Supplementary Information for
High Speed All Optical Nyquist Signal Generation and Full-band Coherent Detection

Junwen Zhang[1,2,] Jianjun Yu[1], Yuan Fang[1,2], and Nan Chi[1]

[1] *Department of Communication Science and Engineering, and Key Laboratory for Information Science of Electromagnetic Waves (MoE), Fudan University, 220 Handan Road, Shanghai 200433, China*

[2] *ZTE (TX) Inc, Morristown, NJ 07960, USA*

Corresponding authors email: J. Y. (jianjun@fudan.edu.cn)

Flattened comb generation with multiple tones of equal phase

This supplementary information is about the detailed explanation on how we realize a flattened comb consisting of multiple tones of equal phase. One of the key aspects of the Nyquist pulse generation is the phase relation. In our previous analysis, it is sufficient that the phases of all frequency components are locked showing a linear relation dependence on frequency. It can be expressed as:

$$E_N(t) = \frac{E_0}{N}\sum_{-(N-1)/2}^{(N-1)/2} \exp[j2\pi(f_0 + n\Delta f)t + j\phi + jn\varphi_0] \quad (1)$$

Here we define a time delay τ_0, which satisfies $\varphi_0 = 2\pi\Delta f\tau_0$, then we have

$$\begin{aligned} E_N(t) &= \frac{E_0}{N}\sum_{-(N-1)/2}^{(N-1)/2} \exp[j2\pi(f_0 + n\Delta f)t + j\phi + jn\varphi_0] = E_0 \exp(j2\pi f_0 + j\phi)\frac{\sin[\pi N\Delta f(t+\tau_0)]}{N\sin[\pi\Delta f(t+\tau_0)]} \quad (2)\\ &= E_0 \exp(j2\pi f_0 + j\phi)\sum_{-\infty}^{+\infty}(-1)^{(N-1)n}\sin c[N\Delta f(t - nT + \tau_0)] \end{aligned}$$

Therefore, this linear phase dependence shows a time delay of the *Sinc*-shaped pulse, and it can be nullified by properly choosing the time origin without the loss of generality. As analyzed in [1]The frequency-locked comb source can be realized by the radio frequency (RF) driven cascaded Mach–Zehnder modulators (MZM), of which each sub-carriers are with linear phase relation.

Back to the single-drive MZM for comb generation case, in [1], they have proved this liner phase relationship can be realized for the two-tone or three-tone case by adjusting the relation of RF-driving voltage and DC bias. Principally, using the cascaded MZMs driving by the RF signals, one can generated a large number of tones with equal amplitude and liner phase. Therefore, each MZM generated 2 or 3 tones.

In our experiment demonstration, we use one single MZM and one BPF to generate the 3 or 5 tones comb with equal amplitude and linear phase. To realize the conditions given by above Eq. 2, we need to adjust the voltage of the RF driving signal and the DC bias for optimal output.

Using the same method in [1], we can expand the output of MZM by Bessel function by using the Jacobi–Anger expansion. For zero-chirp MZ intensity modulator, assuming the RF driver signal voltages represented as $aV_{pi}\cos(2\pi\Delta ft)$, and the DC bias voltages as bV_{pi}, then the output of the single

MZM can be

$$E(t) = \cos\{\frac{\pi}{2}[a\cos(2\pi\Delta ft) + b]\} = \cos\{\frac{\pi}{2}[a\cos(2\pi\Delta ft) + \frac{\pi}{2}b]\}$$
$$= \cos[\frac{\pi}{2}a\cos(2\pi\Delta ft)]\cos(\frac{\pi}{2}b) - \sin[\frac{\pi}{2}a\cos(2\pi\Delta ft)]\sin(\frac{\pi}{2}b) \quad (3)$$

Using the Jacobi–Anger expansion, the above equation can be expanded as

$$E(t) = \sum_{n=-\infty}^{+\infty}(-1)^n\{\cos(\frac{\pi}{2}b)J_{2n}(\frac{\pi}{2}a)\cos[2\pi(f_0 + 2n\Delta f)t]$$
$$+ \sin(\frac{\pi}{2}b)J_{2n-1}(\frac{\pi}{2}a)\cos[2\pi(f_0 + (2n-1)\Delta f)t]\} \quad (4)$$

Here J_n is the n-th order Bessel function of the first kind, V_{pi} is the half-wave voltage of the modulator and a and b are the normalized driving voltage and DC bias. Therefore, two degrees of freedom, modulation and bias index can be used to equalize the amplitude and phase of each frequency tone of generated comb. To obtain the *Sinc*-shaped pulse, the comb should satisfy the equal amplitude and linear phase conditions given by above Eq. 1 and 2.

