SUPPLEMENTARY INFORMATION
Origin and Structure of Dynamic Cooperative Networks

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S1 Phenotype Space

The behavioural type of an individual i is derived from the number of outgoing and incoming links, k_i and l_i, respectively. The altruistic level, L_i, of individual i is given by

$$L_i = \frac{k_i - l_i}{k_i + l_i}, \tag{S1}$$

where $L_i \in [-1, 1]$ such that low levels refer to egoists ($k_i < l_i$), high levels indicate altruists ($k_i > l_i$) and values near zero correspond to fair individuals ($k_i \approx l_i$). The total number of individuals interacting with the node i indicates the activity, A_i, of the node i, which is given by

$$A_i = \frac{k_i + l_i}{2(N-1)} \tag{S2}$$

with $A_i \in [0, 1]$. The normalization is such that in a population of size N an individual i, which links to everyone else ($k_i = N - 1$) and everyone links to the individual ($l_i = N - 1$), has an activity of one. The behavioural type of an individual is then represented by its altruistic level and activity in the phenotype space $L \times A$. Note that phenotype space is discrete because k_i and l_i are integers (Fig. S1).

S2 Phase transitions

The average activity of individuals in the network undergoes phase transitions triggered by the selection intensity, β. In the limit of no selection, $\beta = 0$, a random network forms whereas for small β the network effectively disappears. However, for further increases of β the activity increases again and social networks emerge. The random network and the phase transition for small β are analytically accessible whereas only an intuitive illustration can be derived for the complex dynamics and network formation arising for large β.

A focal individual i considers an individual j as a model depending on the relative success of j. The success of i is defined as the cumulative payoff,

$$P_i = bl_i - ck_i,$$

and the relative success between i and j is defined as

$$\Delta\pi = \frac{\pi_j - \pi_i}{\pi_j + \pi_i},$$

where $\pi_i = 1 + 1/(c(k_i + k_j))P_i$ to ensure positive values. In the same way, the normalized out-degree difference is defined as

$$\Delta\kappa = \frac{k_j - k_i}{k_j + k_i}.$$

The payoff and out-degree differences, $\Delta\pi$ and $\Delta\kappa$, are normalized to prevent interferences of population size and network density with the selection strength, β, because they both affect the possible range for individual payoffs and out-degrees. To ensure positive payoffs in the normalization factor, any positive amount scaling with the local density of links around i and j is enough and does not change the results. The point here is that any non-local normalization is equivalent to change the selection intensity β. Moreover without normalization selection intensity is no longer well defined by β as, for a fixed β, selection can jump from moderate to strong depending only on the payoff difference magnitude.

A focal individual i adds or removes an outgoing link to a model j with probability p or q, respectively:

$$p = f(\Delta\pi) \cdot g(\Delta\kappa) \tag{S3}$$

$$q = f(\Delta\pi) \cdot g(-\Delta\kappa) \tag{S4}$$

where $f(z) = g(z) = 1/(1 + e^{-\beta z})$. Note that, since $-1 \leq \Delta\pi, \Delta\kappa \leq 1$ the range of $f(z), g(z)$ is bounded and determined by the selection strength:

$$\frac{1}{1 + e^{\beta}} \leq f(\Delta\pi), g(\Delta\kappa) \leq \frac{1}{1 + e^{-\beta}}.$$

Hence the focal individual increases its out-degree by one ($k_i \to k_i + 1$) with probability

$$T^+ = (1 - q)\, p. \tag{S5}$$

and decreases it by one ($k_i \to k_i - 1$) with probability

$$T^- = q\,(1 - p)\,. \tag{S6}$$

S2.1 Random network: no selection, $\beta = 0$

In the absence of selection, $\beta = 0$, costs and benefits do not matter and links are added and removed randomly with constant probabilities $p = q = 1/4$. As a consequence, a random, uncorrelated and relentlessly changing network emerges. When adding new links, the recipient is chosen uniformly at random. Hence, the incoming links are evenly distributed and the in-degree distribution is given by a binomial distribution. Therefore, the network can be described by the probability distribution, $P(k)$, that a given node has k outgoing links and the stationary distribution of $P(k)$ is approximated by solving the Master equation

$$P(k) = P(k-1)T^+ + P(k+1)T^- + (1 - T^+ - T^-)P(k). \tag{S7}$$

For $\beta = 0$ we have $T^+ = T^- = 3/16$ and the stationary probability distribution is given by

$$P(k) = \begin{cases} \frac{3}{2}\frac{1}{2N-1} & k = 0 \\ \frac{2}{2N-1} & 1 \leq k < N-1 \\ \frac{3}{2}\frac{1}{2N-1} & k = N-1 \end{cases} \tag{S8}$$

Thus the out-degree distribution is almost uniform and both average in-degree and out-degree equal $N/2$. In equilibrium the random network is very dense with an average activity of $\bar{A} = N/(2N-2)$ or approximately $1/2$ for large N. Fig. S2 shows the phenotype distribution and a network snapshot for $\beta = 0$.

S2.2 No network: weak selection, $\beta \ll 1$

For weak selection, $\beta \ll 1$, correlations between in-degrees and out-degrees are still weak because of weak discrimination. Hence the in-degree distribution can still be approximated by a binomial distribution, with average in-degree l.

