Supplementary Information: Active Dynamics of Semiflexible Filaments

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ANALYTICAL MODEL

The free energy of a semiflexible polymer can be expressed as -

$$F[\vec{r}(s)] = \int_0^L ds \left(\left(\frac{b_o\kappa}{2}\right) \frac{\partial \hat{t}}{\partial s} \cdot \frac{\partial \hat{t}}{\partial s} + g(s)\hat{t}(s)\cdot\hat{t}(s) \right) \tag{S1}$$

where L is the length of the polymer, b_o is the thickness of the chain, κ is the bending rigidity, $\hat{t}(s) = \frac{\partial \hat{r}}{\partial s}$ is the unit tangent vector to the chain at location s along the chain, ζ is the friction per unit length and $g(s)$ is a Lagrange multiplier preserving the local metric; $g(s)$ has the interpretation of local tension, and is assumed to be independent of time (though strictly this is not true).

The functional derivative $\delta F/\delta\vec{r}$ gives us the force on the chain. We perform a variation $\vec{r}(s) \to \vec{r}(s) + \delta\vec{r}(s)$ and obtain the forces due to bending and instretchability separately as shown below -

$$\begin{aligned} \frac{\delta F_{bending}}{\delta \vec{r}(s)} &\simeq 2\int_0^L ds\ \frac{\partial^2 \vec{r}(s)}{\partial s^2} \cdot \frac{\partial^2 \delta\vec{r}(s)}{\partial s^2} \\ &= 2\int_0^L ds\ \frac{\partial^4 \vec{r}(s)}{\partial s^4} \end{aligned} \tag{S2}$$

where integration by parts was used and variations at the boundary neglected. Similarly the stretching term yields

$$\frac{\delta F_{stretching}}{\delta \vec{r}(s)} = -2\int_0^L ds\ \frac{\partial}{\partial s}\left(g(s,t)\ \frac{\partial \vec{r}(s,t)}{\partial s} \right) \tag{S3}$$

The overdamped equation of motion for the polymer can be written as

$$\zeta\frac{\partial \vec{r}(s)}{\partial t} = -b_o\kappa\frac{\partial^4 \vec{r}}{\partial s^4} + 2\frac{\partial}{\partial s}\left(g(s)\frac{\partial \vec{r}}{\partial s}\right) + \vec{f}_T(s,t) + \vec{F}(s,t) \tag{S4}$$

where $\vec{f}_T(s,t)$ is the thermal noise and $\vec{F}(s,t)$ is the active noise. The equation is difficult to solve exactly. Various methods such as gradient expansion have been used in the past to study the dynamics. We resort to a simple approximation in which the tension $g(s)$ along the chain is replaced by the mean tension g; the above equation then simplifies to

$$\zeta\frac{\partial \vec{r}(s)}{\partial t} = -b_o\kappa\frac{\partial^4 \vec{r}}{\partial s^4} + 2g\frac{\partial^2 \vec{r}}{\partial s^2} + \vec{f}_T(s,t) + \vec{F}(s,t) \tag{S5}$$

Following the treatment of Doi and Edwards [1] we introduce modes -

$$\vec{X}_p(t) = \frac{1}{L}\int_0^L ds\ \phi_p(s)\ \vec{r}(s,t) \tag{S6}$$

We want to choose $\phi_p(s)$ such that the modes evolve in time according to

$$\zeta_p\frac{\partial \vec{X}_p}{\partial t} = -k_p\vec{X}_p(t)\ +\ \vec{f}_{T,p}(t) + \vec{F}_p(t) \tag{S7}$$

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$$\zeta_p \frac{\partial \vec{X}_p}{\partial t} = \zeta_p \int_0^L ds \ \phi_p(s) \frac{\partial r(s,t)}{\partial s}$$
$$= \frac{\zeta_p}{\zeta} \int_0^L ds \ \left(-b_o \kappa \frac{\partial^4 \vec{r}}{\partial s^4} + 2g \frac{\partial^2 \vec{r}}{\partial s^2} + \vec{f}_T(s,t) + \vec{F}(s,t) \right) \quad \text{(S8)}$$