From Eq. 4, we can see that the phase of each tone is dependent on the sign of the amplitude, either 0 or π. Therefore, assuming the phase of the center carrier is 0, the phase of the 3 tones case can be {0, 0, 0} or {π, 0, π}, and the phase of the 5 tones can be {0, 0, 0, 0, 0} or {0, π, 0, π, 0}. The even tones should have the same phase.

Using the Eq. 4, we can plot the power difference and also the phase relation of the 5-tone comb by varying the normalized amplitude and DC bias, as shown in the figures S1 below.

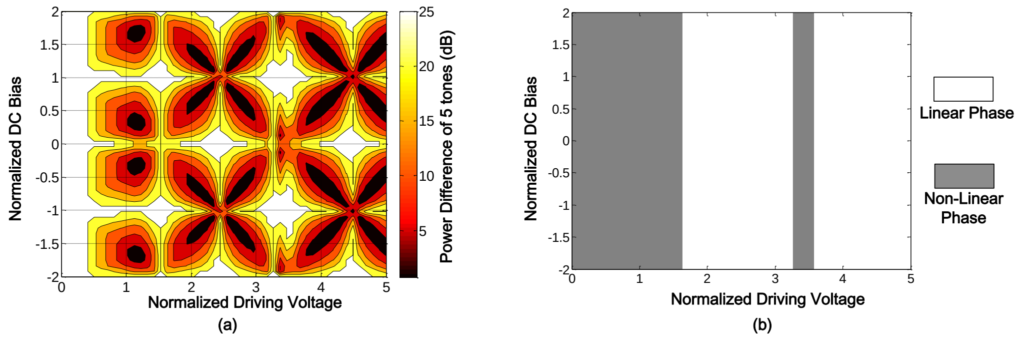

Figure S1. (a) the power difference of the 5 tone-comb versus the normalized driving voltage and the DC bias; (b) the phase relation of 5 tones versus the driving voltage and DC bias. Here the power difference is calculated by the power of the 5 tones. Two phase relations are defined here, called linear phase or non-linear phase. For linear phase relation, the phase of the 5 tones are {0, 0, 0, 0, 0} or {0, π, 0, π, 0}, otherwise, it will be marked as non-linear phase relation.

From Fig. S1, we can see that there are several optimal zones with RF driving voltages and DC bias values (black zones in Fig.S1 (a)) to achieve a flattened 5-tone comb generation with smallest power differences. The power difference is determined by both driving voltage and DC bias. We also find the linear phase relation of generated tones is only determined by the driving voltage. The linear phase relation of the 5 tones is easy to achieve by applying an appropriate driving voltage with a large tunable range. At the same time, the optimal zones in Fig. S1 (a) and linear phase zones in (b) can both be satisfied, since there are several overlapping areas. However, we can see that, the area of optimal zones for flattened comb with smallest power difference is much smaller than that of the linear phase zones. Compared with the phase relation, the power difference is much more sensitive to the value of driving voltage and DC bias. Finally, since the linear-phase condition for 5-tone comb is much easier to achieve, the equal power conditions can be relieved by using power equalizer devices, such as filters or wavelength selective switch (WSS).

In our case, we use a commercial band-pass filter (BPF), which is the wavelength- and bandwidth-tunable optical band-pass filter (Alnair Lab, BVF-200). The filtering bandwidth is variable from 0.1nm to 15nm, and the center wavelength is tunable from 1525nm to 1610nm. It has an ideal flat-top response with sharp roll-off > 150dB/nm and the out-of-band suppression is about 50dB. It has negligible chromatic dispersion less than 0.1ps/nm. Therefore, as analyzed above, the filter does not compensate the phase relation since the linear phase can be easily achieved. However, we can use this filter to suppress the 2^{nd}-order sidebands to equalize the power to achieve more flattened comb. In our case, we choose the driving voltage about $2V_{pi}$ and the DC bias is about 0.52 V_{pi}. The power difference of the generated 5-tone comb for the 5x12.5GHz, 3x25GHz, and 5x25GHz comb after the filter is less than 0.3dB.

Supplementary References

[1] Soto, M. A. et al. Optical sinc-shaped Nyquist pulses of exceptional quality. *Nat. Communications* **4**, 3898-3908(2013).