The phase transition from a random network for $\beta = 0$ to essentially no network for $\beta \ll 1$ can be analyzed as a perturbation of the random network. As an approximation, assume that all players have the same in-degree, l, the normalized payoff difference between a model with j out-going links and a focal player with k out-going links is given by

$$\Delta\pi = \frac{-c(j-k)}{2l}$$

and the normalized out-degree difference is given by

$$\Delta\kappa = \frac{j-k}{j+k}.$$

The probabilities to add and remove links, p and q, now depends on the out-degree k of the focal player:

$$p_k = \sum_j P(j) f(\Delta\pi)\, g(\Delta\kappa) \tag{S9}$$

$$q_k = \sum_j P(j) f(\Delta\pi)\, g(-\Delta\kappa), \tag{S10}$$

where the sum runs over all potential models. For weak selection the functions $f(z), g(z)$ can be approximated by

$$f(z) = g(z) = \frac{1}{1+e^{-\beta z}} \approx \frac{1}{2} + \frac{1}{4}\beta z. \tag{S11}$$

This approximation simplifies the expressions for p_k and q_k and hence the probabilities to increase or decrease the out-degree, $T_k^+ = p_k(1-q_k)$ and $T_k^- = (1-p_k)q_k$, respectively:

$$T_k^+ = \frac{3}{16} - \frac{\beta}{32}\frac{c}{l}(\bar{k}-k) + \frac{\beta}{8}\sum_j \Delta\kappa P(j), \qquad \text{for } k < N-1 \tag{S12}$$

$$T_k^- = \frac{3}{16} - \frac{\beta}{32}\frac{c}{l}(\bar{k}-k) - \frac{\beta}{8}\sum_j \Delta\kappa P(j), \qquad \text{for } k > 0, \tag{S13}$$

to first order in β. First, note that $T_k^+ < T_k^-$ is equivalent to $\sum_j \Delta\kappa P(j) < 0$. Since

$$\begin{aligned}
\sum_j \Delta\kappa P(j) &= \sum_{j<k} \frac{j-k}{j+k}P(j) + \sum_{j>k}\frac{j-k}{j+k}P(j) &\text{(S14)}\\
&< \sum_{j<k} \frac{j-k}{2k}P(j) + \sum_{j>k}\frac{j-k}{2k}P(j) &\text{(S15)}\\
&= \frac{\bar{k}-k}{2k} &\text{(S16)}
\end{aligned}$$

we have $\sum_j \Delta\kappa P(j) < 0$ for $k > \bar{k}$, which means that for those individuals that provide more help than on average, the probability to increase help further is lowered while the probability to reduce help is increased. Second, the probability that the out-degree of a node remains unchanged is

$$T_k^0 = 1 - T_k^+ - T_k^- = \frac{10}{16} + \frac{\beta c}{16l}(\bar{k} - k) + o(\beta^2), \tag{S17}$$

which decreases for increasing k. Consequently, individuals with high out-degrees remove links at a higher rate, which in turn lowers the average degree $\bar{k}$ and hence creates a positive feedback and more individuals start removing links. The numerical solution for the stationary distribution of the master equation S7 is shown in Fig. S3.

S2.3 Social network: strong selection, $\beta \gg 1$

For increasing selection strengths analytical approximations become challenging due to the complex network dynamics. However, the basic mechanisms driving the emergence and structure of social networks can be intuitively understood based on the probabilities to add, attract, retain or remove links. More specifically, in the following we provide a detailed argument illustrating that in sparse networks the activity increases and a social network emerges for sufficiently small costs and sufficiently strong selection. In contrast, in dense networks the activity decreases and hence equilibrates at intermediate activities. In equilibrium the in- and out-degrees of all individuals are close to the average and consequently individuals are fair. Finally, increasing costs weakens the correlation between in- and out-degrees and as a consequence the range of behavioural types increases and the network becomes stratified.

***F1*: Average activity increases for strong selection and small costs**

Consider an individual i and the change in the probabilities to attract a new link and to loose an existing one after increasing its level of cooperation by adding a new link. The probability that i attracts a link from u is given by

$$p = \frac{1}{1 + e^{-\beta\Delta\pi}} \frac{1}{1 + e^{-\beta\Delta\kappa}}.$$

Now suppose that individual i has added an outgoing link to another player v ($v \neq u$). The out-degree of individual i is increased by 1 and its payoff is reduced by $-c$. The probability that i attracts now a link from u is given by

$$p' = \frac{1}{1 + e^{-\beta\Delta\pi'}} \frac{1}{1 + e^{-\beta\Delta\kappa'}}.$$

Note that $\Delta\pi' > \Delta\pi$ and hence the first factor decreases but because of $\Delta\kappa' < \Delta\kappa$ the second factor increases. For sufficiently small costs of cooperation, c, the second factor outweighs the first such that $p' > p$, which means that the probability of i to attract further links effectively increases by adding outgoing links. An analogous argument shows that the probability that individual i looses an incoming link decreases after adding a new outgoing link. Thus, helping others can effectively increases the probability of both attracting additional benefits as well as retaining existing ones. This mechanism drives the emergence of social networks and promotes dense networks at large β.

Note that the converse is true too: decreasing cooperation by removing an outgoing link increases the probability to loose existing benefits because others are more likely to withdraw their support. However, those individuals are now less successful *and* less cooperative and hence readily imitate others, which restores their out-degree. Conversely, individuals that increase their help and successfully attract further benefits become more successful *and* more cooperative. As a consequence they are less likely to adjust their strategy but more likely to serve as models and inspire other to equally increase cooperation.

***F2*: Distribution of out-degrees is narrow**

On average the payoff of each individual is $\bar{P}$ and its out-degree is $\bar{k}$. Consider a focal individual with out-degree k, in-degree l and hence payoff P. The focal individual can differ in four distinct ways:

1. $k > \bar{k}$, $P > \bar{P}$: the focal individual is more successful and provides more. Thus, the focal is unlikely to change its strategy but likely attracts another link, which makes it even more successful and likely to be imitated while slowly increasing $\bar{k}$.
2. $k < \bar{k}$, $P > \bar{P}$: the focal is more successful and provides less and hence it likely gets imitated and thereby looses benefits from incoming links. This reduces P and with it the chances to be imitated decrease. Removing links also slightly decreases $\bar{k}$.
3. $k > \bar{k}$, $P < \bar{P}$ and (iv) $k < \bar{k}$, $P < \bar{P}$: the focal is less successful and provides more (less). Consequently, the focal is readily imitating the strategy of more successful models, which means decreasing (increasing) the number of outgoing links such that k ends up getting closer to $\bar{k}$.