Consider the first integral term apart from the factor $-\kappa b_0 \zeta_p/\zeta$ -

$$\int_0^L ds \ \phi_p(s) \ \frac{\partial^4 \vec{r}}{\partial s^4} = \left[\phi_p(s) \frac{\partial^3 \vec{r}}{\partial s^3} \right]_0^L - \left[\frac{\partial \phi_p(s)}{\partial s} \ \frac{\partial^2 \vec{r}}{\partial s^2} \right]_0^L + \left[\frac{\partial^2 \phi_p(s)}{\partial s^2} \frac{\partial \vec{r}}{\partial s} \right]_0^L + \left[\frac{\partial^3 \phi_p(s)}{\partial s^3} \vec{r} \right]_0^L$$
$$+ \int_0^L ds \ \frac{\partial^4 \phi_p(s)}{\partial s^4} \ \vec{r}(s,t) \quad \text{(S9)}$$

using integration by parts.

Similarly apart from the factor $2g\zeta_p/\zeta$ the second term -

$$\int_0^L ds \ \phi_p(s) \ \frac{\partial^2 \vec{r}}{\partial s^2} = \left[\phi_p(s) \frac{\partial \vec{r}}{\partial s} \right]_0^L - \left[\frac{\partial \phi_p(s)}{\partial s} \ \vec{r}(s) \right]_0^L + \int_0^L ds \ \frac{\partial^2 \phi_p(s)}{\partial s^2} \ \vec{r}(s,t) \quad \text{(S10)}$$

Provided we can neglect the boundary terms with appropriate boundary conditions, comparing with the desired equation for mode evolution we get

$$\kappa \ b_0 \ \frac{\zeta_p}{\zeta} \frac{\partial^4 \phi_p(s)}{\partial s^4} = 2g \ \frac{\partial^2 \phi_p(s)}{\partial s^2} + k_p \phi_p(s) \quad \text{(S11)}$$

Choosing $\phi = \frac{1}{L} \ \cos\left(\frac{\pi p s}{L}\right)$, Eq. (S11) is satisfied provided

$$k_p = \kappa \ b_0 \ \frac{\zeta_p}{\zeta} \ \left(\frac{\pi p}{L}\right)^4 + 2g \ \frac{\zeta_p}{\zeta} \ \left(\frac{\pi p}{L}\right)^2$$
$$= a \ p^4 + b \ p^2 \quad \text{(S12)}$$

The noise terms become

$$\vec{f}_{Tp}(t) = \frac{\zeta_p}{L \ \zeta} \int_0^L ds \ \cos\left(\frac{\pi p s}{L}\right) \ \vec{f}_T(s,t) \quad \text{(S13)}$$

and

$$\vec{F}_p(t) = \frac{\zeta_p}{L \ \zeta} \int_0^L ds \ \cos\left(\frac{\pi p s}{L}\right) \ \vec{F}(s,t) \quad \text{(S14)}$$

Since the thermal and the active noise occur as a sum, we shall treat them separately.

MSD FOR THE THERMAL NOISE

Thermal forces have zero mean and obey the fluctuation-dissipation theorem

$$\langle f_{T\alpha}(s,t) f_{T\beta}(s',t') \rangle = 2K_B T \zeta \ \delta_{\alpha\beta} \delta(t-t') \ \delta(s-s') \quad \text{(S15)}$$

We choose ζ_p such that their Fourier counterparts also obey the same form of the fluctuation-dissipation theorem -

$$\langle f_{Tp,\alpha}(t) f_{Tq,\beta}(t') \rangle = 2K_B T \zeta_p \ \delta_{\alpha\beta} \delta(t-t') \ \delta_{pq} \quad \text{(S16)}$$

