Supplementary Information: "Message spreading in networks with stickiness and persistence: Large clustering does not always facilitate large-scale diffusion"

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1. MESSAGE DIFFUSION ON SQUARE LATTICE

We first present additional results to support the arguments for message diffusion on square lattice.

In Fig. S1, we can observe that the size of message spreading depends more on the stickiness of the message (values of a with fixed n_s) than on the persistence b. We observe that the information can reach the vast majority of population (more than 80%) for $a \gtrsim 0.45$.

Fig. S2 shows the dependence of the size of recovered population on the parameters a and b. For $a \lesssim 0.3$, $\alpha_0 \approx 0.7$, whose value is much larger than the other three indices α_n $(n = 1, 2, 3)$, which indicates that the information has not yet outbreak. As a increases, transmission events E_m $(m > 0)$ contribute a lot to the message spreading, hinting the large scale outbreak of the message. Accordingly, α_0 (α_n) decreases (increases) sharply, as shown in Figs. S2(b)–(d). In addition, it is found that E_0 and E_1 (depending more closely on a) constitute most of the transmission events, whereas E_2 and E_3 rarely occurs during the spreading process.

To provide support for the real phenomena "three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth"), in what follows we investigate the changes of the accumulative indices $\eta_i(a, b)$ by evaluating verification indices α_i as $\eta_i = \sum_{j<i} \alpha_j$ $(i = 1, 2, 3, 4)$. Here the accumulative indices η_i represents the proportions of individuals that have adopted the message when they heard it from at most i informed neighbors. Like the results in Fig. S2, high values of η_1 reflect that the occurrences of E_0 and E_1 account for most of the transmission events. In addition, the results in Figs. S3(b)(c) show that the vast majority will accept a message as truth if it is mentioned or reported by at least two or three neighbors, which supports the mechanisms "three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth"). Moreover, it also indicates that the reinforcement begins to work as the information is bursting and prevailing on the square lattice, where the growth of η_i keep increasing with n_i (Fig. S3(b)). To make the point clear, we plot the global graphs of the four accumulative indices $\eta_i(a, b)$ for different values of inflection point (n_s) in Fig. S4, Fig. S5, and Fig. S6, respectively. Similar behaviors can be detected in the three figures. The results, especially for positive persistence, also reveal the more important roles of E_0 and E_1, and simultaneously provide the evidence for the real phenomenon "three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

2. MESSAGE DIFFUSION ON RANDOM REGULAR NETWORKS (RRNS) AND REGULAR LATTICE NETWORKS (RLS)

In this section, we present additional results to support the arguments for message diffusion on RRNs and RLs.

The sizes of message diffusion are presented in Fig. S7 as a function of a and b. In the presence of social reinforcement $(b > 0)$, we can observe that the message can more easily invade and reach the majority of the population in larger parameter regions beyond the thresholds on RLs, by comparing with that on RRNs with the same average degree. The denser regular lattices (Moore lattice in Fig. S7(b) (d)) are much more efficient in promoting information spreading [1] when the message outbreaks. The reason is that there are more local clustering links can be used for transmission with smaller n_s [1–3] and positive persistence. Instead, the message can more easily diffuse in the RRNs in the presence of strong decay effects $(b < 0)$.

The results summarized in Fig. S8 and Fig. S9 show that the peaks of $E_i(t)$ $(i = 0, 1, 2, 3)$ brings about the peaks of subsequent transmission event $E_{i+1}(t)$, which indicates that transmission events $E_i(t)$ with $i > 2$ fail to last stably simultaneously, even at the critical points throughout the spreading processes. There are thus no time correlations among different events. The time correlations of different transmission events can be completely ruled out in estimating the critical behaviors of the message spreading.

By comparison, the occurrences of transmission events in Hexagonal lattices and Moore lattices can last stably for a long time at critical points (see Fig. S10(b) and Fig. S11(b)). This suggests the existence of the time correlations among different transmission events $E_i(t)$. Besides, the observed huge disparities between $\alpha_i(a, b, t)$ and the corresponding $\beta_i(a, b, t)$ illustrate that the time correlations among transmission events in the both lattices are considerable.

As observed in Fig. S12(b)(c), Fig. S13(b)(c), and the figures ranging from Fig. S14 to Fig. S21, the message captures the vast majority of the population until $E_i(t)$ $(i > 2)$ happens when message outbreaks and prevails. And the spreading reaches a saturation state for the case where $\frac{n_s}{\langle k \rangle} > \frac{1}{2}$, which means that the transmission events E_i $(i > \frac{1}{2}\langle k \rangle)$ rarely happen in the spreading process. Therefore, the results in Fig. S12–Fig. S21 can be regarded as the evidence of the mechanism "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth"), in addition to the results for positive persistence illustrated in Fig. S22, and Fig. S23.

3. MESSAGE DIFFUSION ON ER AND SF NETWORKS

In this section, we provide additional results to support the arguments for message diffusion on ER networks and SF networks.

We observe in Fig. S24 that transmission events $E_i(t)$ $(i > 2)$ fail to last simultaneously and stably over the long time at the critical points throughout the spreading process. Therefore, the time correlations need not to be taken into consideration in theoretical analysis. We have found the same phenomena in the ER networks and the SF with other average degrees.

As illustrated in Fig. S25, the theoretical estimations are sufficient to give fairly precise value of the thresholds. It is apparent that the critical behaviors of the message spreading are completely determined by stickiness (a) of message. Moreover, in comparison with the case of ER networks, the analytical solutions are in better agreement with the simulation in SF networks, attributing to shorter shortest paths and hubs [6–8]. More interestingly, both the analytical boundaries and the numerical thresholds are shifting to left with average degree $\langle k \rangle$, instead of size of the populations as stressed in [9]. This demonstrates that message can easily reach and infect a larger amount of susceptible individuals through paths from those high-degree vertices whose links increase rapidly with $\langle k \rangle$, although they can be infected only once.

In Fig. S26, we find that the persistence also boosts its reasonable impact on the spreading as $\langle k \rangle$ gets larger. It implies that hub nodes of larger size and shorter shortest paths in the networks can transmit the information more efficiently [6, 8, 9]. It is in accordance with the conclusion in [4] that higher degrees and densities are relevant factors in improving the global spread of information.

It should be noted that the both ER and SF networks with small average degrees (such as $\langle k \rangle = 6, 8$) are tree-like, with few short loops, indicating that the critical transmissibility T_C can be derived from equation $T_c = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$ [4, 5]. More specifically, $T_c = \frac{1}{\langle k \rangle - 1}$ for ER networks. Qualitatively, the contributions of the infection events E_n $(n > \langle k \rangle - 2)$ to the spread are insignificant (see Fig. S27 and Fig. S28). Therefore we neglect the contributions of the transmission events $E_l(t)$ $(l > \langle k \rangle)$ to message spreading, and further assume that the following relationship $T_C = \langle T \rangle = \sum_{n=0}^{\langle k \rangle - 1} q_n(1 - e^{-\lambda_{n+1}(a,b)})$ is always satisfied in estimating the analytical thresholds.

Similar to what has been demonstrated in Sec. 2, both Fig. S27 (ER networks) and Fig. S28 (SF networks) show that the results nearby the critical points for $b > 0$ can also be considered as the evidence of the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

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Figures

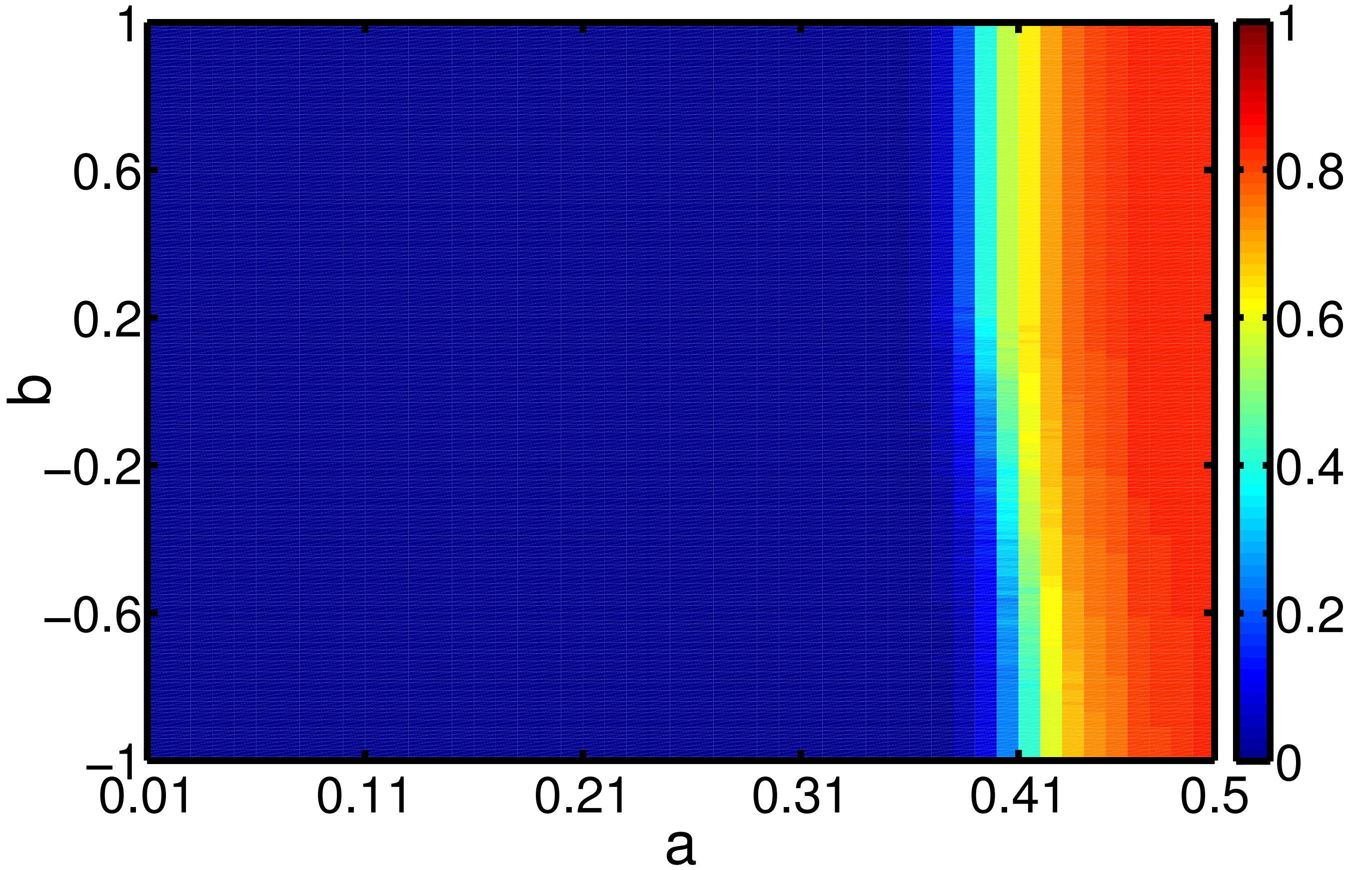

Fig S1: **The densities of recovered individuals as a function of a and b.** Each data point is obtained by averaging 100 independent realizations. The other parameters are $n_s = 2$ and $L = 101$.

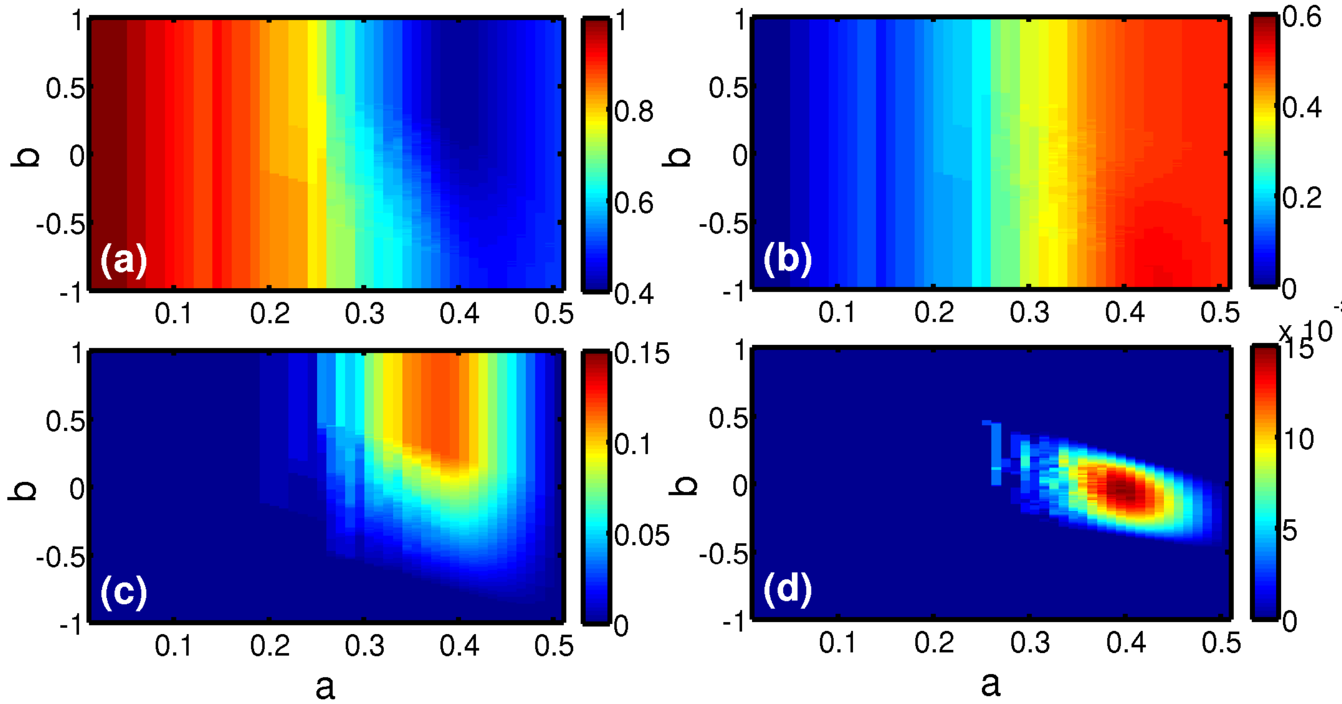

Fig S2: **The four indices (a)** $\alpha_0(a,b)$**, (b)** $\alpha_1(a,b)$**, (c)** $\alpha_2(a,b)$**, and (d)** $\alpha_3(a,b)$ **as a function of** a **and** b**.** The other parameters are token as $n_s = 2$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. As message outbreaks ($a \gtrsim 0.32$), it is clear in (a) and (b) that the transmission events E_0 and E_1 contribute the most to the whole spreading process.