In all four cases, there is a tendency that differences between k and $\bar{k}$ are levelled out either by adjusting k accordingly or by slowly adjusting $\bar{k}$ if the network has not yet reached its equilibrium density (activity).

***F3*: Fairness is maintained in equilibrium**

In equilibrium the average out-degree $\bar{k}$ and average in-degree $\bar{l}$ remain constant. Since every outgoing link is an incoming link elsewhere $\bar{k} = \bar{l}$ must hold at all times. Consider a focal individual with $l \neq \bar{l}$. Whenever a representative node with $\bar{k}$ and $\bar{l}$ updates its strategy, the probability that the focal node looses in incoming link, L^-, is

$$L^- = \frac{l}{N}\frac{1}{\bar{k}}f(\Delta\pi)\frac{1}{2}.$$

The first term indicates the probability that the selected node provides benefits to the focal; the second term denotes the probability that the link directed to the focal is under consideration to be removed; the third and fourth terms represent the probability q that the link is indeed removed. Note that according to *F1* a dense network emerges (large $\bar{k}$) and *F2* states that the variation in k is small. As a consequence, $\Delta\kappa \approx 0$ and the fourth term is simply $1/2$. Similarly, the probability that the focal attracts another in-coming link, L^+, is

$$L^+ = \frac{N-l}{N}\frac{1}{N-\bar{k}}f(\Delta\pi)\frac{1}{2}.$$

The first term now indicates the probability that the selected node does *not* provide benefits to the focal and the second term denotes the probability that the focal is serving as a model. The other terms refer to the probability p for establishing a new link with $\Delta\kappa \approx 0$. Thus

$$L^- > L^+ \iff l > \bar{k}.$$

Consequently, if the focal is an egoist, $l > \bar{k}$, it is more likely to loose incoming links but if the focal is an altruist, $l < \bar{k}$, it is more likely to attract incoming links. In either case the in-degree of the focal converges to $\bar{k}$ and hence reduces the variance in l, which increases fairness.

***F4*: High activity is unsustainable**

According to *F1-F3* a fair network emerges with little variation in out- and in-degrees. For a dense network, payoffs and degrees are both large but similar and hence $\Delta\pi \approx 0$ and $\Delta\kappa \approx 0$ even if the network is not quite in equilibrium. Consequently, the link dynamics becomes more noisy and we can apply the same reasoning as in the weak selection case (c.f. S2.2). More specifically, the dynamics of dense *and* fair networks represent a perturbation of the random network with $p = q = \frac{1}{4}$. According to eq. S12 and S13, individuals with high out-degrees tend to remove links, while those with low out-degrees are more inert and unlikely to change their out-degree. The two effects generate a trend towards lowering the average degree $\bar{k}$, which results in a positive feedback such that more individuals start removing links. Hence, in dense networks the activity decreases and, for low costs (*F1*), approaches an intermediate equilibrium value.

***F5*: Large costs induce stratification**

This is a corollary to *F1*. According to *F1* the probability of a node to attract benefits increases by helping others for sufficiently small costs. If costs are large, helping others not only reduces the success of an individual but also decreases the probability to attract benefits. However, unsuccessful individuals with out-degrees below the average $\bar{k}$ tend to establish further links and hence become more altruistic and even less successful. Consequently an altruist class emerges, which provides benefits to fair players and egoists. In particular, note that egoists with $k < \bar{k}$ are unstable because others would withdraw their help by imitating the egoist and hence the in-degree of the egoist decreases until eventually turning the individual into a fair player. Egoists survive only if their selfishness is a consequence of receiving much more but not as a consequence of not giving. Thus large costs not only reduces activity, but also promotes co-existence of altruists, fair players and egoists.

S3 Node and Network Reciprocity

Richard D. Alexander[1] has proposed that cooperation in large groups could be sustained by a network of "indirect reciprocity" instead of direct reciprocation. An individual x helps an individual y, who in turn helps another individual z, and so on, until benefits are returned to individual x. However, in a directed network, we need to distinguish between two types of reciprocity reflecting the extent to which actions of a focal node are reciprocated by others (the network) and the extent to which the focal node reciprocates actions of its providers. The two measures of reciprocity we refer to as *network reciprocity* and *node reciprocity*, respectively. In both cases we apply the concept introduced by Alexander[1] to explicitly account for direct as well as indirect components of reciprocity.

S3.1 Network reciprocity

Network reciprocity arises through a series of directed links that create directed cycles starting and returning to a focal individual x. A cycle of length l is represented by a path $\{x \to v_1 \to v_2 \to \ldots \to v_{l-1} \to x\}$. The direct component of network reciprocity aimed at the focal individual x by another individual y is represented by the bi-directional link (or two-cycle) $\{x \to y \to x\}$. Cycles of length $l > 2$ represent paths of indirect reciprocation – if x helps y, benefits are returned not necessarily by the original recipient, but by one or more other individuals. Naturally, short cycles and direct reciprocation, in particular, are more robust than long cycles[2]. Long cycles are easily broken as individuals far away likely have other, closer interactions, which shape their behaviour more directly.