The correlation function in the Fourier space is

$$
\begin{aligned}
\langle f_{Tp,\alpha}(t) f_{Tq,\beta}(t')\rangle &= \frac{\zeta_p\zeta_q}{L^2\zeta^2}\int_0^L\int_0^L dsds' \;\cos\left(\frac{\pi ps}{L}\right)\cos\left(\frac{\pi qs'}{L}\right)\;\langle f_{T\alpha}(s,t) f_{T\beta}(s',t')\rangle \\
&= \frac{\zeta_p\zeta_q}{L^2\zeta^2}\int_0^L\int_0^L dsds' \;\cos\left(\frac{\pi ps}{L}\right)\cos\left(\frac{\pi qs'}{L}\right)\; 2K_BT\zeta\;\delta_{\alpha\beta}\delta(t-t')\;\delta(s-s') \\
&= \frac{\zeta_p\zeta_q}{L^2\zeta^2}\;2K_BT\zeta\;\delta_{\alpha\beta}\delta(t-t')\;\int_0^L ds\cos\left(\frac{\pi ps}{L}\right)\cos\left(\frac{\pi qs}{L}\right) \\
&= \frac{\zeta_p\zeta_q}{L^2\zeta^2}\;2K_BT\zeta\;\delta_{\alpha\beta}\delta(t-t')\;\delta_{pq}\frac{(1+\delta_{p,0})}{2}
\end{aligned}
\tag{S17}
$$

Demanding that the RHS in the last line of the above equation has the same form as the fluctuation dissipation theorem, we have $\zeta_0 = L\zeta$ and $\zeta_{p\neq 0} = 2L\zeta$.

First we calculate the MSD for the zero-th mode which corresponds to the centre-of-mass

$$
\vec{X}_0(t) = \frac{1}{L}\int_0^L ds\vec{r}(s,t) \tag{S18}
$$

We have $k_{p=0} = 0$ and so the equation of motion of the zero-th mode is

$$
\zeta_0\frac{\partial\vec{X}_0(t)}{\partial t} = \vec{f}_{T0}(t) \tag{S19}
$$

The formal solution to the above equation is

$$
\vec{X}_0(t) = \frac{1}{\zeta_0}\int_{-\infty}^t dt'\vec{f}_{T0}(t') \tag{S20}
$$

The MSD for the centre-of-mass is given by

$$
\begin{aligned}
(\vec{X}_0(t) - \vec{X}_0(0))^2 &= \frac{1}{\zeta_0^2}\int_0^t dt_1\int_0^t dt_2\langle\vec{f}_{T0}(t_1)\cdot\vec{f}_{T0}(t_2)\rangle \\
&= \frac{1}{\zeta_0^2}\int_0^t dt_1\int_0^t dt_2\;2\cdot 3\;k_BT\zeta_0\;\delta(t-t') \\
&= 6k_BTt/\zeta_0 \\
&= 6k_BTt/(L\zeta) \\
&= 6k_BTt/(N\gamma) \\
&= 6D_{COM}t
\end{aligned}
\tag{S21}
$$

where $D_{COM} = k_BT/(N\gamma)$ is the diffusion coefficient as expected for the centre of mass using the definition $\zeta = \gamma/b_0$ and $L = Nb_o$.

Now we proceed to calculate the MSD for an arbitrary monomer of the chain. The solution to the equation of motion (Eq.S10) is

$$
\vec{X}_0(t) = \frac{\exp(-k_pt/\zeta_p)}{\zeta_p}\int_{-\infty}^t dt'\;\exp(k_pt'/\zeta_p)\;\cos\left(\frac{\pi ps}{L}\right)\vec{f}_{Tp}(t') \tag{S22}
$$

Corresponding to the adopted choice of modes, the inverse transform is given by

$$
\vec{r}(s,t) = \vec{X}_0(t) + 2\sum_{p=1}^{\infty}\vec{X}_p(t) \tag{S23}
$$

The MSD of an arbitrary monomer is given by

$$
\begin{aligned}
(\langle\vec{r}(s,t)\;-\;\vec{r}(s,0))^2\rangle &= MSD_{CM} + 4\sum_{p=1}^{\infty}\cos\left(\frac{\pi ps}{L}\right)\langle(\vec{X}_p(t)-\vec{X}_p(0))\cdot(\vec{X}_0(t)-\vec{X}_0(0))\rangle \\
&+ 4\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\cos\left(\frac{\pi ps}{L}\right)\cos\left(\frac{\pi qs}{L}\right)\langle(\vec{X}_p(t)-\vec{X}_p(0))\cdot(\vec{X}_q(t)-\vec{X}_q(0))\rangle
\end{aligned}
\tag{S24}
$$