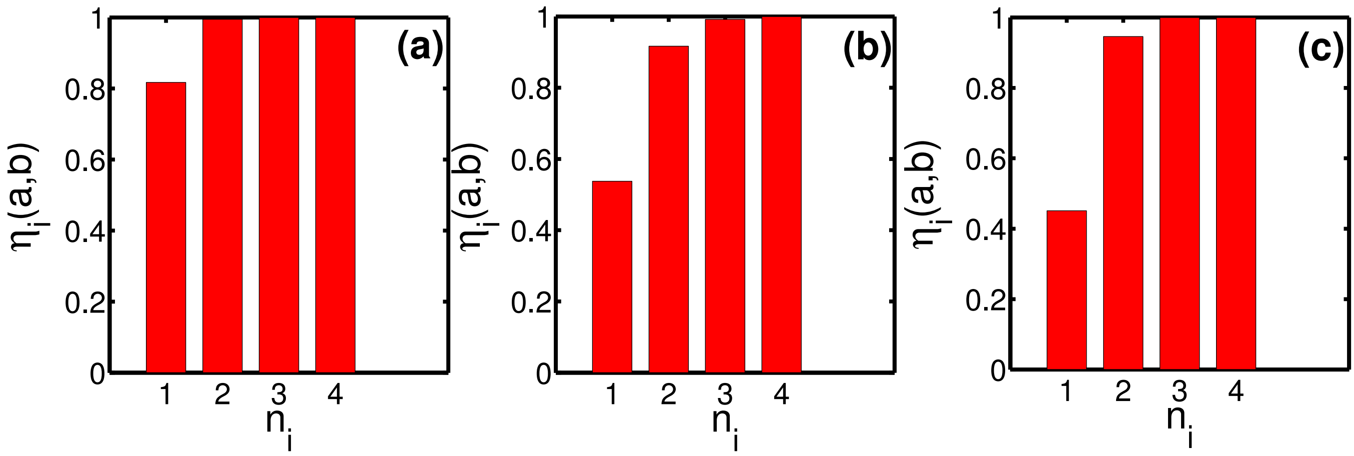

Fig S3: **The four accumulative indices** $\eta_i(a,b)$ **in square latttice.** n_i denotes the number of informed neighbors an individual has had at most when it approves the message. Analysis is performed at (a) subcritical point $a = 0.20$, $b = 0.20$; (b) critical point $a = 0.33$, $b = 0.20$; and (c) supercritical point $a = 0.45$, $b = 0.20$. The other parameters are token as $n_s = 2$ and $L = 101$. Results are obtained by averaging 100 independent realizations. No matter which case, the diffusion of the message owes much to $E_0(t)$ and $E_1(t)$. In (b) and (c), the value of η_3 approaches to 1 rather than η_i ($i < 4$) and η_i ($i < 3$), and the gaps between η_1 and η_2 are obvious. The results in (b) and (c) can be considered as an evidence of the emergence of “Three men make a tiger” (or “A lie, if repeated often enough, will be accepted as truth”) – the vast majority of the population accept the message as truth only when it is repeated more than two times in their ears.

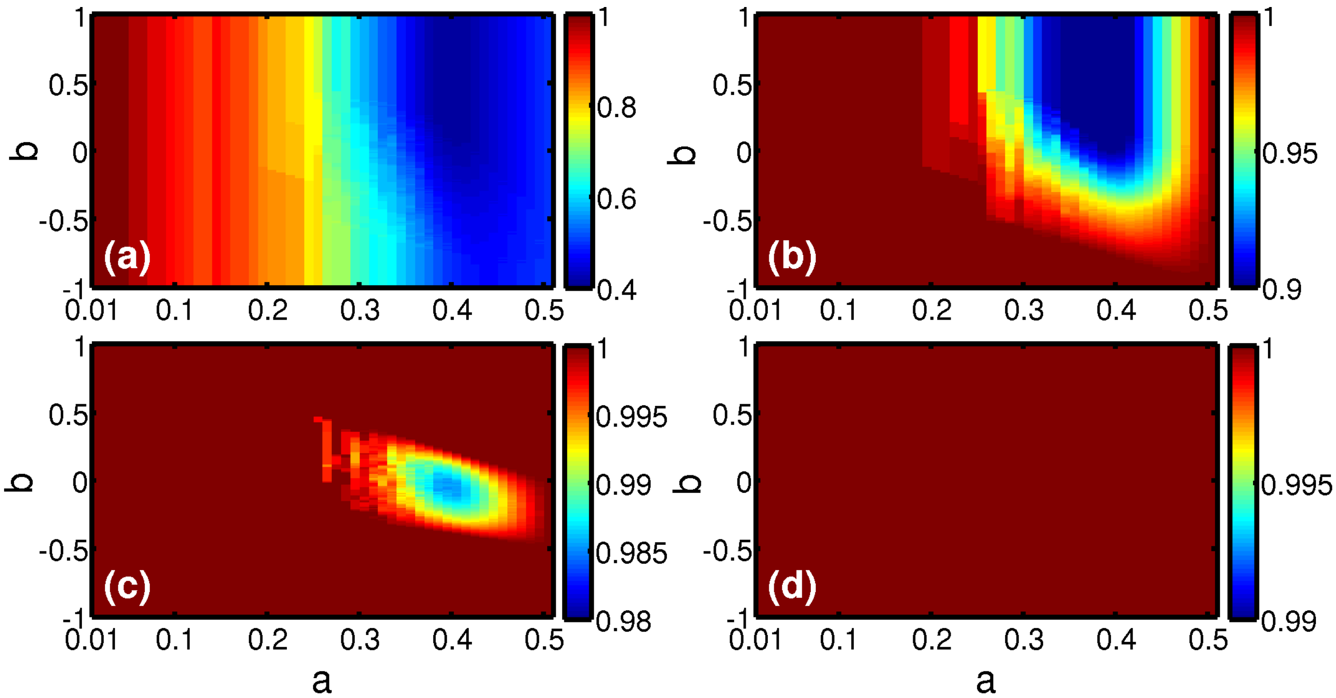

Fig S4: **The four accumulative indices η_0 (a), η_1 (b), η_2 (c), and η_3 (d) as a function of a and b.** The other parameters are token as $n_s = 2$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 90\%$ $(i \geq 2)$ rather than η_1, where transmission events $E_i(t)$ $(i \geq 2)$ contribute more than 60% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidence for the real phenomenon “Three men make a tiger” (or “A lie, if repeated often enough, will be accepted as truth”).

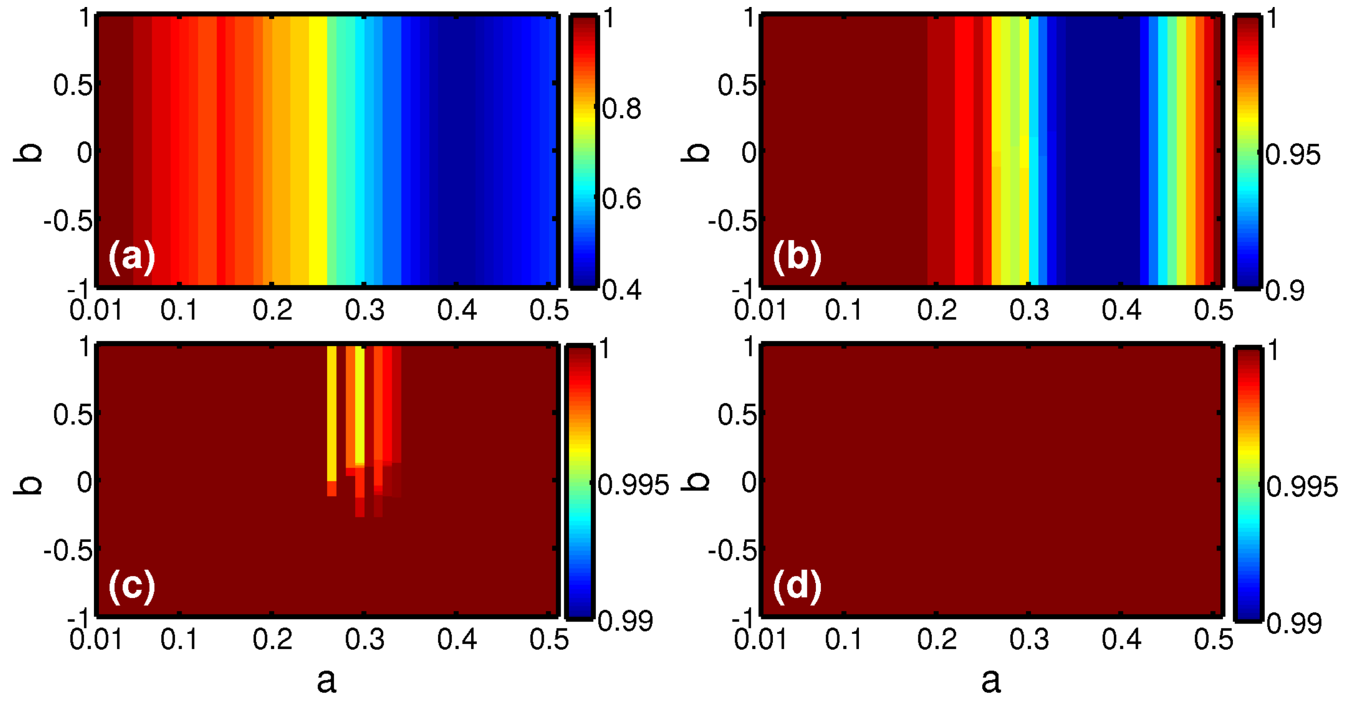

Fig S5: **The four accumulative indices η_0 (a), η_1 (b), η_2 (c), and η_3 (d) as a function of a and b.** The other parameters are token as $n_s = 3$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 90\%$ $(i \geq 2)$ rather than η_1, where transmission events $E_i(t)$ $(i \geq 2)$ contribute more than 60% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidence for the real phenomenon “Three men make a tiger” (or “A lie, if repeated often enough, will be accepted as truth”).

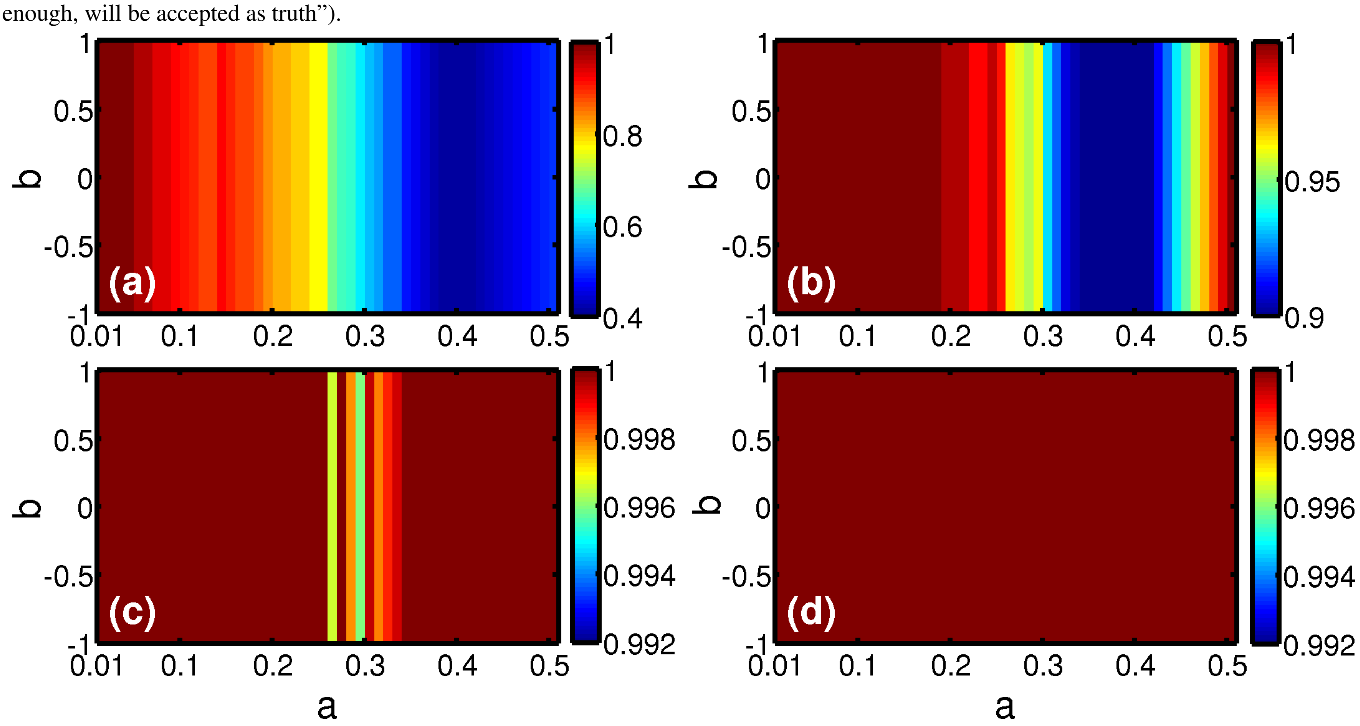

Fig S6: **The four accumulative indices η_0 (a), η_1 (b), η_2 (c), and η_3 (d) as a function of a and b.** The other parameters are token as $n_s = 4$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 90\%$ $(i \geq 2)$ rather than η_1, where transmission events $E_i(t)$ $(i \geq 2)$ contribute more than 60% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidence for the real phenomenon “Three men make a tiger” (or “A lie, if repeated often enough, will be accepted as truth”).

