## Filter Algorithms

## Filter for $\tilde{G}^L$

The filter algorithm below is designed to prevent the propagation of FP edges in any  $\underline{G}_{\{i\}}$  to the lower bound  $\tilde{G}^L$ . The pseudocode for the union with filter  $\bigcup$  is given below:

$$\begin{aligned} Adj\left(\tilde{G}^{L}\right) &= Adj\left(\underline{\tilde{G}}_{\emptyset}\right) \\ A &= Acc(\overline{\tilde{G}}_{\emptyset}) \\ \text{FOR every k} \\ A_{test} &= \sum_{i \in V_{KO}^{k}} A_{i} \otimes A^{i} \\ Adj\left(\overline{G}^{L}\right) &= Adj\left(\overline{G}^{L}\right) + A_{test} \odot Adj\left(\underline{G}_{V_{KO}^{k}}\right) \\ \text{END FOR} \end{aligned}$$

where  $\otimes$  denotes the outer product and  $\odot$  denotes the Hadamard multiplication (element wise multiplication). Note that the elements of the matrices are either 0 or 1, and the addition operation + denotes a Boolean sum.

## Filter for $\tilde{G}^U$

The following filter is created to avoid propagating FN edges in any  $\bar{G}_{\{i\}}$  to  $\tilde{G}^U$ . The pseudocode for the intersection with filter  $\dot{\bigcap}$  is given below:

$$\begin{aligned} Adj\left(\bar{G}^{U}\right) &= Acc(\bar{G}_{\emptyset})\\ A &= Acc(\overline{\tilde{G}_{\emptyset}})\\ \text{FOR every } k\\ A_{test} &= \sum_{i \in V_{KO}^{k}} A_{i} \otimes A^{i}\\ Adj\left(\tilde{G}^{U}\right) &= Adj\left(\tilde{G}^{U}\right) \odot A_{test} \odot Adj\left(\bar{G}_{V_{KO}^{k}}\right)\\ \text{END FOR} \end{aligned}$$