The first term has already been evaluated and the second term is trivially seen to be zero using Eq.(S17). We proceed to evaluate the last term which involves the evaluation of the following four terms -

$$\begin{aligned}
&\langle \vec{X}_p(t)\cdot\vec{X}_q(t)\rangle \\
&= \frac{\exp(-k_p t/\zeta_p)\exp(-k_q t/\zeta_q)}{\zeta_p\zeta_q}\int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2\ \exp(k_p t_1/\zeta_p)\ \exp(k_q t_2/\zeta_q)\ \langle \vec{f}_{Tp}(t_1)\cdot\vec{f}_{Tq}(t_2)\rangle \\
&= \frac{\exp(-k_p t/\zeta_p)\exp(-k_q t/\zeta_q)}{\zeta_p\zeta_q}\int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2\ \exp(k_p t_1/\zeta_p)\ \exp(k_q t_2/\zeta_q)\ 6\zeta_p K_B T\delta_{pq}\delta(t_1-t_2) \\
&= \frac{6\zeta_p K_B T\delta_{pq}}{\zeta_p\zeta_q}\times\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)}
\end{aligned}\tag{S25}$$

The last term can be evaluated simply by setting $t = 0$ in the last eqation -

$$\langle \vec{X}_p(0)\cdot\vec{X}_q(0)\rangle = \frac{6\zeta_p K_B T\delta_{pq}}{\zeta_p\zeta_q}\times\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)}\tag{S26}$$

The third and the fourth terms viz.$\langle \vec{X}_p(t)\cdot\vec{X}_q(t)\rangle$ and $\langle \vec{X}_q(t)\cdot\vec{X}_p(t)\rangle$ have a $p\to q$ symmetry that will be exploited; one of the terms will be calculated and the other will be obtained by interchanging p with q.

$$\begin{aligned}
&\langle \vec{X}_p(t)\cdot\vec{X}_q(0)\rangle \\
&= \frac{\exp(-k_p t/\zeta_p)}{\zeta_p\zeta_q}\int_{-\infty}^{t}\int_{-\infty}^{0} dt_1 dt_2\ \exp(k_p t_1/\zeta_p)\ \exp(k_q t_2/\zeta_q)\ \langle \vec{f}_{Tp}(t_1)\cdot\vec{f}_{Tq}(t_2)\rangle \\
&= \frac{\exp(-k_p t/\zeta_p)}{\zeta_p\zeta_q}\int_{-\infty}^{t}\int_{-\infty}^{0} dt_1 dt_2\ \exp(k_p t_1/\zeta_p)\ \exp(k_q t_2/\zeta_q)\ 6\zeta_p K_B T\delta_{pq}\delta(t_1-t_2) \\
&= \exp(-k_p t/\zeta_p)\frac{6\zeta_p K_B T\delta_{pq}}{\zeta_p\zeta_q}\times\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)}
\end{aligned}$$

So the counterpart term is $\exp(-k_q t/\zeta_q)\frac{6\zeta_q K_B T\delta_{pq}}{\zeta_q\zeta_p}\times\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)}$.

Gathering all the above terms, we finally obtain

$$\begin{aligned}
(\langle \vec{r}(s,t)\ -\ \vec{r}(s,0))^2\rangle &= MSD_{CM} +\ 4\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\cos\left(\frac{\pi q s}{L}\right)\cos\left(\frac{\pi p s}{L}\right)\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)}\times \\
&\left[\frac{12\zeta_p K_B T\delta_{pq}}{\zeta_p\zeta_q} - \exp(-k_p t/\zeta_p)\frac{6\zeta_p K_B T\delta_{pq}}{\zeta_p\zeta_q} - \exp(-k_q t/\zeta_q)\frac{6\zeta_q K_B T\delta_{pq}}{\zeta_q\zeta_p}\right] \\
&= 24\sum_{p=1}^{\infty}\cos^2\left(\frac{\pi p s}{L}\right)\ \frac{K_B T(1-\exp(-k_p t/\zeta_p))}{k_p}
\end{aligned}\tag{S27}$$

RECOVERING WINKLER'S RESULT FOR THE ABOVE CHOICE OF MODES

Using Eq.(S23), we get the unit tangent vector to the polymer is given by

$$\hat{t}(s,t) = -2\sum_{p=1}^{\infty}\vec{X}_p(t)\ \left(\frac{p\pi}{L}\right)\ \sin\left(\frac{p\pi s}{L}\right)\tag{S28}$$

