 (3.6)	29.5 (3.5)	31.9 (4.5)
	Rank	5.2, 76%	5.7, 35%	5.1, 79%	5.5, 57%	4.8, 77%
0.1	Est	4.5 (0.7)	4.5 (0.7)	4.5 (0.7)	4.5 (0.7)	4.5 (0.7)
	Pred	29.7 (3.5)	29.8 (3.5)	30.0 (3.5)	30.1 (3.5)	30.3 (3.7)
	Rank	5.2, 79%	5.6, 43%	5.1, 89%	5.4, 67%	4.10, 99%
0.3	Est	40.5 (5.6)	40.5 (5.6)	40.5 (5.6)	40.5 (5.6)	40.5 (5.6)
	Pred	29.1 (3.4)	30.1 (3.4)	30.3 (3.5)	30.3 (3.5)	30.1 (3.5)
	Rank	5.2, 76%	5.5, 55%	5.2, 87%	5.3, 79%	5.0, 100%
Time		0.15	0.15	0.26	0.26	0.08