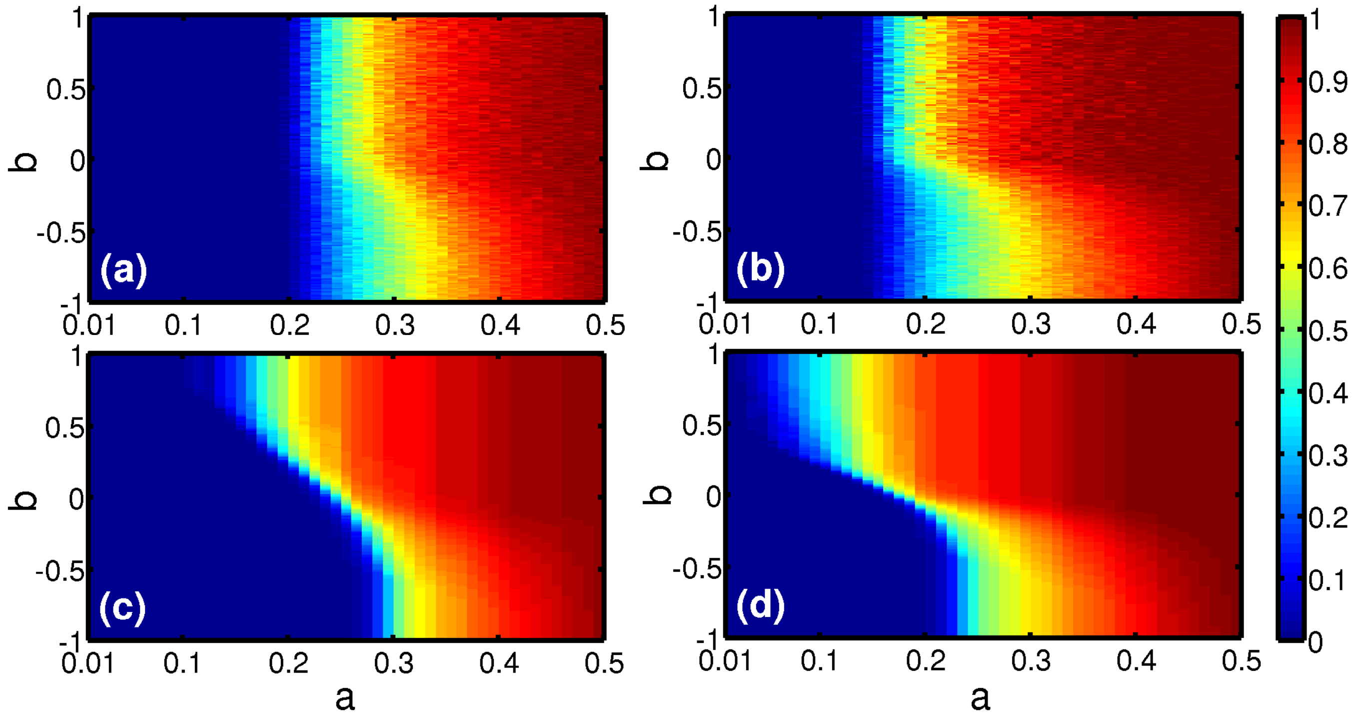

Fig S7: **The densities of recovered individuals as a function of a and b.** The other parameters are token as $n_s = 2$, $N = 1000$ for the RRNs, and $L = 101$ for the RLs. The scales of spreading on RLs ((a) (b)) are compared with that in RRNs ((a) (b)). The degrees of the networks are $< k >= 6$ (a, c) and $< k >= 8$ (b, d), respectively. Each data is obtained by averaging 100 independent realizations. By comparing (a) with (c), for $b > 0$, it is clear in (c) that the message can more easily outbreak and capture a larger population, in contrast to what is observed in (a), owing to the function of social reinforcement effects. However, more individuals accept the message in RRNs for $b < 0$, indicating the advantage of the RRNs in facilitating the diffusion of message in the presence of strong decay effects. As expected, the same conclusion can also be reached by comparing the plots in (b) with that in (d).

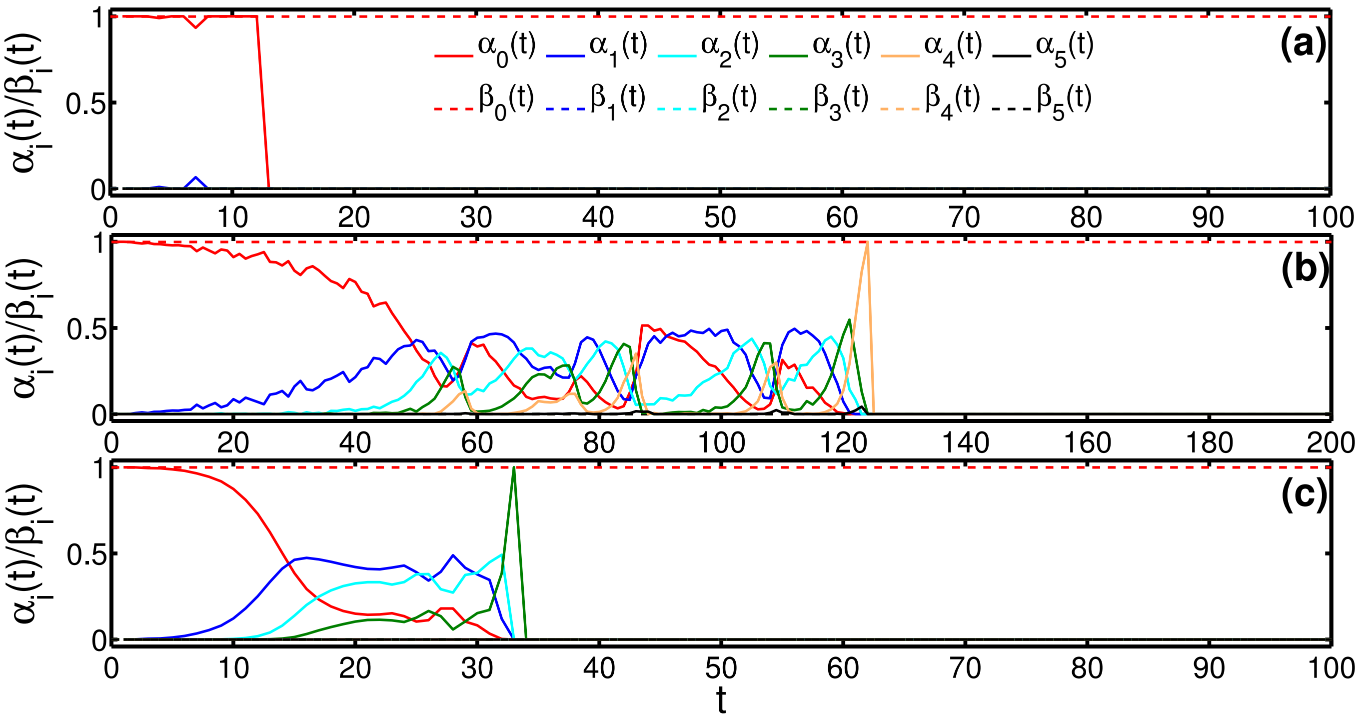

Fig S8: **The evolution of proportions of the transmission events.** The evolution of indices $\alpha_i(a, b, t)$ from simulation (solid lines) and $\beta_i(a, b, t)$ from prediction of percolation theory (dashed lines) on the RRN with $\langle k \rangle = 6$ are presented. Three different cases are considered here: (a) the information vanishes for $a = 0.10$, $b = 0.20$, (b) it outbreaks for $a = 0.19$, $b = 0.20$; and prevails for $a = 0.30$, $b = 0.20$ (c). The other parameters are token as $n_s = 2$ and $N = 1000$. It can be observed that the occurrences of all the transmission events fail to last stably. The time correlations among the transmission events can thus be neglected in estimating the critical behavior of the message spreading.

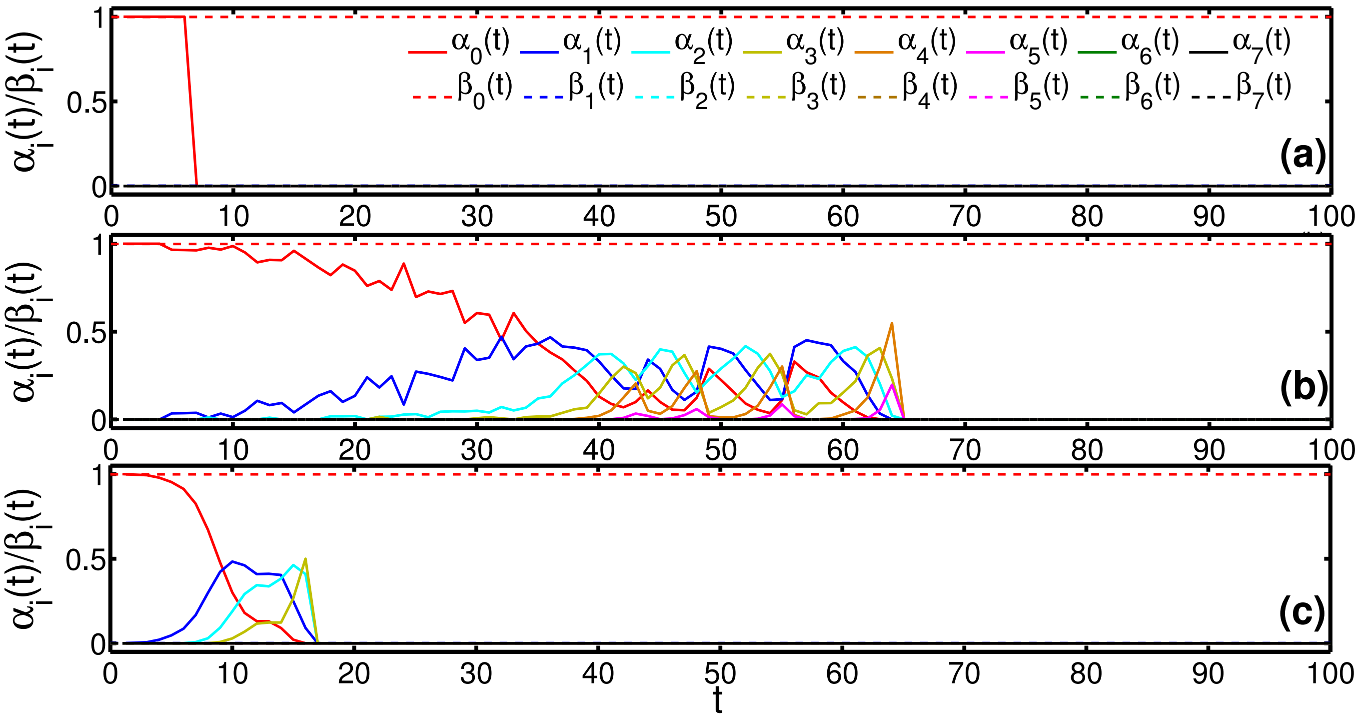

Fig S9: **The evolution of proportions of the transmission events.** The evolution of indices $\alpha_i(a, b, t)$ from simulation (solid lines) and $\beta_i(a, b, t)$ from prediction of percolation theory (dashed lines) on the RRN with $\langle k \rangle = 8$ are presented. Three different cases are considered here: (a) the information vanishes for $a = 0.10$, $b = 0.20$, (b) it outbreaks for $a = 0.13$, $b = 0.20$; and prevails for $a = 0.30$, $b = 0.20$ (c).The other parameters are token as $n_s = 2$ and $N = 1000$. It can be observed that the occurrences of all the transmission events fail to last stably throughout the whole spreading process. The time correlations among the transmission events can thus be neglected in estimating the critical behavior of the message spreading.

