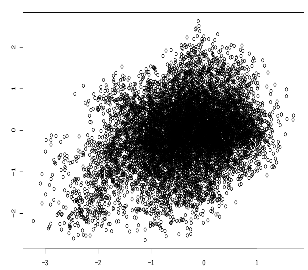
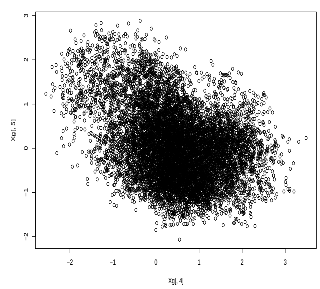
Web-based Supplementary Materials for "Assessing the Significance of Global and Local Correlations under Spatial Autocorrelation; a Nonparametric Approach", by Viladomat, Mazumder, McInturff, McCauley, and Hastie.

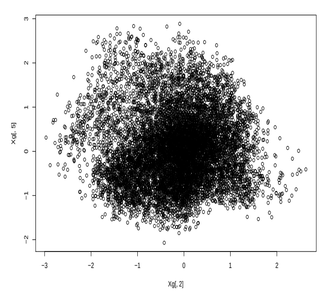
1 Web Figure 1



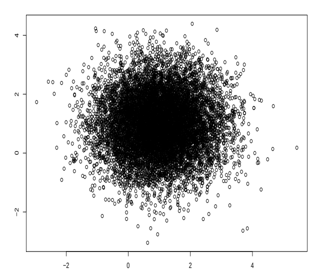
(a) $\widehat{r}_{\mathbf{X_s},\mathbf{Y_s}} = 0.3$



(b) $\widehat{r}_{\mathbf{X_s},\mathbf{Y_s}} = -0.36$



(c) $\widehat{r}_{\mathbf{X_s},\mathbf{Y_s}} = 0.003$



(d) mutually independent samples, $\widehat{r}_{\text{ind}} = 0.01$

Figure 1: Top left and right, bottom left: three realizations of two pairs of independent and spatially autocorrelated Gaussian random fields $\mathbf{X_s}$ and $\mathbf{Y_s}$, with $\widehat{r}_{\mathbf{X_s},\mathbf{Y_s}} = 0.30$, $\widehat{r}_{\mathbf{X_s},\mathbf{Y_s}} = -0.36$ and $\widehat{r}_{\mathbf{X_s},\mathbf{Y_s}} = 0.003$ respectively. The consequences of spatial autocorrelation is that, by chance, the variance of $r_{\mathbf{X_s},\mathbf{Y_s}}$ is larger. Bottom, right: two independent samples, each mutually independent and normally distributed (no autocorrelation), with $\widehat{r}_{\text{ind}} = 0.01$.

2 Web Figure 2

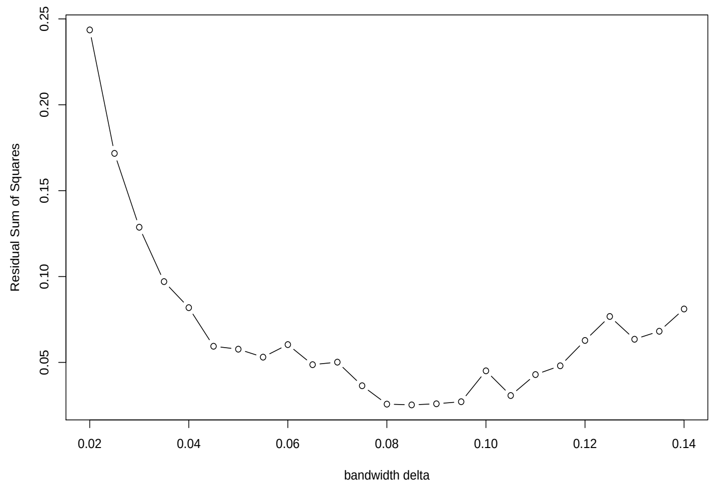


Figure 2: Residual sum of squares of the linear fit between $\widehat{\gamma}(\mathbf{X}_\mathbf{s}^\delta)$ and $\widehat{\gamma}(\mathbf{X}_\mathbf{s})$ as a function of the bandwidth δ. The minimum value is reached at $\delta = 0.085$.