In order to formalize the concept of network reciprocity, let us first introduce some definitions. A path is an ordered sequence of nodes connected by directed links such that the predecessor points to the successor node. The shortest path between two nodes is given by the path with the minimum number of links and the length of the shortest path indicates the distance between them. Let $D = [d_{i,j}]$ be the $N \times N$ matrix representing the distance between any pair of nodes i, j with $d_{i,i} = 0$ in a network of size N. If no path exists between i and j then the distance is set to an arbitrarily large number (at least

$d_{i,j} \gg N$). Let $\mathbf{O}_x = \{o_1, \ldots, o_k\}$ be the set of nodes to whom the focal individual x is providing benefits and let $\mathbf{I}_x = \{i_1, \ldots, i_l\}$ be the set of nodes from whom the player x receives benefits, where k and l denote the out-degree and the in-degree of x.

Network reciprocity determines conversion rate of costs expended into benefits received by tracking all directed cycles that connect an outgoing link of the focal individual x to one (or several) of its incoming links. For each outgoing link the benefits that return to x are discounted by the length of the cycle that leads back to x and summed over all cycles. Therefore, bi-directional links (direct reciprocity) have a high efficiency and contribute heavily to network reciprocity as opposed to low efficiencies of long, fragile cycles that contribute little – although large numbers of such cycles may exist. More specifically, for a cycle starting with $x \to o$ and ending with $i \to x$ its length is $d_{o,i} + 2$ and we set its reciprocity weight essentially to the reciprocal length of the cycle

$$r_{o,i} = \frac{1}{1 + d_{o,i}}$$

such that direct reciprocation through bi-directional links $\{x \to o \to x\}$ has weight $r_{o,i} = 1$ (Fig. S4a).

An indirect reciprocity cycle $\{x \to o \to i \to x\}$ has weight $r_{o,i} = 1/2$ and $\{x \to o \to v \to i \to x\}$ corresponds to $r_{o,i} = 1/3$ (Fig. S4b & c) and converges to zero for very long cycles. While no issues arise if one outgoing link admits multiple cycles that loop back to x (Fig. S4e), the converse situation where multiple outgoing links return to x via one common incoming link requires more careful attention (Fig. S4f). In that case the reciprocity generated by each of those outgoing links must be further discounted by $w_{o,i}$ as they jointly generate benefits:

$$w_{o,i} = \frac{r_{o,i}}{\sum_{v \in \mathbf{O}_o} r_{v,i}}.$$

The network reciprocity of the focal node x can then be defined as

$$\mathcal{R}_x^{\downarrow} = \frac{1}{k} \sum_{o \in \mathbf{O}_x} \sum_{i \in \mathbf{I}_x} r_{o,i} \cdot w_{o,i}$$

where the index $\overset{\downarrow}{x}$ indicates that benefits return to the focal individual x. The population average is then given by

$$\bar{\mathcal{R}}^{\downarrow} = \frac{1}{N} \sum_{i=1}^{N} \mathcal{R}_i^{\downarrow}.$$

The network reciprocity includes both direct and indirect components. The set $\mathbf{O}_x$ can be partitioned into two subsets:

$$\begin{aligned} \mathbf{O}_x^D &= \{o \in \mathbf{O}_x \; ; \; \exists \, (x \to o \to x)\} \\ \mathbf{O}_x^I &= \mathbf{O}_x - \mathbf{O}_x^D. \end{aligned}$$

The network reciprocity of node x can then be rewritten as

$$\mathcal{R}_x^{\downarrow} = \frac{1}{k} \sum_{o \in \mathbf{O}_x^D} \sum_{i \in \mathbf{I}_x} r_{o,i} \cdot w_{o,i} + \frac{1}{k} \sum_{o \in \mathbf{O}_x^I} \sum_{i \in \mathbf{I}_x} r_{o,i} \cdot w_{o,i}.$$

The first term is the direct reciprocity component and the second term is the indirect reciprocity component. Since each link in $\mathbf{O}_x^D$ is bi-directional and hence has an efficiency of 1, the first term simply denotes the fraction of outgoing links with direct reciprocation.

S3.2 Node reciprocity

Node reciprocity arises through a series of directed links that create directed cycles starting at a provider y of the focal individual x and returning to the provider y through x. A cycle of length l is represented by a path $\{y \to x \to v_1 \to v_2 \to \cdots \to v_{l-1} \to y\}$. The direct component of node reciprocity aimed at y by the focal individual x is represented by the bi-directional link (or two-cycle) $\{y \to x \to y\}$. Cycles of length $l > 2$ represent paths of indirect reciprocation – if y helps x, benefits are returned not necessarily directly to y, but through one or more other individuals.

Following an analogous line of arguments used to define network reciprocity, node reciprocity of the focal node x is defined as

$$\mathcal{R}_x^{\uparrow} = \frac{1}{l} \sum_{i \in \mathbf{I}_x} \sum_{o \in \mathbf{O}_x} r_{i,o} \cdot w_{i,o},$$

where

$$w_{i,o} = \frac{r_{i,o}}{\sum_{v \in \mathbf{I}_i} r_{v,o}}.$$

The population average is then given by

$$\bar{\mathcal{R}}^{\uparrow} = \frac{1}{N} \sum_{i=1}^{N} \mathcal{R}_i^{\uparrow}.$$

The average node reciprocity is depicted in Fig. 4 of the main text as well as for the largest connected component of the economical networks in rural Indian villages in Fig. 5 of the main text.