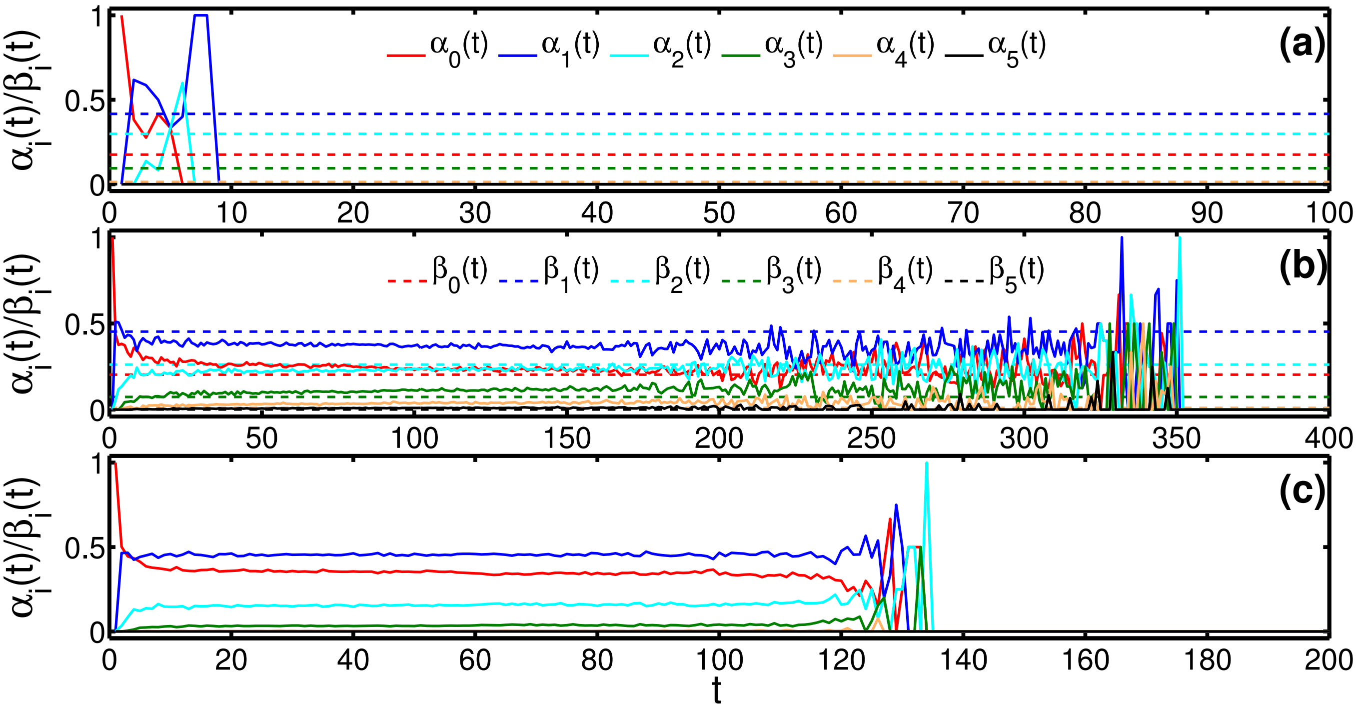

Fig S10: **The evolution of proportions of the transmission events.** The evolution of indices $\alpha_i(a, b, t)$ from simulation (solid lines) and $\beta_i(a, b, t)$ from prediction of percolation theory (dashed lines) on the Hexagonal lattice are presented. Three difference cases are considered here: (a) the information vanishes for $a = 0.10$, $b = 0.20$; (b) it outbreaks for $a = 0.19$, $b = 0.20$; and prevails for $a = 0.30$, $b = 0.20$ (c). The other parameters are token as $n_s = 2$ and $L = 101$. In comparison with the dynamic behaviors of the corresponding RRNs shown in Fig. S8, the occurrences of transmission events in Hexagonal lattice can last stably for over 350 MCs at the critical point, hence the time correlations among the events cannot be neglected in estimating the critical behavior of the message spreading. The inconsistencies between $\alpha_i(a, b, t)$ and the corresponding $\beta_i(a, b, t)$ have been chosen to demonstrate the existence of the time correlations between different transmission events $E_i(t)$. The phenomenon $\alpha_i(t) > 0$ in (b) indicates that almost all transmission events involve in the spreading process at the critical point. In addition, $\beta_i(t)$ gets close to corresponding $\alpha_i(t)$ in (b) and (c).

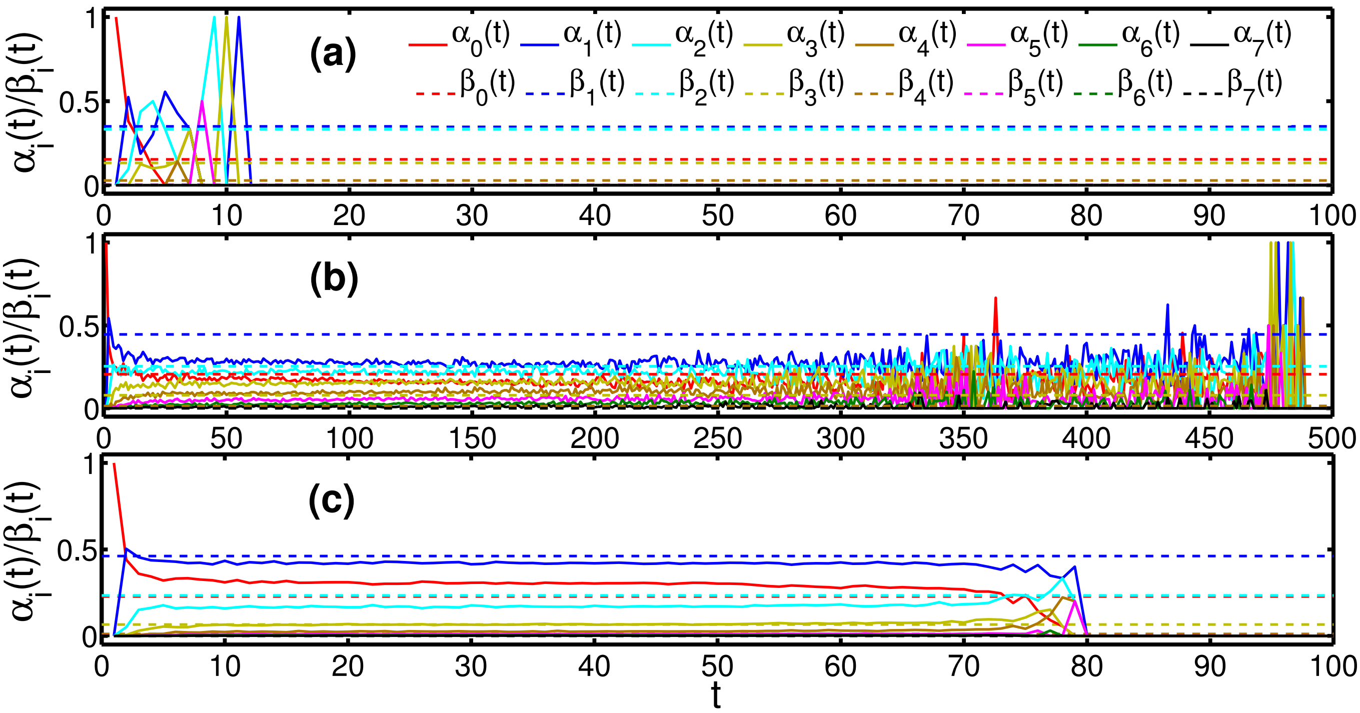

Fig S11: **The evolution of proportions of the transmission events.** The evolution of indices $\alpha_i(a, b, t)$ from simulation (solid lines) and $\beta_i(a, b, t)$ from prediction of percolation theory (dashed lines) on the Moore lattice are presented. Three different cases are considered here: (a) the information vanishes for $a = 0.05$, $b = 0.20$; (b) it outbreaks for $a = 0.10$, $b = 0.20$ and prevails for $a = 0.30$, $b = 0.20$ (c). The other parameters are token as $n_s = 2$ and $L = 101$. In comparison with the dynamic behaviors of corresponding RRNs shown in Fig. S9, the occurrences of transmission events in Moore lattice can last stably for over 450 MCs at the critical point, hence the time correlation among the events cannot be neglected in estimating the critical behavior of the message spreading. The inconsistencies between $\alpha_i(a, b, t)$ and the corresponding $\beta_i(a, b, t)$ have been chosen to demonstrate the existence of the time correlations between different transmisssion events $E_i(t)$. The phenomenon $\alpha_i(t) > 0$ in (b) indicates that almost all transmission events involve in the spreading process at the critical point. In addition, $\beta_i(t)$ gets close to corresponding $\alpha_i(t)$ in (b) and (c).

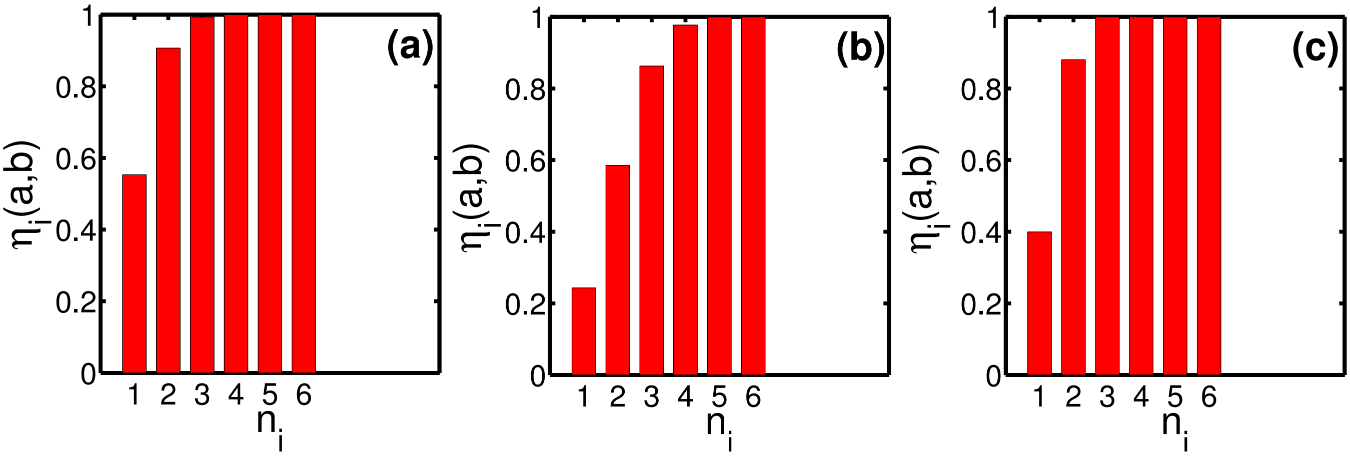

Fig S12: **The six accumulative indices $\eta_i(a,b)$ in Hexagonal lattice.** n_i denotes the number of informed neighbors an individual has had at most when it approves the message. The parameters are chosen at three selected parameter points: (a) subcritical point $a = 0.10$, $b = 0.20$; (b) critical point $a = 0.18$, $b = 0.20$ and (c) supercritical point $a = 0.40$, $b = 0.20$. The other parameters are token as $n_s = 2$ and $L = 101$. Results are obtained by averaging 100 independent realizations. In (b), the gaps between η_i and η_{i+1} $(i < 4)$ are apparent, which indicates that almost all transmission events involve in the spreading process. In (b) and (c), the value of η_4 or η_3 approach to 1, rather than η_i $(i < 4)$ and η_i $(i < 3)$. That can be considered as an evidence of the emergence of "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth") – the vast majority of the population accept the message as truth only when it is repeated more than two times in their ears.

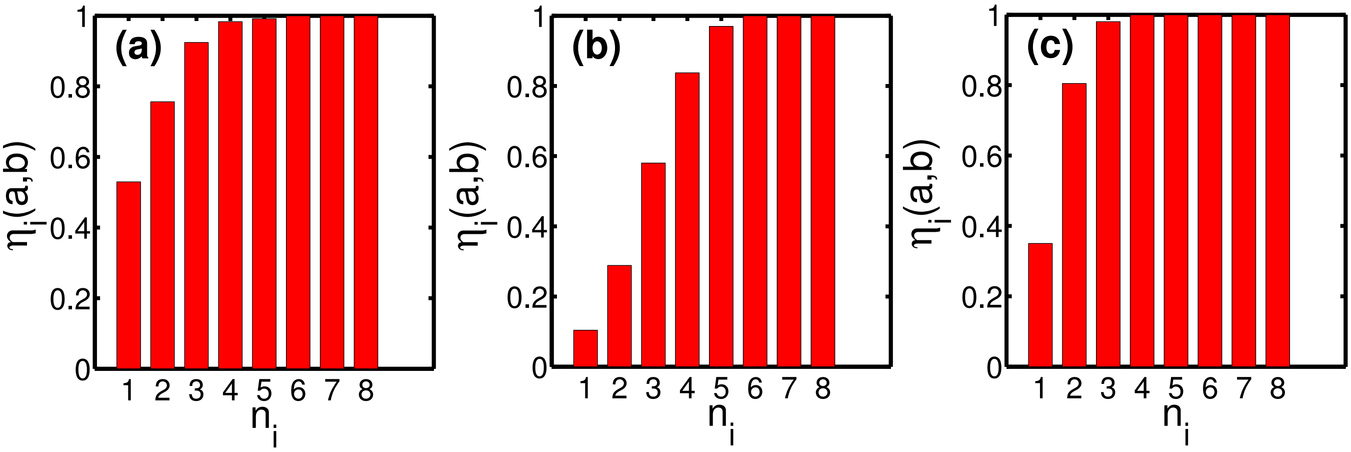

Fig S13: **The eight accumulative indices $\eta_i(a,b)$ in Moore lattice.** n_i denotes the number of informed neighbors an individual has had at most when it approves the message. The parameters are chosen at three selected parameter points: (a) subcritical point $a = 0.10$, $b = 0.20$; (b) critical point $a = 0.18$, $b = 0.20$ and (c) supercritical point $a = 0.40$, $b = 0.20$. The other parameters are token as $n_s = 2$ and $L = 101$. Results are obtained by averaging 100 independent realizations. In (b), the gaps between η_i and η_{i+1} $(i < 4)$ are apparent, which indicates that almost all transmission events involve in the spreading process. In (b) and (c), the value of η_4 or η_3 approach to 1, rather than η_i $(i < 6)$ and η_i $(i < 4)$. That can be considered as an evidence of the emergence of "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth") – the vast majority of the population accept the message as truth only when it is repeated more than two times in their ears.