The equal time tangent-tangent correlation is given by

$$\begin{aligned}
\langle \hat{t}(s)\cdot\hat{t}(s')\rangle &= 4\sum_{p,q=1}^{\infty}\left(\frac{\pi}{L}\right)^2 pq\sin\left(\frac{p\pi s}{L}\right)\sin\left(\frac{q\pi s'}{L}\right)\langle \vec{X}_p(t)\cdot\vec{X}_q(t)\rangle \\
&= 4\sum_{p,q=1}^{\infty}\left(\frac{\pi}{L}\right)^2 pq\sin\left(\frac{p\pi s}{L}\right)\sin\left(\frac{q\pi s'}{L}\right)\ \frac{6\zeta_p K_B T\delta_{pq}}{\zeta_p\zeta_q}\times\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)} \\
&\simeq 12\left(\frac{\pi}{L}\right)^2 K_B T\ \int_0^\infty dp\ p^2/k_p\ \sin\left(\frac{p\pi s}{L}\right)\sin\left(\frac{p\pi s'}{L}\right) \\
&= 12\left(\frac{\pi}{L}\right)^2 K_B T\ \int_0^\infty dp\ p^2/(a\ p^4+b\ p^2)\ \sin\left(\frac{p\pi s}{L}\right)\sin\left(\frac{p\pi s'}{L}\right) \\
&= 12K_bT\ \left(\frac{\pi}{L}\right)^2\pi/(4\sqrt{(ab)}\ \exp(-\sqrt{(b/a)}(\pi/L)\ |s-s'|)
\end{aligned}\tag{S29}$$

where Eq(S25) has been used. Since the denominator in the exponent should give us the persistent length L_p and the correlation function of two unit vectors can at most be unity in magnitude, we demand, using the definitions of a and b (See Eq.(S12)), $\sqrt{2\kappa b_0 g} = 3K_BT/2$ and $\sqrt{(2g/\kappa b_0)} = 1/L_p$. This gives us $2g = 3K_BT/2L_p$ and $\kappa b_0 = 3K_BTL_p/2$. This is a result familiar from the work of Winkler and co-workers and this self-consistent determination establishes the efficacy of our simple model.

DYNAMICS WITH ACTIVE NOISE

Derivation of the MSD for the centre of mass and an arbitrary monomer basically proceeds along similar lines as above; the only additional ingredient is the correlations of the active forces. We assume, contrary to earlier studies by Liverpool that the average of the active force is zero. This is not artificial because in the simulations that we perform, the active forces act in the direction of the local normal to the polymer. The azimuthal symmetry about the axis of the filament (assumed stiff) justifies the above assumption though the average in the longitudinal direction need not be zero because of the polarity of the track. The active forces do not obey the fluctuation-dissipation theorem and the active force correlations are given by

$$\langle \vec{F}(s,t)\cdot\vec{F}(s',t')\rangle = C\exp(-|t-t'|/\tau)\ \delta(s-s') \tag{S30}$$

The prefactor is not dependent on ambient temperature and friction; it is expressed as $C = F^2 p_{on}/b_o$ where $F(s,t)$ is the magnitude of the active force, τ is the time period of motor activity and p_{on} is the probability for a single motor to remain active. The correlation function in the Fourier space is

$$\begin{aligned}
\langle \vec{F}_p(t)\cdot\vec{F}_q(t')\rangle &= \frac{\zeta_p\zeta_q}{L^2\zeta^2}\int_0^L\int_0^L dsds'\ \cos\left(\frac{\pi ps}{L}\right)\cos\left(\frac{\pi qs'}{L}\right)\langle \vec{F}(s,t)\cdot\vec{F}(s',t')\rangle \\
&= \frac{\zeta_p\zeta_q}{L^2\zeta^2}\int_0^L\int_0^L dsds'\ \cos\left(\frac{\pi ps}{L}\right)\cos\left(\frac{\pi qs'}{L}\right)\ C\exp(-|t-t'|/\tau)\ \delta(s-s') \\
&= C\ \frac{\zeta_p\zeta_q}{L^2\zeta^2}\ \delta_{\alpha\beta}\ \exp(-|t-t'|/\tau)\delta_{pq}(1+\delta_{p,0})L/2
\end{aligned}\tag{S31}$$