Again, two components of node reciprocity, direct and indirect, can be identified by partitioning the set **I** into two subsets:

$$\begin{aligned} \mathbf{I}_x^D &= \{i \in \mathbf{I}_x \; ; \; \exists \, (i \to x \to i)\} \\ \mathbf{I}_x^I &= \mathbf{I}_x - \mathbf{I}_x^D. \end{aligned}$$

The node reciprocity of node x can then be rewritten as

$$\mathcal{R}_x^{\uparrow} = \frac{1}{l} \sum_{i \in \mathbf{I}_x^D} \sum_{o \in \mathbf{O}_x} r_{i,o} \cdot w_{i,o} + \frac{1}{l} \sum_{i \in \mathbf{I}_x^I} \sum_{o \in \mathbf{O}_x} r_{i,o} \cdot w_{i,o}.$$

In summary, network reciprocity provides a measure to which degree benefits return to the actor through the structure of the social network. Conversely, node reciprocity provides a measure to which degree benefits received by a node are returned to the actor through the structure of the social network. An altruist with many outgoing but few incoming links tends to have a high node reciprocity if the numerous benefits provided to others indeed find their way back to its few providers. Conversely, the same individual would tend to have a low network reciprocity because few of the recipients return benefits directly or indirectly. Similarly, egoists tend to have low node reciprocity but high network reciprocity. However, any level of reciprocity is contingent on the existence of loops in the social structure that facilitate reciprocation otherwise both node and network reciprocity are negligible. Hence network and node reciprocity both serve as an indicator of social stratification. In a network with high activity but low network and node reciprocity, all benefits flow in a preferred direction from producers (altruists) to consumers (egoists) possibly involving a chain of fair players that just channel the flux of benefits.

S4 Empirical data

The empirical data is based on extensive surveys in 75 villages in rural India, which were designed as a part of the deployment of a micro-finance program[3]. The information is used to build networks where each individual is represented by a node and the act of providing a good is represented by a directed link. In particular we borrowed data from the exchange network of rice and kerosene as well as from the network for providing advice. The data is publicly available at *http://www.stanford.edu/ jacksonm/Data.html*. The act of providing material or immaterial goods is costly to the actor in terms of money and/or time and provides benefits to the recipient. Thus, the network topology represents a snapshot in time of the cooperative behaviour. The surveys covered numerous questions but the only ones that are relevant in the present context are

Q.A.1 "Who would come to you if he/she needed to borrow kerosene or rice?"

Q.A.2 "If you needed to borrow kerosene or rice, to whom would you go?"

for the rice/kerosene network and

Q.B.1 "Who comes to you for advice?"

Q.B.2. "If you had to make a difficult personal decision, whom would you ask for advice?"

for the advice network. Our modelling framework assumes that whether or not to provide benefits is solely a decision of the donor. In order to reflect this, the empirical networks are only based on the answers to the questions Q.A.1 and Q.B.1. Thus the links point from the donor to the individual borrowing rice/kerosene or seeking advice. The complete networks are shown in Fig. S5 and Fig. S6.

References

[1] Alexander, R. *The Biology of Moral Systems* (Aldine de Gruyter, New York, 1987).

[2] Boyd, R. & Richerson, P. J. The evolution of indirect reciprocity. *Social Networks* **11**, 213–236 (1989).

[3] Jackson, M. O., Rodriguez-Barraquer, T. & Tan, X. Social capital and social quilts: Network patterns of favor exchange. *American Economic Review* **102**, 1857–976 (2012).

Figures

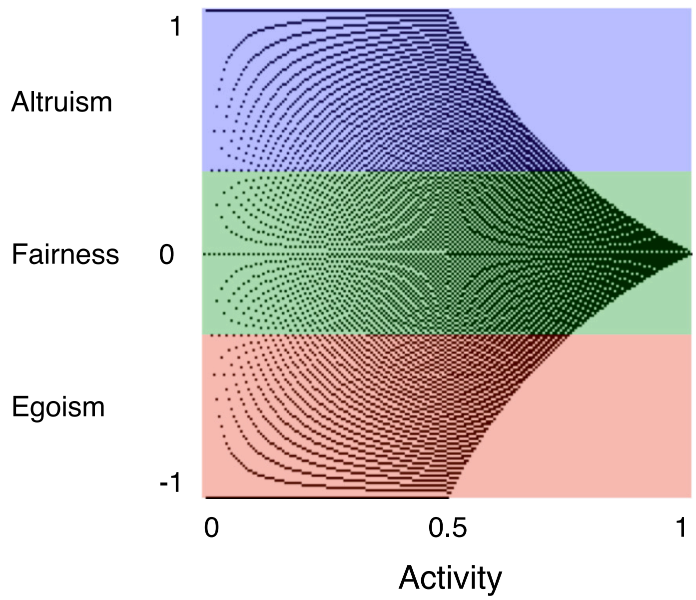

Figure S1: Phenotype space $L \times A$ of individual behaviour. All possible, discrete phenotypes are shown for a population of size $N = 100$ (black dots). For example, only two levels of altruism, $L = -1$ and $L = 1$, admit an activity of $A = 1/(2N - 2)$ – either a single incoming and no outgoing links or vice versa. Conversely, an activity of $A = 1/2$ can be achieved for almost any level of altruism. The level of altruism is divided into three regions: egoists ($-1 \leq L < -1/3$, red), fair players ($-1/3 \leq L \leq 1/3$, green), and altruists ($1/3 < L \leq 1$, blue).