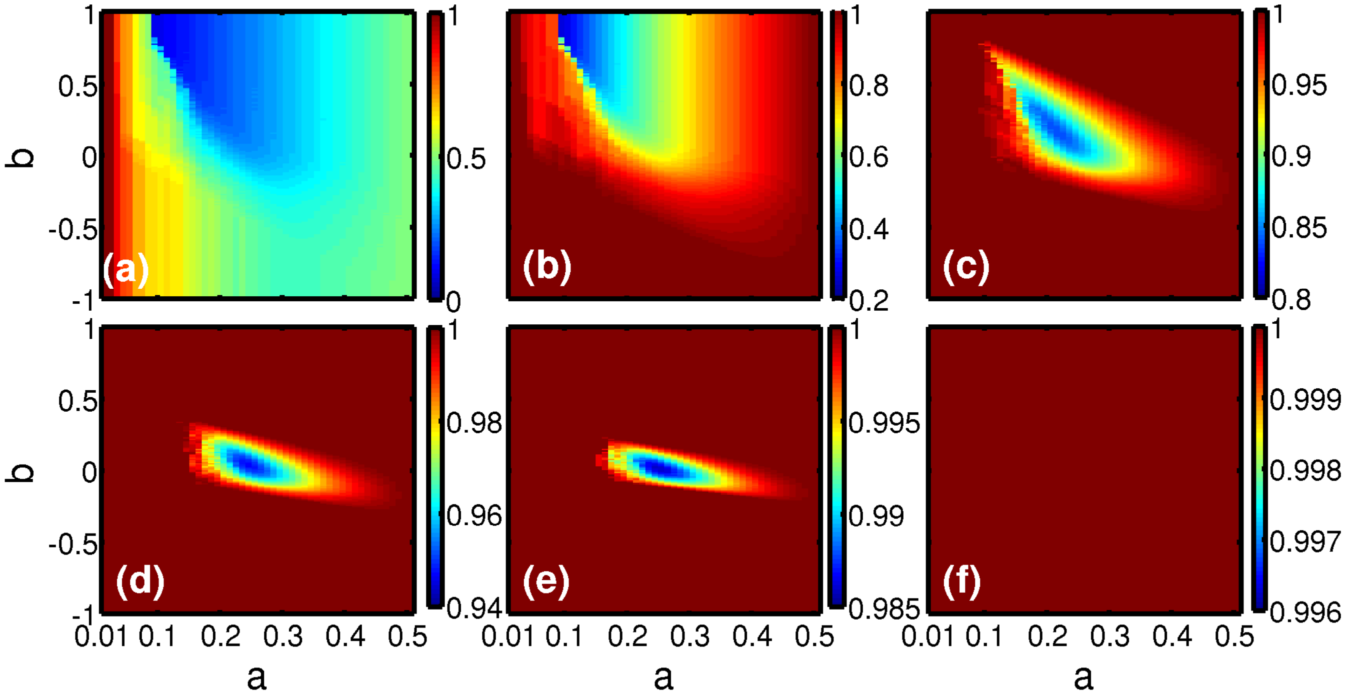

Fig S14: **The six accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), and η_6 (f) as a function of a and b for Hexagonal lattice.** The other parameters are token as $n_s = 2$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ $(i \geq frac\langle k\rangle 2)$ rather than η_j $(j < 3)$, where transmission events $E_i(t)$ $(i \geq 2)$ contribute more than 70% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidence for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

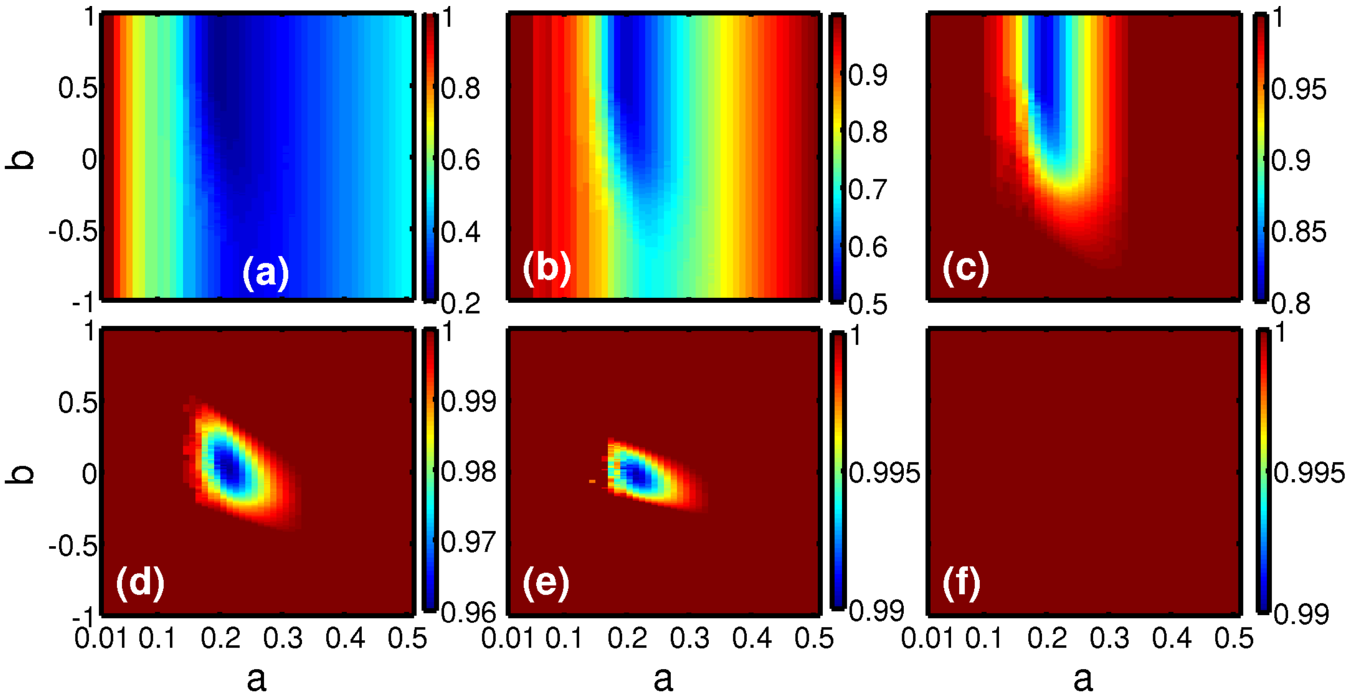

Fig S15: **The six accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), and η_6 (f) as a function of a and b for Hexagonal lattice.** The other parameters are token as $n_s = 3$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions with positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ $(i \geq \frac{\langle k\rangle}{2})$ rather than η_j $(j < 3)$, where transmission events $E_i(t)$ $(i \geq 2)$ contribute more than 80% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidence for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

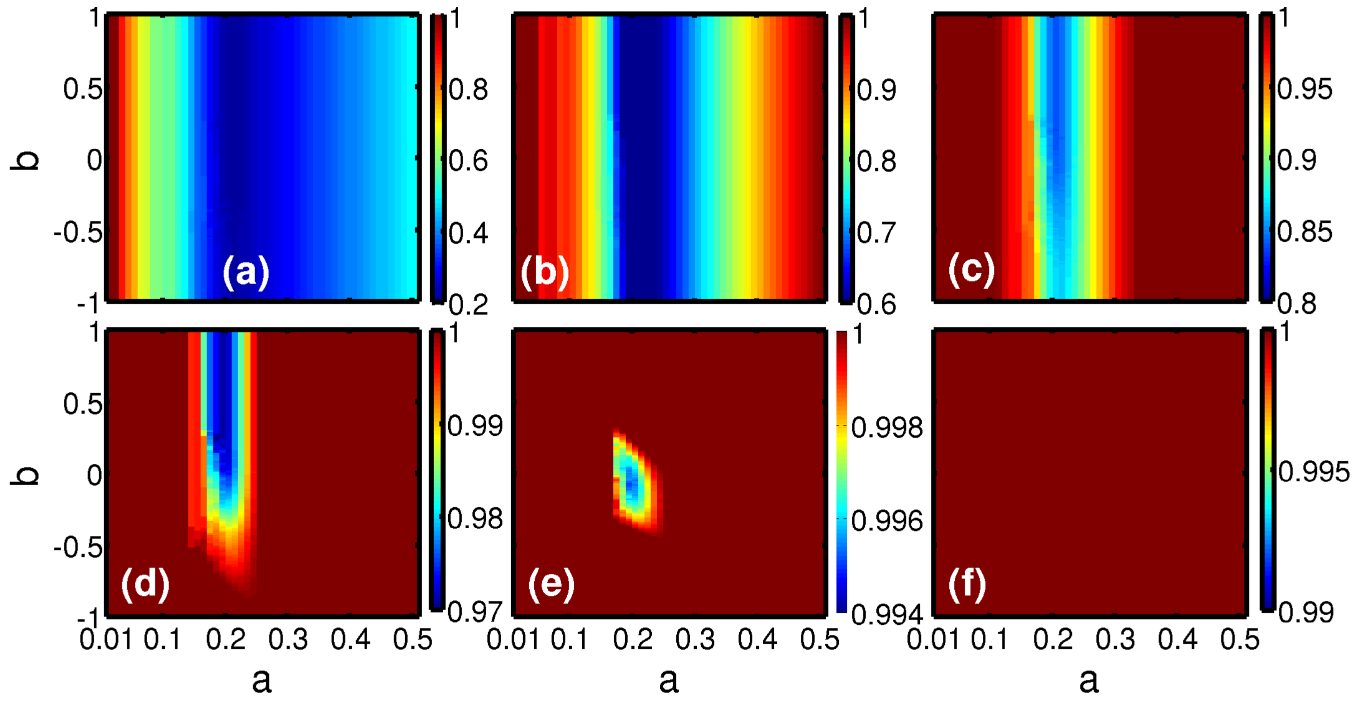

Fig S16: **The six accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), and η_6 (f) as a function of a and b for Hexagonal lattice.** The other parameters are token as $n_s = 4$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i \geq 2$) contribute more than 80% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidence for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

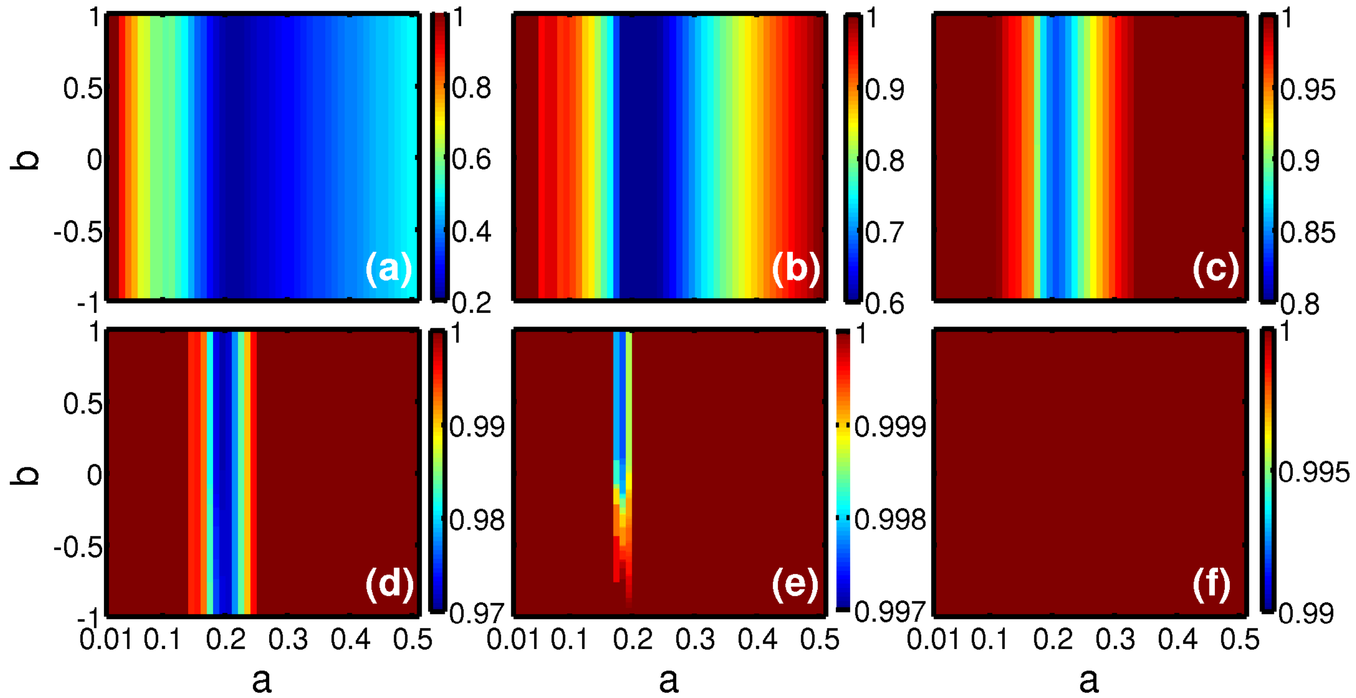

Fig S17: **The six accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), and η_6 (f) as a function of a and b for Hexagonal lattice.** The other parameters are token as $n_s = 5$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i \geq 2$) contribute more than 80% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results nearby the threshold also provide the evidence for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