First we evaluate the MSD for the centre of mass. As before the MSD for the centre-of-mass is given by

$$\begin{aligned}
(\vec{X}_0(t)-\vec{X}_0(0))^2 &= \frac{1}{\zeta_0^2}\int_0^t dt_1\int_0^t dt_2\langle \vec{F}_0(t_1)\cdot\vec{F}_0(t_2)\rangle \\
&= \frac{1}{\zeta_0^2}\int_0^t dt_1\int_0^t dt_2 LC\ \exp(-|t-t'|/\tau) \\
&= \frac{2C\tau}{L\zeta^2}(t+\tau(\exp[-t/\tau]-1))
\end{aligned}\tag{S32}$$

For $t \ll \tau$, the exponential can be expanded and we get

$$\begin{aligned}
(\vec{X}_0(t)-\vec{X}_0(0))^2 &= \frac{C}{L\zeta^2}t^2 \\
&= N(\frac{F}{N\gamma})^2\ t^2
\end{aligned}\tag{S33}$$

which offers the scope of defining an active velocity $v_{activ} \sim \sqrt{N}\ (F/(N\gamma))$

The general solution for the active modes can be written as

$$\vec{X}_0(t) = \frac{\exp(-k_p t/\zeta_p)}{\zeta_p} \int_{-\infty}^{t} dt' \ \exp(k_p t'/\zeta_p)\ \vec{F}_p(t') \tag{S34}$$

As before the MSD for an arbitrary monomer is evaluated using the same structure as Eq.(S24). For active noise, evaluation of the different terms proceeds as follows. First term in the summation -

$$\begin{aligned}
&\langle \vec{X}_p(t)\cdot\vec{X}_q(t)\rangle \\
&= \frac{\exp(-k_p t/\zeta_p)\exp(-k_q t/\zeta_q)}{\zeta_p\zeta_q} \int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2\ \exp(k_p t_1)/\zeta_p\ \exp(k_q t_2)/\zeta_q\ \langle \vec{F}_p(t_1)\cdot\vec{F}_q(t_2)\rangle \\
&= \frac{\exp(-k_p t/\zeta_p)\exp(-k_q t/\zeta_q)}{\zeta_p\zeta_q} \int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2\ \exp(k_p t_1)/\zeta_p\ \exp(k_q t_2)/\zeta_q\ C\ \frac{\zeta_p\zeta_q}{L^2\zeta^2}\ \exp(-|t-t'|/\tau)\delta_{pq}\ \frac{L}{2} \\
&= \frac{C\delta_{pq}}{2L\zeta^2} \times \frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)} \times \left(\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{1}{\tau}\right)} + \frac{1}{\left(\frac{k_q}{\zeta_q}+\frac{1}{\tau}\right)}\right)
\end{aligned} \tag{S35}$$

The last term in the summation is the same as above using similar arguements as with the thermal noise.

Let us evaluate the second term and the third term will follow from $p \to q$ symmetry.

The second term -

$$\begin{aligned}
&\langle \vec{X}_p(t)\cdot\vec{X}_q(0)\rangle \\
&= \frac{\exp(-k_p t/\zeta_p)}{\zeta_p\zeta_q} \int_{-\infty}^{t}\int_{-\infty}^{0} dt_1 dt_2\ \exp(k_p t_1/\zeta_p)\ \exp(k_q t_2/\zeta_q)\ \langle \vec{F}_p(t_1)\cdot\vec{F}_q(t_2)\rangle \\
&= C\ \delta_{pq}\frac{\exp(-k_p t/\zeta_p)}{2L\zeta^2} \int_{-\infty}^{t}\int_{-\infty}^{0} dt_1 dt_2\ \exp(k_p t_1/\zeta_p)\ \exp(k_q t_2/\zeta_q)\ \exp(-|t-t'|/\tau) \\
&= \exp(-\frac{k_p t}{\zeta_p})\frac{C\delta_{pq}}{2L\zeta^2} \times \left(\frac{(\exp(k_p/\zeta_p - 1/\tau)t - 1)}{\left(\left(\frac{k_p}{\zeta_p}-\frac{1}{\tau}\right)+\left(\frac{k_q}{\zeta_q}+\frac{1}{\tau}\right)\right)} + \frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{k_q}{\zeta_q}\right)} \times \left(\frac{1}{\left(\frac{k_p}{\zeta_p}+\frac{1}{\tau}\right)} + \frac{1}{\left(\frac{k_q}{\zeta_q}+\frac{1}{\tau}\right)}\right)\right)
\end{aligned}$$