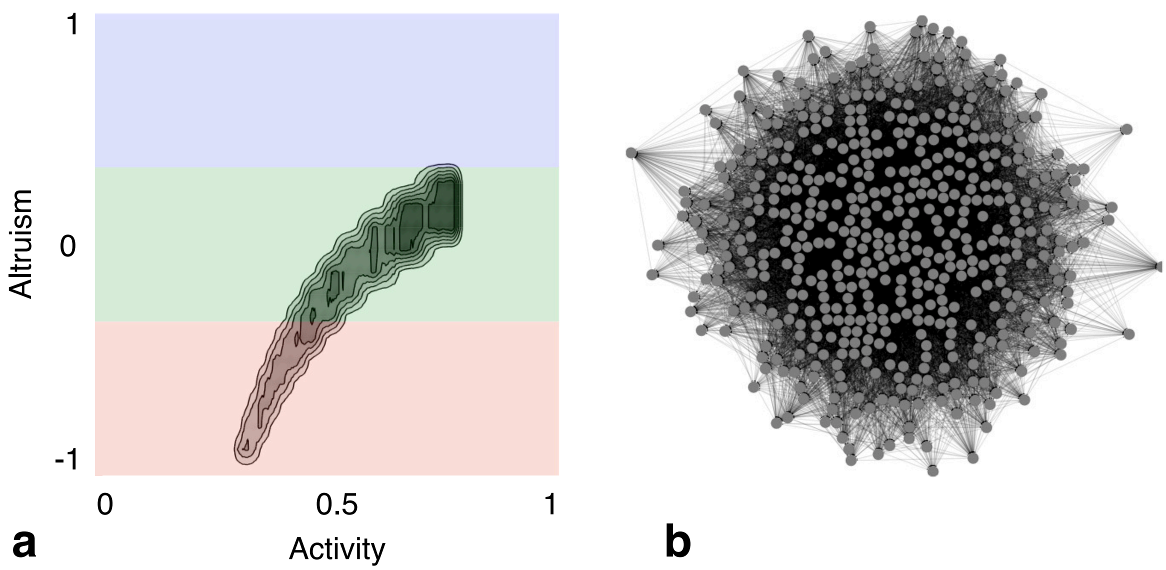

Figure S2: Random network, $\beta = 0$: **a** Phenotype distribution for $\beta = 0$ and **b** snapshot of network with only 10% of the links shown due to the high activity. Because neither differences in payoffs nor strategies affect the dynamic of the network discussing behavioural types is not meaningful.

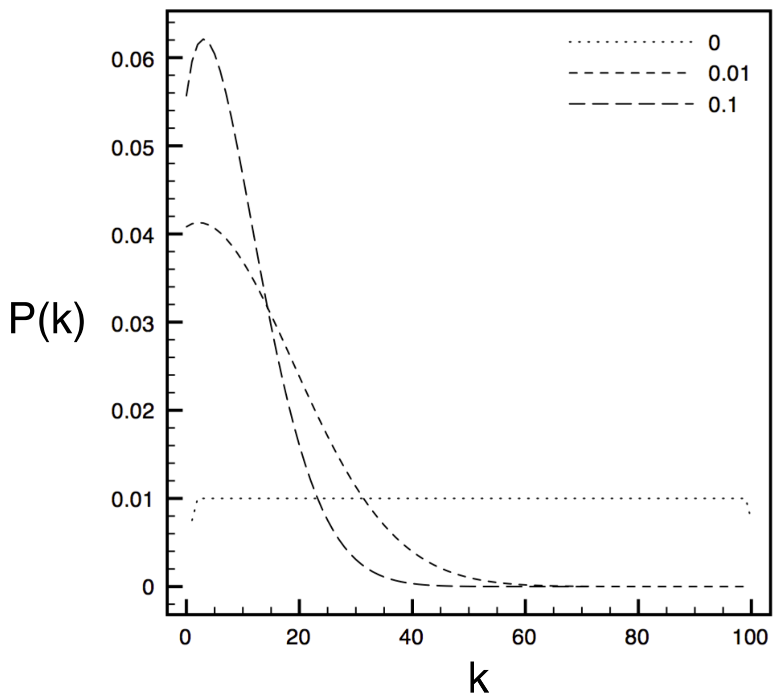

Figure S3: Analytical approximation of the degree distribution, $P(k)$, in the weak selection limit $0 < \beta \ll 1$ for $N = 100$. The dotted line is the analytical approximation for $\beta = 0$ and the dashed lines are numerical solutions of the Master S7. As β increases the probability of small k values increases and the probability of large k values decreases. Thus the average activity decreases as β increases.