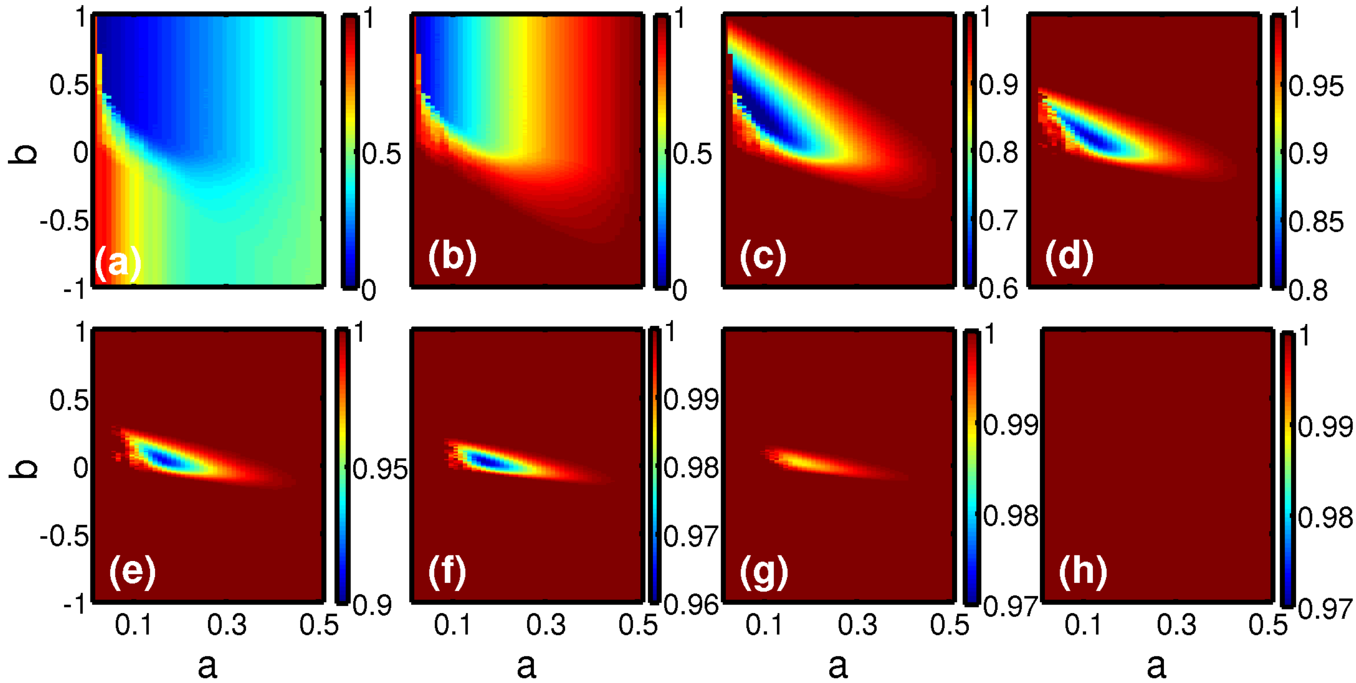

Fig S18: **The eight accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), η_6 (f), η_7 (g), and η_8 (h) as a function of a and b for Moore lattice.** The other parameters are token as $n_s = 2$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i > 2$) contribute more than 80% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive nearby the threshold also provide the evidence for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

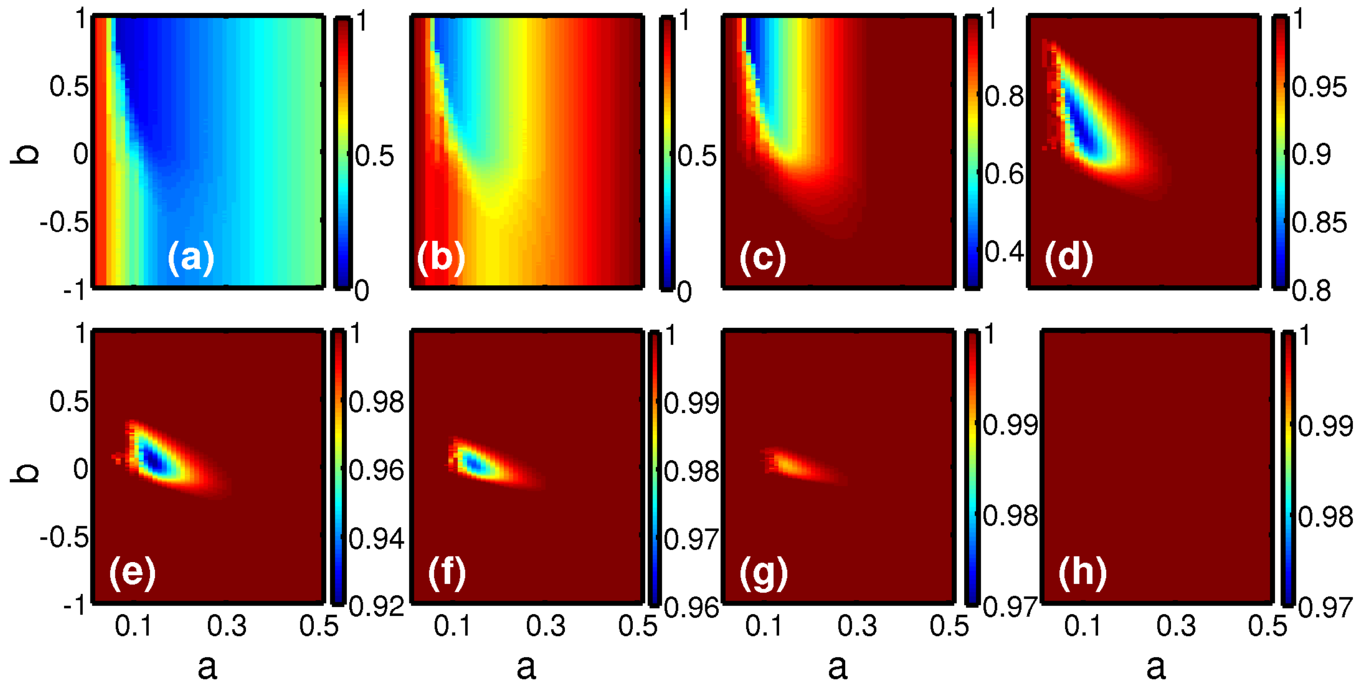

Fig S19: **The eight accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), η_6 (f), η_7 (g), and η_8 (h) as a function of a and b for Moore lattice.** The other parameters are token as $n_s = 3$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i > 2$) contribute more than 70% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive nearby the threshold also provide the evidences for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

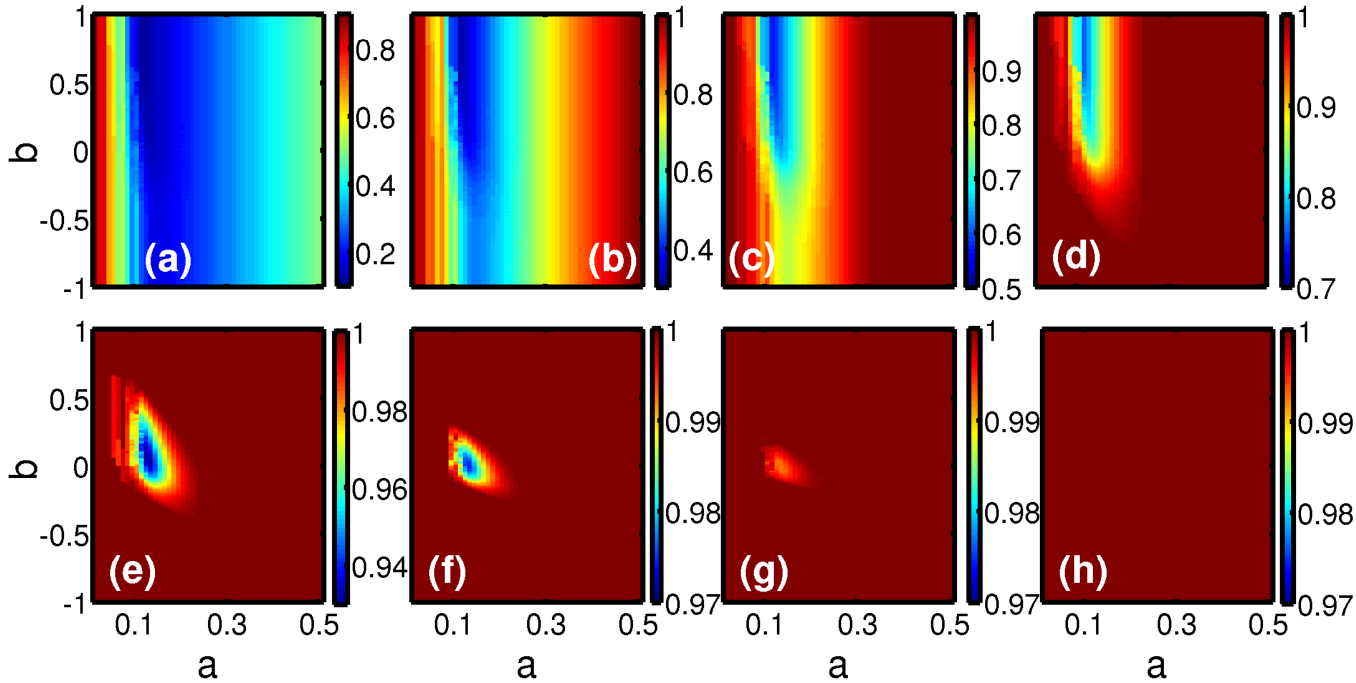

Fig S20: **The eight accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), η_6 (f), η_7 (g), and η_8 (h) as a function of a and b for Moore lattice.** The other parameters are token as $n_s = 4$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 70\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i > 2$) contribute more than 70% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive nearby the threshold also provide the evidences for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

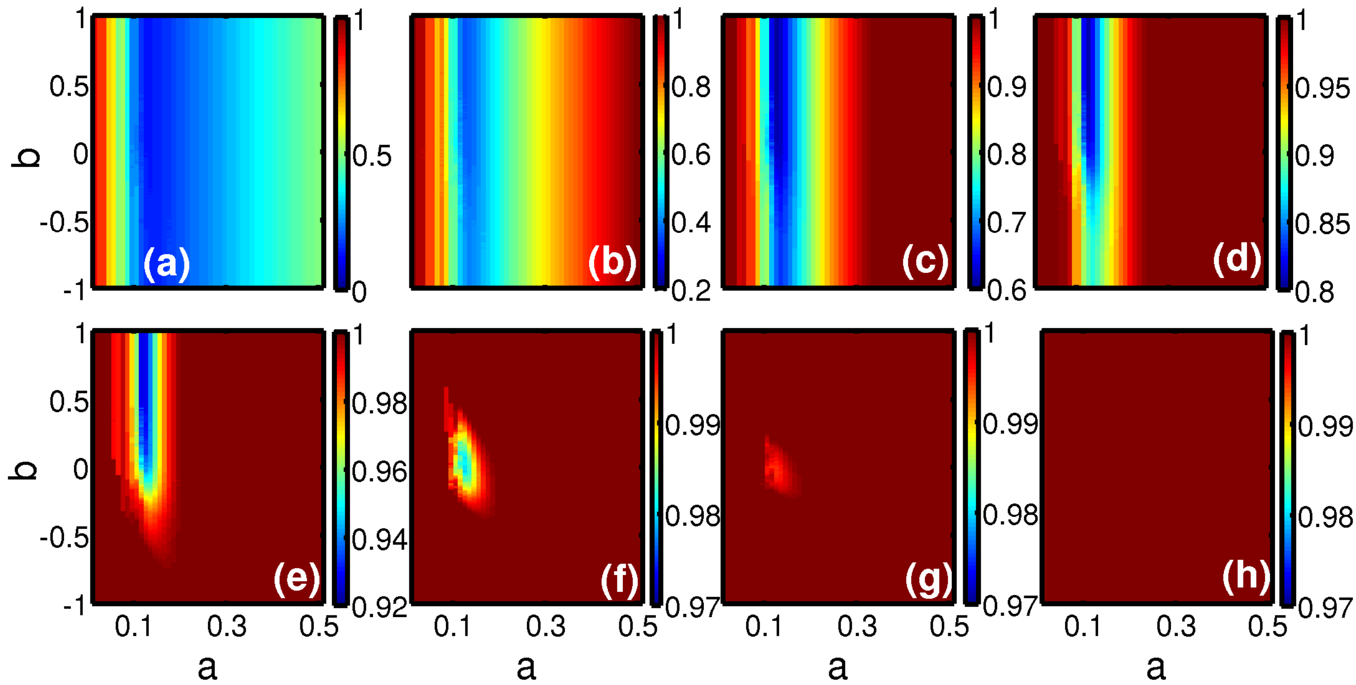

Fig S21: **The eight accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), η_6 (f), η_7 (g), and η_8 (h) as a function of a and b for Moore lattice.** The other parameters are token as $n_s = 5$ and $L = 101$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i > 2$) contribute more than 70% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results nearby the threshold also provide the evidences for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