Finally we gather all the terms to get

$$\begin{aligned}
(\langle \vec{r}(s,t)\ -\ \vec{r}(s,0))^2\rangle &= MSD_{CM} \\
&+4C/(L\zeta^2)\sum_{p=1}^{\infty}\cos^2(\pi ps/L)\ (T1+T2)
\end{aligned} \tag{S36}$$

where

$$T1 = (1-\exp(-k_p t/\zeta_p))/\left(\frac{k_p}{\zeta_p}\left(\frac{k_p}{\zeta_p}+\frac{1}{\tau}\right)\right) \tag{S37}$$

and

$$T2 = (\exp(-t/\tau)-\exp(-k_p t/\zeta_p))/\left(\left(\frac{k_p}{\zeta_p}\right)^2 - \left(\frac{1}{\tau}\right)^2\right) \tag{S38}$$

ESTIMATE OF THE CROSSOVER TIME

As seen from Fig.(2) of the main text, very small times are thermal noise dominated and there is a crossover time for the system to be adequately excited by the active noise. By comparing the MSD terms (thermal and active) for the first mode, we make an approximate estimate of the crossover time t_c as follows -

We expand the exponentials in the corresponding expressions Eqs.(S27 and S35) to get

$$\frac{24K_BT}{k_p}(k_p t/\zeta_p - k_p^2t^2/2\zeta_p^2) = \frac{4C}{L\zeta^2}\frac{1}{1+k_p\tau/\zeta_p}t^2/2 \tag{S39}$$

After some algebra, we get

$$\frac{6K_BT\zeta}{C}(1 - tk_p/2\zeta_p) = t/(1 + k_p\tau/\zeta_p) \tag{S40}$$

This can be simplified to yield

$$t_c = \frac{1}{\frac{C}{6K_BT\zeta(k_p\tau/\zeta_p+1)} + \frac{k_p}{2\zeta_p}} \tag{S41}$$

In the higly overdamped case where $k_p\tau/\zeta_p$ is small, the crossover time $t_c \sim \frac{6K_BT\zeta}{C}$. In a somewhat intuitive way, the same result can be obtained as follows - for the thermal motion $\langle r^2 \rangle \sim 6Dt$ and for the active motion for $t \ll \tau$, $\langle r^2 \rangle \sim (Ft/\gamma)^2$. Equating the two, one gets $t_c \sim 6D/(F/\gamma)^2$.

VELOCITY FLUCTUATIONS OF THE CENTRE OF MASS

The overdamped equation for the centre of mass is given by

$$M\frac{dv}{dt} = -N\gamma v + f + F \tag{S42}$$

Fourier transforming and conjugating, we get

$$\langle \tilde{v}^2 \rangle = \frac{1}{M}\frac{\langle \tilde{f}^2 \rangle + \langle \tilde{F}^2 \rangle}{(\gamma/m)^2 + \omega^2} \tag{S43}$$

where M is the total mass of the polymer and m is the mass of a segment of the polyer. For thermal noise $\langle \tilde{f}^2 \rangle = N\gamma K_BT/\pi$. For active noise

$$\begin{aligned}
\langle \tilde{f}^2(\omega) \rangle &= \left(\frac{NF^2}{2\pi b_o}\right) \int_{-\infty}^{\infty} dt \exp(i\omega t)\exp(-|t|/\tau) \\
&= \left(\frac{NF^2}{2\pi b_o}\right) (\int_{-\infty}^{0} dt \exp(-i\omega t)\exp(t/\tau) + \int_{0}^{\infty} dt \exp(-i\omega t)\exp(-t/\tau)) \\
&= \left(\frac{NF^2}{\pi b_o}\right) Re(\int_{0}^{\infty} dt \exp(-i\omega t)\exp(-|t|/\tau)) \\
&= \left(\frac{NF^2}{\pi b_o}\right) \frac{\tau}{(1 + (\omega\tau)^2)}
\end{aligned}$$

Integrating over all frequencies, we get

$$\langle \tilde{v}^2 \rangle = K_BT/M + \frac{N}{b_o}\frac{F^2}{(N\gamma)^2}\frac{1}{1 + m\gamma/\tau} \tag{S44}$$

The first term corresponds to the equipartition theorem while the second term represents the active contribution.