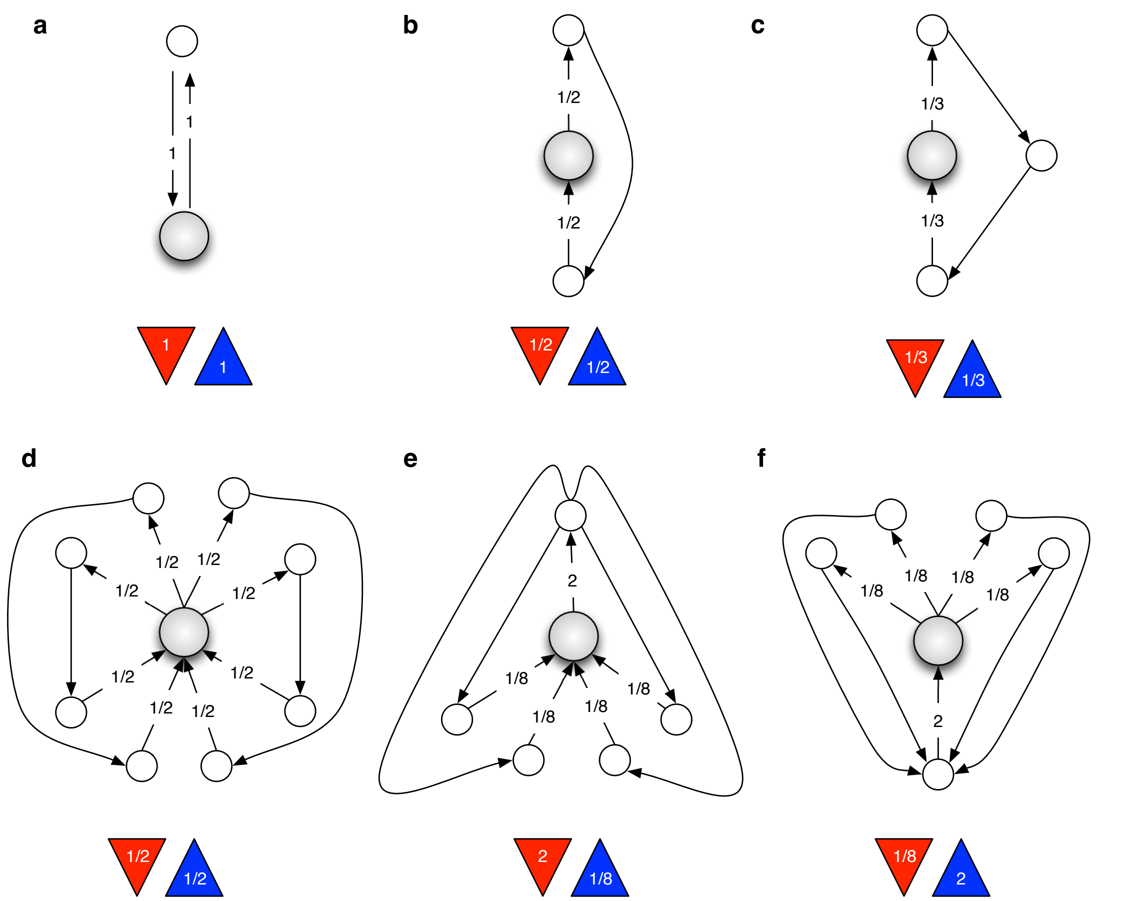

Figure S4: Network reciprocity $\mathcal{R}_x^{\downarrow}$ (red triangle) and node reciprocity $\mathcal{R}_x^{\uparrow}$ (blue triangle) for the central node x (grey) together with the weighted effectivity of each outgoing link (the average is the network reciprocity) and the weighted return for each incoming link (the average is the node reciprocity) for different configurations. **a** Direct reciprocity, $\mathcal{R}_x^{\downarrow} = \mathcal{R}_x^{\uparrow} = 1$. **b** Indirect reciprocity with two intermediate nodes, $\mathcal{R}_x^{\downarrow} = \mathcal{R}_x^{\uparrow} = 1/2$. **c** Indirect reciprocity through a cycle of length four, $\mathcal{R}_x^{\downarrow} = \mathcal{R}_x^{\uparrow} = 1/3$. **d** Fair player receiving and providing indirect benefits through four cycles of length three, $\mathcal{R}_x^{\downarrow} = \mathcal{R}_x^{\uparrow} = 2$. **e** Egoist receiving indirect benefits from a single outgoing link, which results in a high efficiency, $\mathcal{R}_x^{\downarrow} = 2$, while poorly reciprocating its four providers indirectly through a single outgoing link, which results in a low node reciprocity $\mathcal{R}_x^{\uparrow} = 1/8$. **f** Altruist receiving indirect benefits through a single incoming link, which decreases the effectivity of each outgoing link, $\mathcal{R}_x^{\downarrow} = 1/8$, while generously reciprocating its single provider, $\mathcal{R}_x^{\uparrow} = 2$.