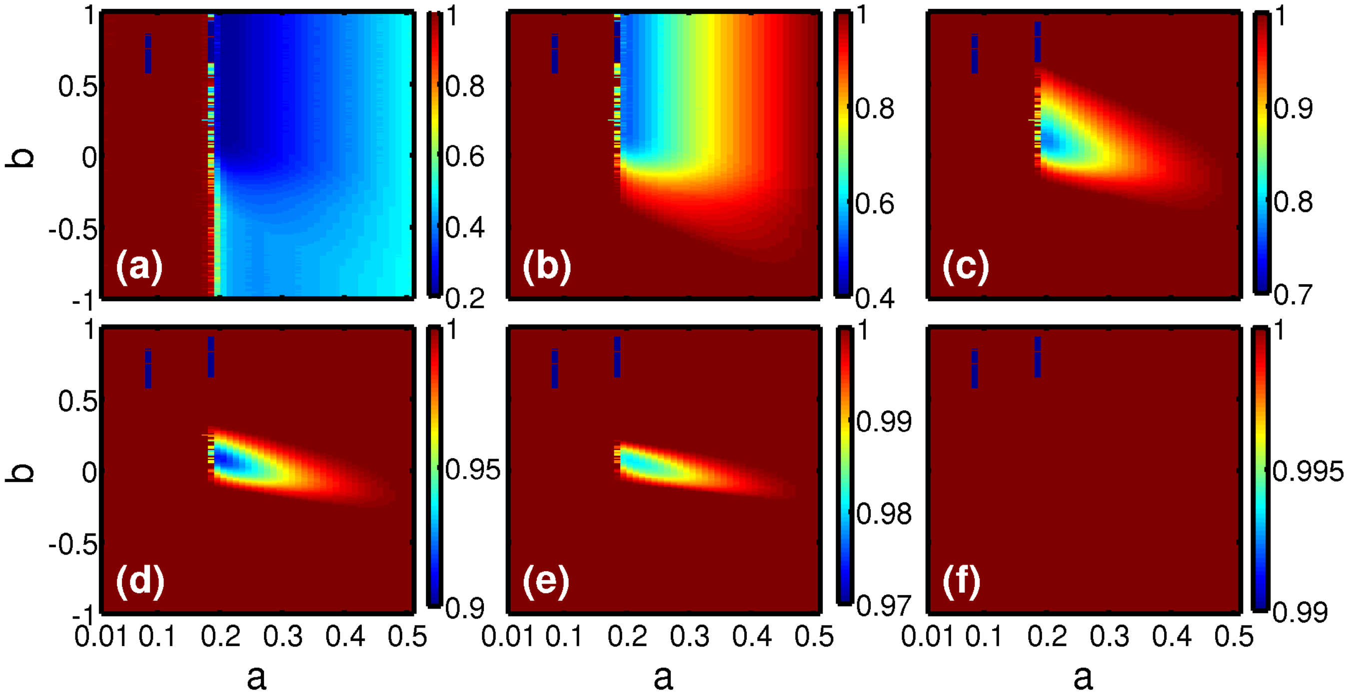

Fig S22: **The six accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), and η_6 (f) as function of a and b for RRs with $\langle k \rangle = 6$.** The other parameters are token as $n_s = 5$ and $N = 10000$. Each data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ $(i \geq \frac{\langle k \rangle}{2})$ rather than η_j $(j < 3)$, , where transmission events $E_i(t)$ $(i > 2)$ contribute more than 60% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidences for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

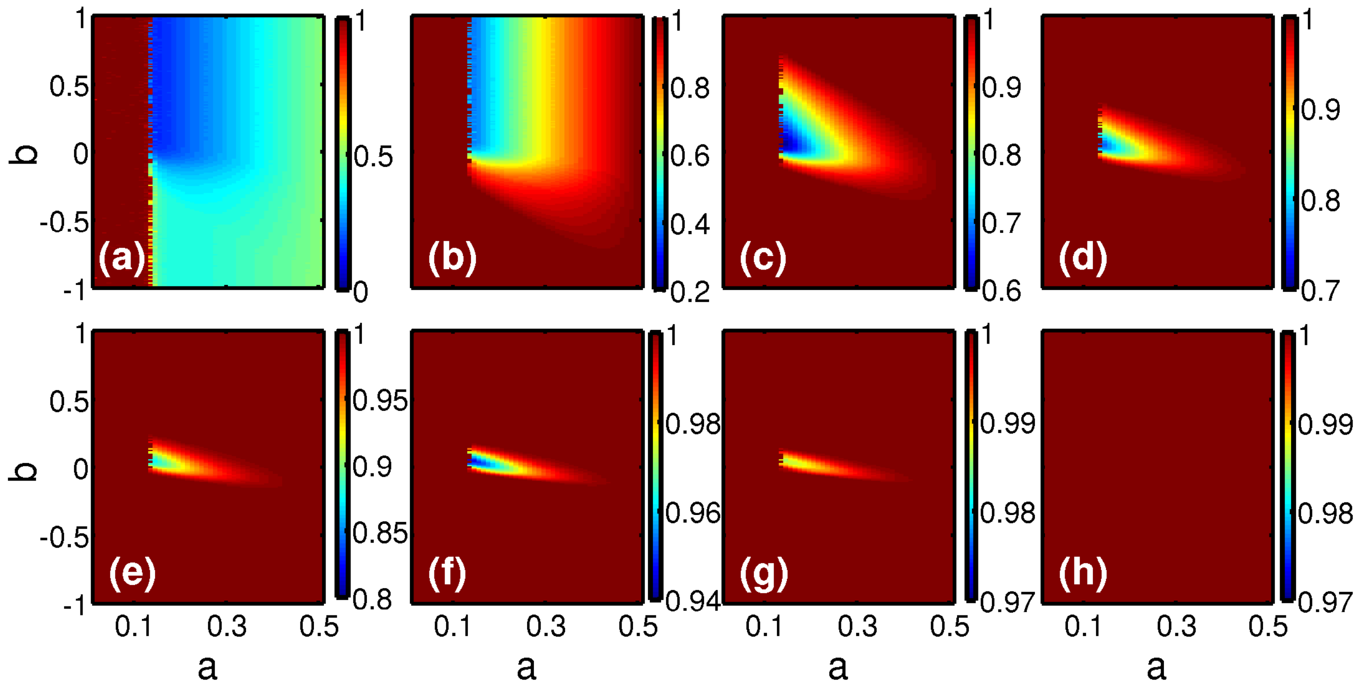

Fig S23: **The eight accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), η_6 (f), η_7 (g), and η_8 (h) as function of a and b for RRs with $\langle k \rangle = 8$.** The other parameters are token as $n_s = 5$ and $N = 10000$. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ $(i \geq \frac{\langle k \rangle}{2})$ rather than η_j $(j < 3)$, where transmission events $E_i(t)$ $(i > 2)$ contribute more than 60% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results with positive persistence nearby the threshold also provide the evidences for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth").

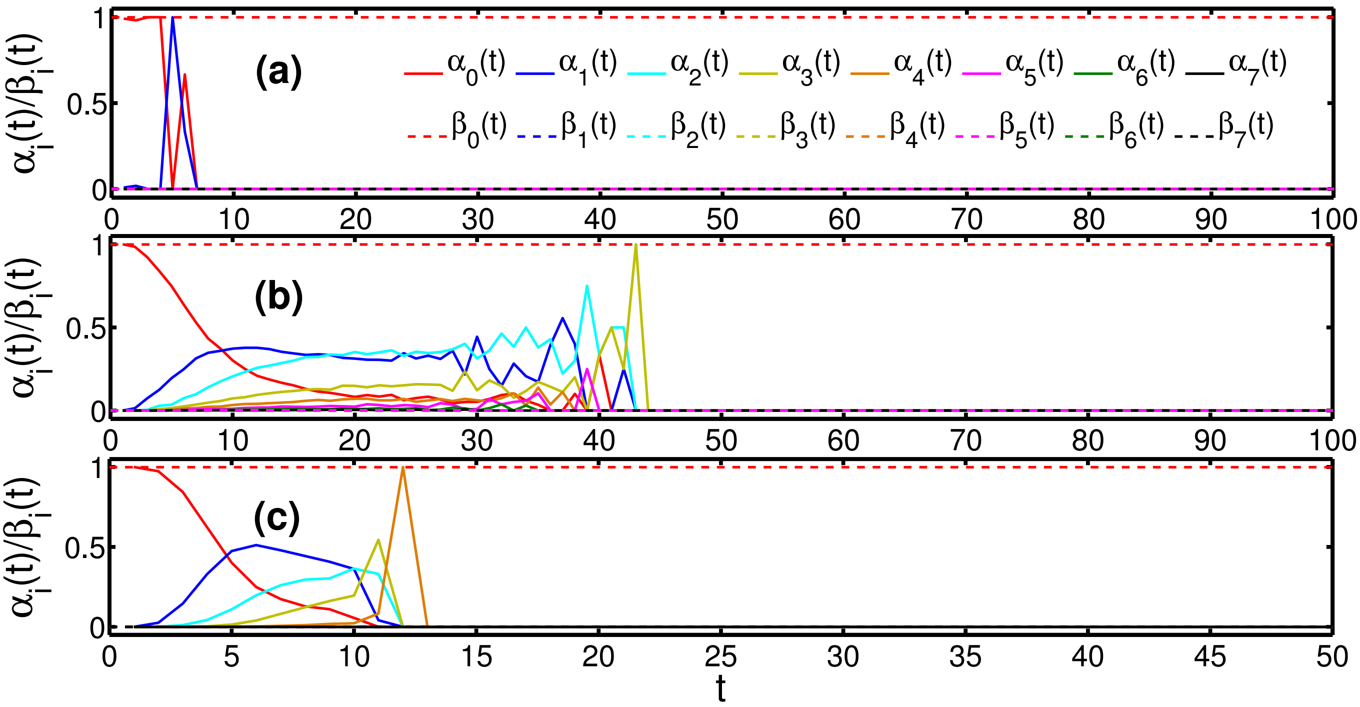

Fig S24: **The evolution of proportions of the transmission events.** The evolution of indices $\alpha_i(a,b,t)$ from simulation (solid lines) and $\beta_i(a,b,t)$ from predications of percolation theory (dashed lines) are presented for SF with $\langle k \rangle = 8$. Three difference cases are considered here: (a) the information vanishes for $a = 0.10$, $b = 0.20$, (b) it outbreaks for $a = 0.19$, $b = 0.20$ and prevails for $a = 0.30$, $b = 0.20$ (c). The other parameters are token as $n_s = 5$ and $N = 10000$. It can be observed that the occurrences of all the transmission events fail to last simultaneously and stably in the whole spreading process, departing from what exhibited in Fig. S11 for moore lattice. The time correlations among the transmission events can thus be neglected in estimating the critical behaviors of the message spreading.