AUTOCORRELATION FUNCTIONS FOR PURELY TANGENTIAL FORCES

Simulations performed with tagential active forces also show similar stabilisation of bending mode in resonance with active forces as shown below (Fig.S1). However the effect is less pronounced as compared to purely normal active forces and for shorter filaments.

ATUOCORRELATION FUNCTIONS FOR PURLEY NORMAL FORCES FOR $L \sim L_p$

We performed simulations with a chain of length $L = 100.0$ and $L_p = 100.0$ and found the resonance effect to be present (Fig.S2); the qualitative features look very much the same as for the case $L < L_p$.

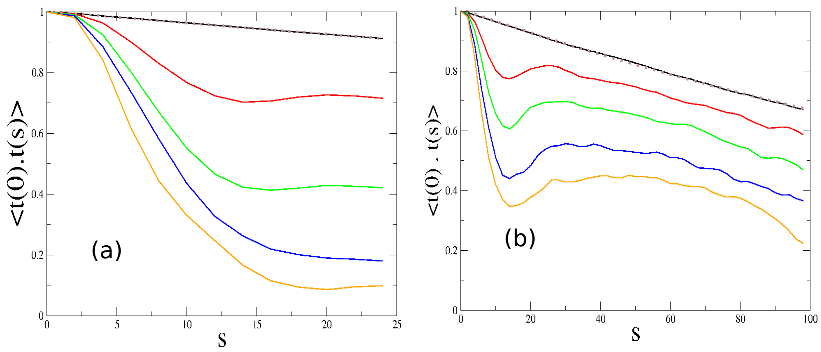

FIG. S1: Tangent-tangent autocorrelation functions for magnitudes of tangential active forces = 0.0 (black), 90.0 (red), 130.0 (green), 170.0 (blue) and 200.0 (orange) for two different chain lengths $L = 25, 100$ (left and right panel respectively), $L_p = 250.0$. The bending resonance effect is present for tangential active forces also. For shorter filaments, the effect is less pronounced as they effectively are more stiff and hence more difficult to bend. Tangential forces on nearby beads effectively add up to point in the local normal direction because of curvature of the polymer.

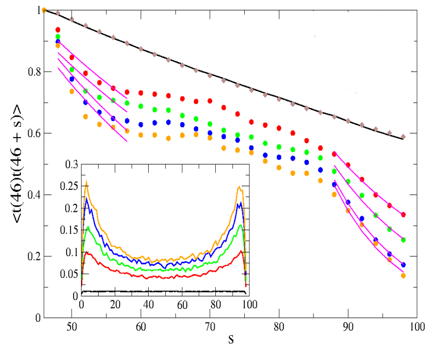

FIG. S2: Equal time tangent autocorrelation function for $L_p = 100.0$ and chain length $L = 100.0$ for magnitudes of active forces $F_a = 0.0$, 90.0, 130.0, 170.0 and 200.0 from top down. The resonance-like effect is found to be persist in this regime.

AUTOCORRELATION FUNCTION WITH CORRELATIONS MEASURED FROM THE END OF THE CHAIN

We show below the results for measurement of the tangent-tangent autocorrelation function measured from one end of a polymer chain of length $L = 100$ and $\kappa = 250.0$. The results have been discussed in the context of Fig.(3b) of the main text.

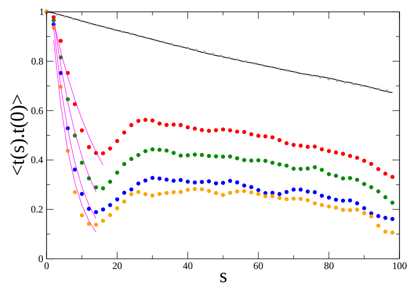

FIG. S3: The tangent-tangent autocorrelation function measured from one end of the polymer with $\kappa = 250.0$ and contour length $L = 100.0$ for different values of the active forces $F = 0.0, 90.0, 130.0, 170.0$ and 200.0 (from top down).

[1] Doi, Masao and S.F.Edwards. 1986. Theory of Polymer Dynamics. OUP.