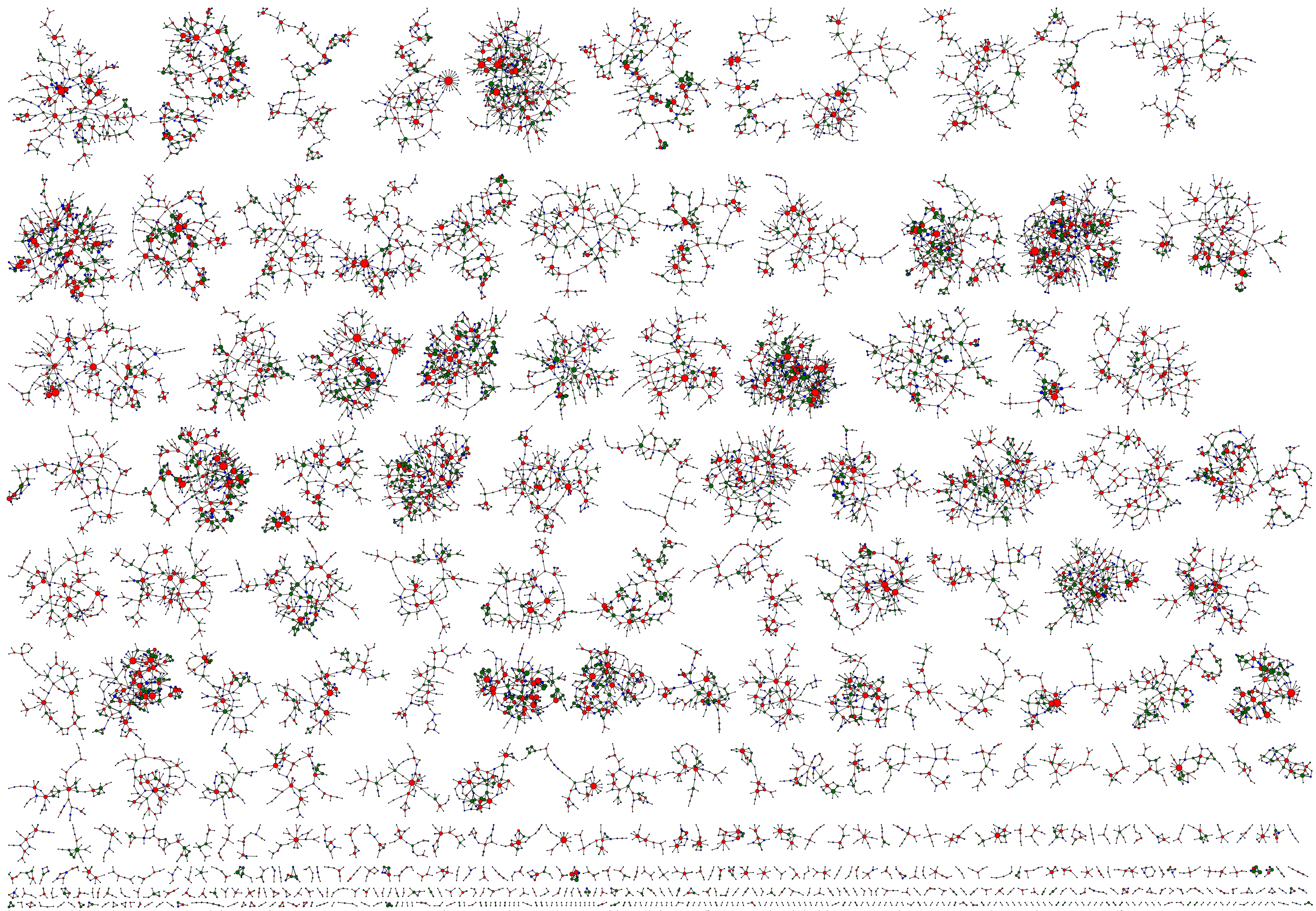

Figure S5: The rice/kerosene network comprises $N = 14771$ nodes, which are arranged in 651 connected components. The largest connected component contains 372 nodes.

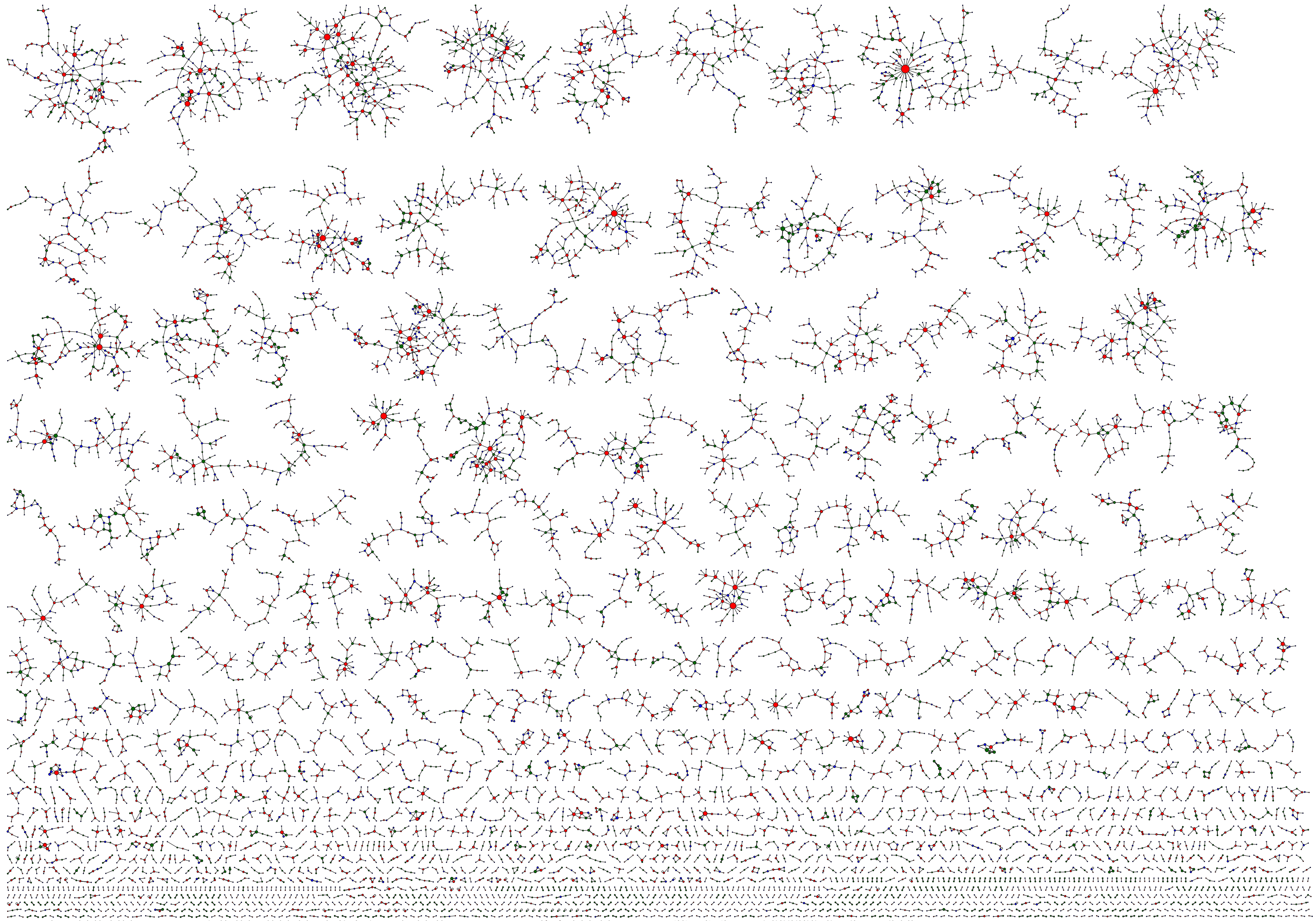

Figure S6: The advice network consists of $N = 13895$ nodes arranged in 2034 connected components. The largest connected component contains 262 nodes.