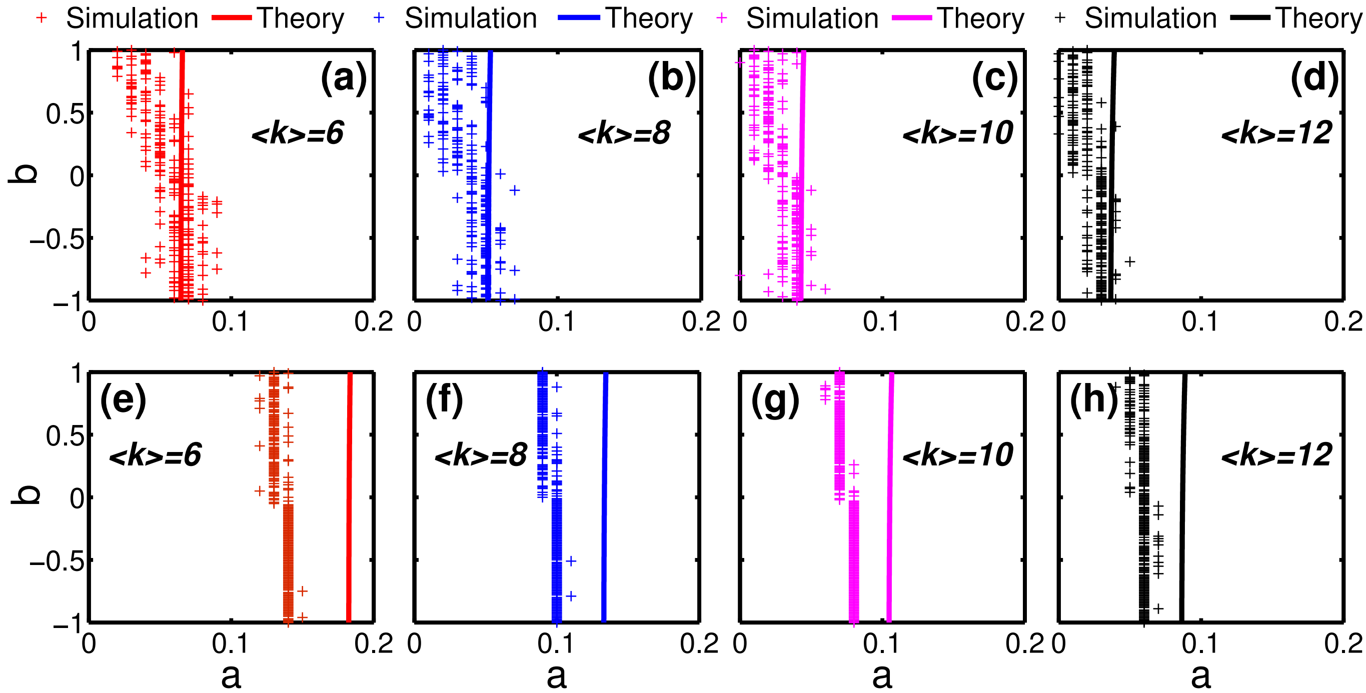

Fig S25: **Locus of thresholds of message diffusion on SF networks and ER networks**. Respectively, analytical solutions (solid lines) for four SF networks (top panel) and four ER networks (bottom panel) with different average degrees are plotted to compare with the corresponding exact numerical data (markers). The other parameters are token as $n_s = 2$ and $N = 10000$. Each numerical data point is obtained by averaging 1000 independent realizations. And (a)(e) $\langle k \rangle = 6$, (b)(f) $\langle k \rangle = 8$, (c)(h) $\langle k \rangle = 10$ and (d)(i) $\langle k \rangle = 12$. Both the simulations and the analytical predications show that the critical behaviors of the spreading are dominated by the stickiness of the message (b).

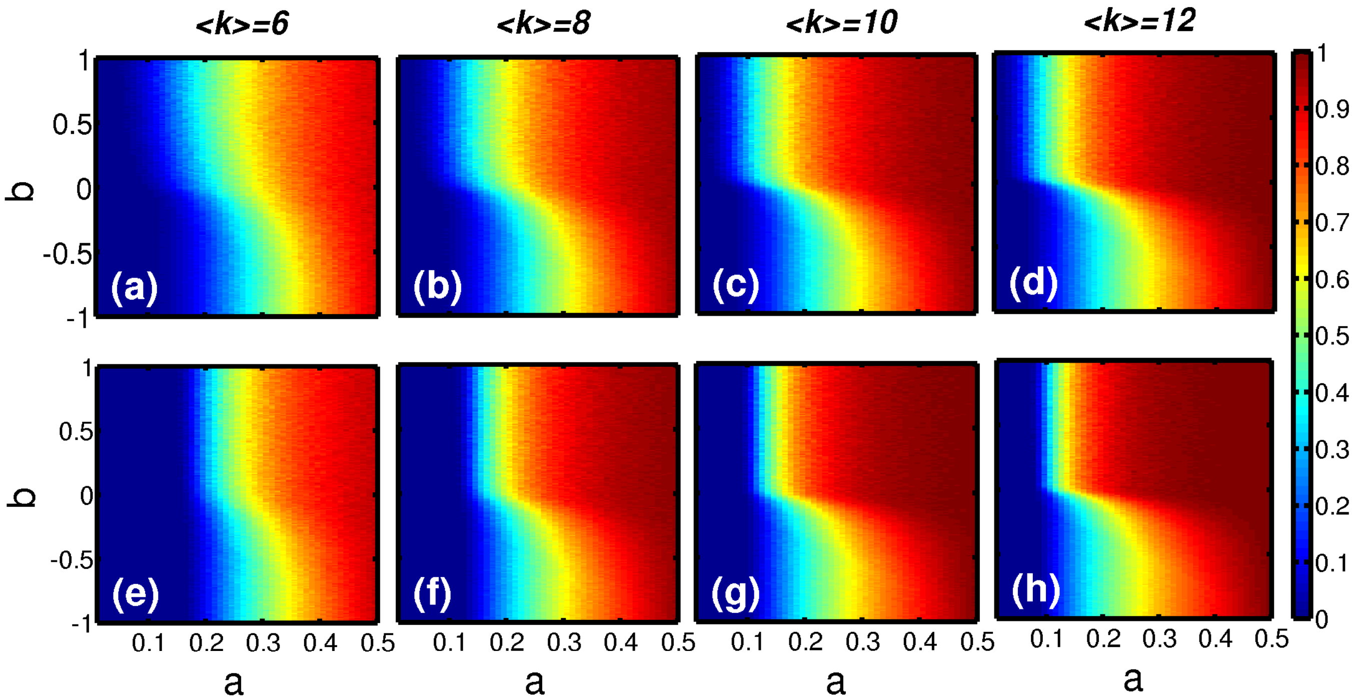

Fig S26: **The densities of recovered individuals as function of a and b,** on SF networks (top panel) and Erdos-Renyi networks (bottom panel) with different average degrees. The other parameters are token as $n_s = 2$ and $N = 10000$. Each numerical data point is obtained by averaging 100 independent realizations. Precisely, (a)(e) $\langle k \rangle = 6$, (b)(f) $\langle k \rangle = 8$, (c)(h) $\langle k \rangle = 10$ and (d)(i) $\langle k \rangle = 12$. Although the critical behaviors of the message spreading on SF and ER are dominated by the stickiness, the persistence of the message (i.e., b) has a reasonable impact on the sizes of message spreading, especially at the parameter regions nearby $b = 0$.

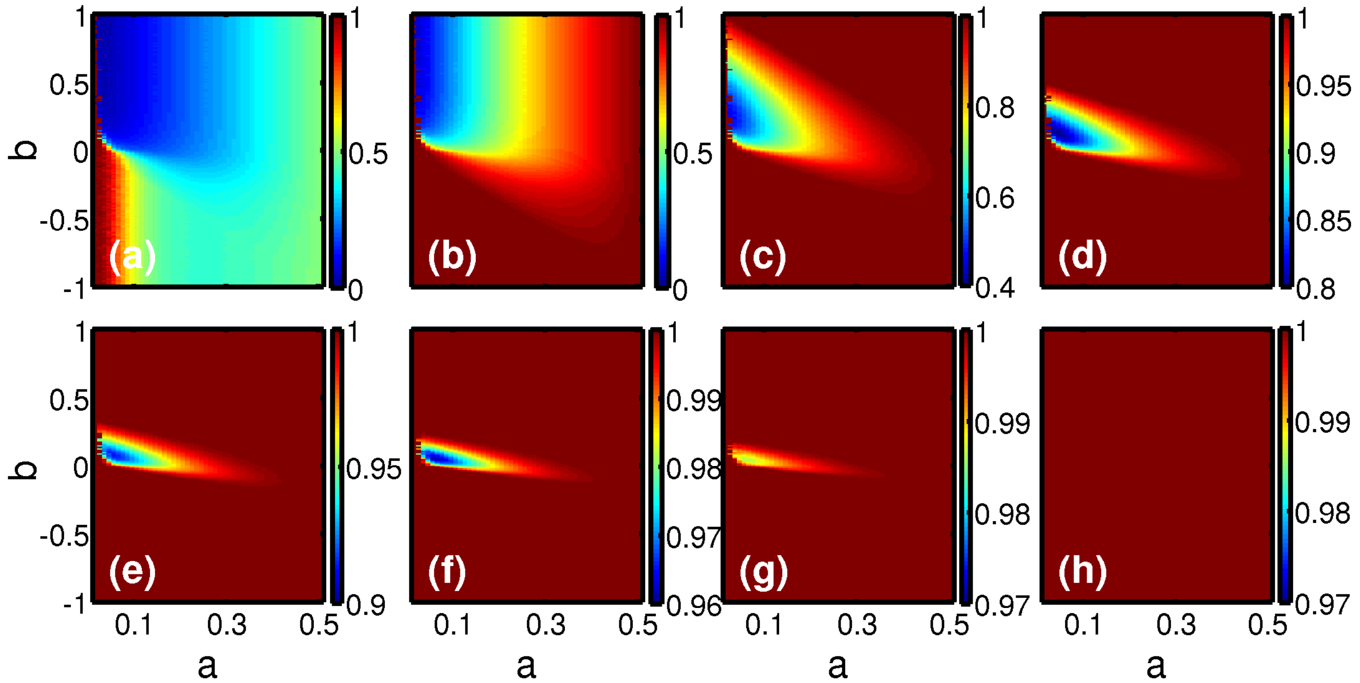

Fig S27: **The eight accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), η_6 (f), η_7 (g) η_8 (h) as function of a and b for SF networks with** $\langle k \rangle = 8$**.** The other parameters are token as $n_s = 2$ and $N = 10000$. Each numerical data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i > 2$) contribute more than 80% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results for $b > 0$ nearby the threshold also provide the evidences for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth"). Similar results are obtained for SF networks with the same average degree.

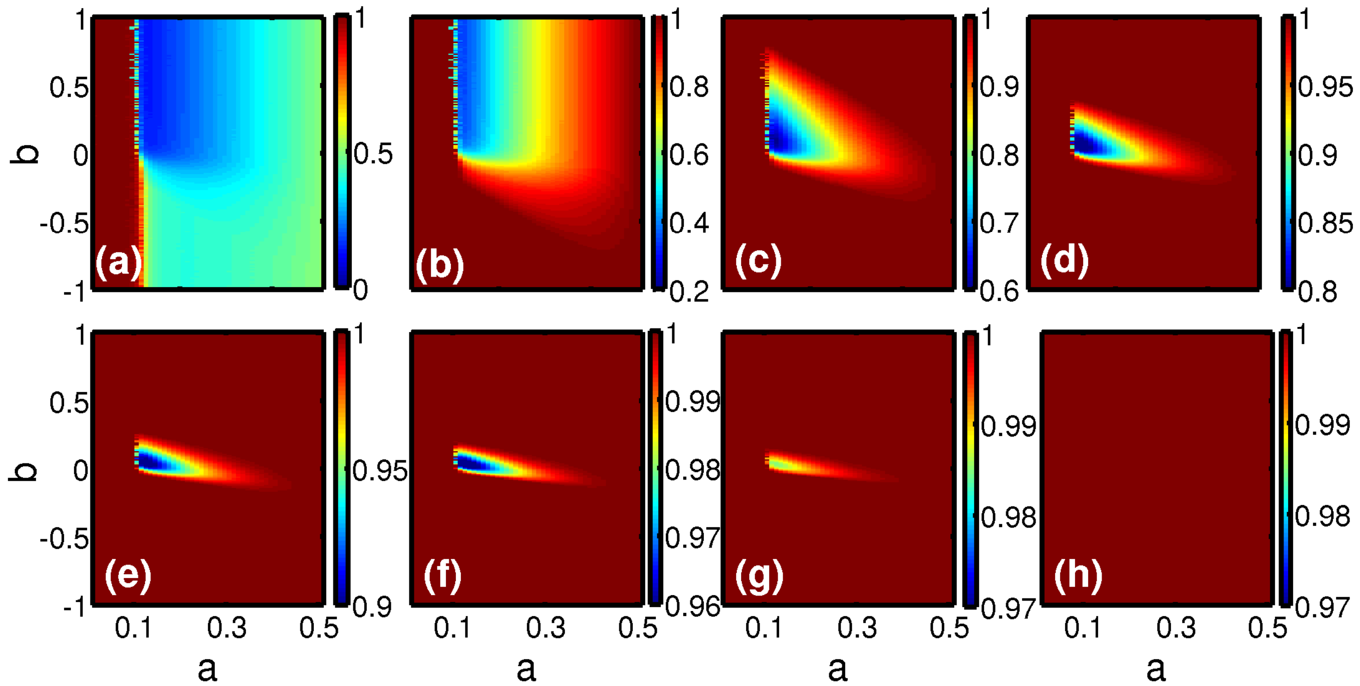

Fig S28: **The eight accumulative indices η_1 (a), η_2 (b), η_3 (c), η_4 (d), η_5 (e), η_6 (f), η_7 (g) η_8 (h) as function of a and b for ER networks with** $\langle k \rangle = 6$**.** The other parameters are token as $n_s = 2$ and $N = 10000$. Each numerical data point is obtained by averaging 100 independent realizations. In the wide parameter regions for positive persistence, especially nearby the critical boundary (threshold), only $\eta_i > 80\%$ ($i \geq \frac{\langle k \rangle}{2}$) rather than η_j ($j < 3$), where transmission events $E_i(t)$ ($i > 2$) contribute more than 70% of the scale of message spreading (i.e., the spreading thus reaches a saturation level). It also implies that the majority of the population accept the message as truth only if the message is repeated more than three times in their ears. Therefore, the results for $b > 0$ nearby the threshold also provide the evidences for the real phenomenon "Three men make a tiger" (or "A lie, if repeated often enough, will be accepted as truth"). Similar results are obtained for SF networks with the same average degree.