3 Web Figure 3

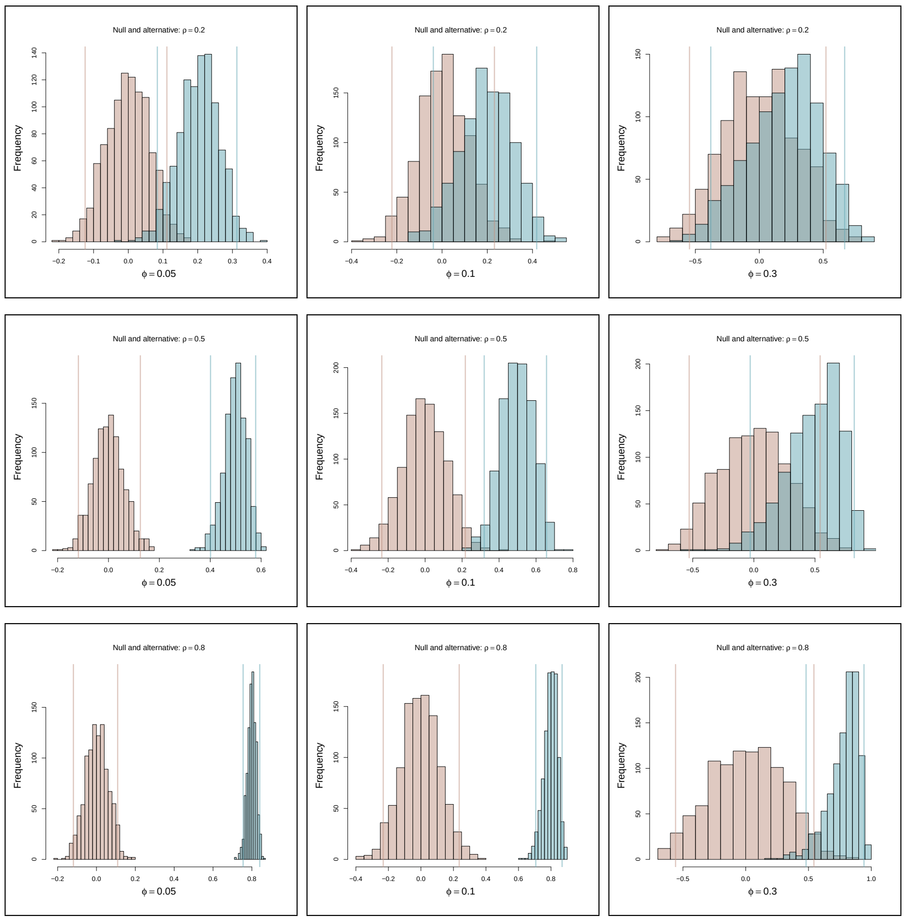


Figure 3: Simulated distributions for $r_{\mathbf{X_s},\mathbf{Y_s}}$ under $H_0 : \rho_{\mathbf{X_s},\mathbf{Y_s}} = 0$ (pink) and $H_1 : \rho_{\mathbf{X_s},\mathbf{Y_s}} = \rho$ (blue). The 9 scenarios correspond to different effect size and spatial autocorrelation: $\rho \in (0.2, 0.5, 0.8)$, $\phi \in (0.05, 0.1, 0.3)$. The lines in the plots indicate the 95% interval points. The grid size is set to $g = 0.05$ ($N = 441$), but the results are identical for $g = 0.01$ ($N = 10,201$).

4 Web Figure 4

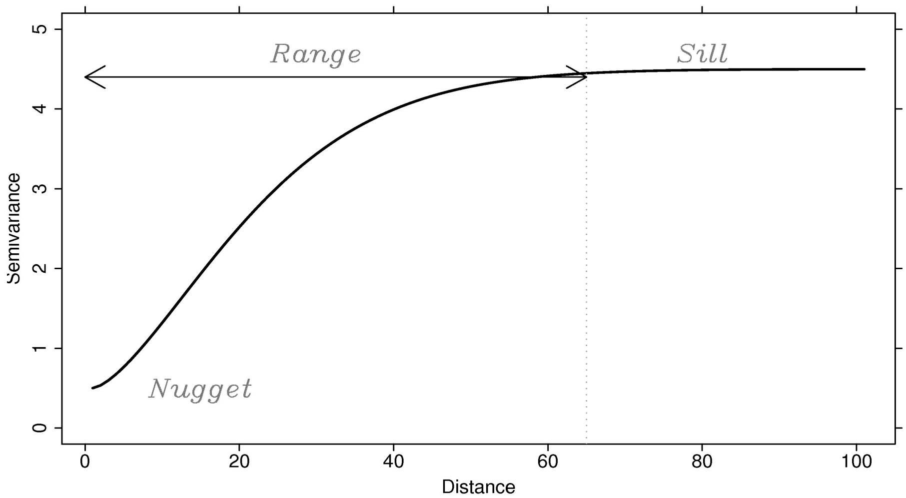


Figure 4: Typical semivariogram of a stationary spatial process: $\gamma(u) = \sigma^2\{1 - \varphi(u)\} + \tau^2$. The range is the distance u at which $\varphi(u) = 0$; i.e. where the autocorrelation function fades away. The intercept is the nugget variance τ^2, and the sill is the variance of the process $\tau^2 + \sigma^2$.