# Randomness in highly reflective silver nanoparticles and their localized optical fields

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# Supplementary information

**Angular spectrum representation.** The angular spectrum representation of an electromagnetic field expresses an optical field as a superposition of plane waves including evanescent components[15,16], which allows us to explicitly represent and quantify optical near-fields. Suppose that the transverse electric (TE) and transverse magnetic (TM) vectors and the unit wavevector are given in a Cartesian coordinate system shown in Fig. 6a and are respectively given by

$$\boldsymbol{\varepsilon}(\boldsymbol{s}^{(+)}, \mathrm{TE}) = (-\sin\beta, \cos\beta, 0) \tag{1}$$

$$\boldsymbol{\varepsilon}(\boldsymbol{s}^{(+)}, \mathrm{TM}) = (\cos\alpha\cos\beta, \cos\alpha\sin\beta, -\sin\alpha) \tag{2}$$

$$\boldsymbol{s}^{(+)} = (s_x, s_y, s_z) = (\sin\alpha\cos\beta, \sin\alpha\sin\beta, \cos\alpha). \tag{3}$$

What should be noted is that while the angle $\beta$ is real ($-\pi \le \beta < \pi$), $\alpha$ takes a complex value. This allows us to explicitly and intuitively deal with optical near-fields. It is useful to introduce $s_{\|} = \sin\alpha$, which allows us to write the parameter $s_z$ of the wavevector $\boldsymbol{s}^{(+)}$ as

$$s_z = \begin{cases} \sqrt{1-s_{\|}^2} & \text{for } 0 \le s_{\|} < 1 \\ i\sqrt{s_{\|}^2 - 1} & \text{for } 1 \le s_{\|} < +\infty. \end{cases} \tag{4}$$

The value of $s_{\|}$ specifies the property of a plane wave as a homogeneous wave ($0 \le s_{\|} < 1$) or an evanescent wave ($1 \le s_{\|} < +\infty$). The electric field $\boldsymbol{E}(\boldsymbol{r})$ originated by a point dipole $\boldsymbol{d}^{(i)}$ with frequency $K$ is given in the form[16]

$$\boldsymbol{E}(\boldsymbol{r}) = \left(\frac{iK^3}{8\pi^2\varepsilon_0}\right) \sum_{\mu=TE}^{TM} \iint_{-\infty}^{-\infty} ds_x ds_y \frac{1}{s_z} \left[\boldsymbol{\varepsilon}(\boldsymbol{s}^{(+)}, \mu) \bullet \boldsymbol{d}^{(i)}\right] \boldsymbol{\varepsilon}(\boldsymbol{s}^{(+)}, \mu) \exp(iK\boldsymbol{s}^{(+)} \bullet \boldsymbol{r}) \tag{5}$$

which corresponds to eq. (2) in the main text.