Optical Peaking Enhancement in High-Speed Ring Modulators

Supplementary Information

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In section I, the figure of merit of the resonant enhancement in RRMs is derived to support arguments in the main text. In section II, equations (3)-(5) are derived, as well as an analytical expression for $Im(\delta\omega_r)/Re(\delta\omega_r)$. In section III, the equivalence between a RRM and a peaking amplifier is exemplified by providing explicit numerical values for the laser detuning dependent values of the electrical components in the equivalent circuit model of fig. 6 (R_1, R_2, L, C, A_v).

I. Resonant enhancement of a RRM relative to lumped element linear modulators

In the following, the quasi-static E/O responses of RRM and Mach-Zehnder interferometer (MZI) modulators is derived in order to compare the two devices and derive the figure for merit of the resonant enhancement.

The power circulating in a waveguide coupled ring resonator is given by

$$P_{Circ} = \frac{\kappa^2}{\left|1 - t e^{\left(i\beta - \frac{\alpha}{2}\right)L}\right|^2} \tag{8}$$

where $t = \sqrt{1-\kappa^2}$ is the transmission coefficient of the coupling junction, α is the waveguide power loss coefficient, β is the wave number of the waveguide and L is the circumference of the resonator. κ is the coupling parameter between the bus waveguide and the resonator already defined in the main text. At resonance, for a critically coupled ring $\left(t = e^{-\frac{\alpha L}{2}}\right)$ this reduces to

$$P_{Circ} = \frac{1}{\kappa^2} = \frac{\mathcal{F}}{\pi} \tag{9}$$

In other words, the power circulating inside the resonator is enhanced by a factor proportional to the finesse of the cavity. One may think of this as a consequence of the light circulating $\tau_a \cdot \frac{c_0}{n_g L} = \frac{\mathcal{F}}{\pi}$ times inside the resonator before being dissipated, where c_0 is the speed of light in vacuum and n_g is the group index of the waveguide.

For a critically coupled RRM, the power transmitted through the bus waveguide is given by

$$P_{Out} = \left|\frac{t\left(1 - e^{i\beta L}\right)}{1 - t^2 e^{i\beta L}}\right|^2 \tag{10}$$

After some algebra, we find that the maximum small signal optical modulation amplitude $\frac{\partial P_{Out}}{\partial n_{eff}} \Delta n_{eff}$ is obtained for a laser detuning given by

$$\Delta\omega = \pm \frac{c_0}{L n_g} acos\left(\frac{-1 - t^4 + \sqrt{1 + 34t^4 + t^8}}{4t^2}\right) \approx \pm \frac{\omega_r}{2\sqrt{3}Q} \tag{11}$$

where Q is the loaded Q-factor of the cavity and ω_r/Q is the full width at half maximum of the resonance. The second equality strictly only holds for small αL (or assuming a perfectly lorentzian transfer function) but is accurate within 0.1% for round trip power losses lower than 50%. For this laser detuning and small αL (or assuming a perfectly lorentzian resonator response), the maximum small signal modulation amplitude is given by

$$\frac{1}{P_{In}}\frac{\partial P_{Out}}{\partial n_{eff}} = \frac{9}{4\sqrt{3}}\frac{Q}{n_g} \approx 1.3\frac{Q}{n_g} \tag{12}$$

where n_g is the group index of the waveguide. In other words, the modulation efficiency of a RRM is a direct function of the Q-factor and of the waveguide losses of the phase shifter. The same however also holds for a linear lumped element modulator: For modulator lengths on the order of $1/\alpha$ waveguide losses become significant and limit the achievable modulation efficiency.

A Mach-Zehnder modulator with a total phase shifter length L_{MZI} equally split over both arms has a transfer function

$$\frac{P_{Out}}{P_{In}} = \frac{1+sin\left(\frac{2\pi\Delta n_{eff}}{\lambda}L_{MZI}\right)}{2}e^{-\frac{\alpha}{2}L_{MZI}} \tag{13}$$

The small signal modulation amplitude is then given by

$$\frac{1}{P_{In}}\frac{\partial P_{Out}}{\partial n_{eff}} = \frac{\pi L_{MZI}}{\lambda}e^{-\frac{\alpha}{2}L_{MZI}} = \frac{Q}{n_g}\frac{\pi}{\mathcal{F}}\frac{L_{MZI}}{L}e^{-\frac{\alpha}{2}L_{MZI}} \tag{14}$$

where the second equality was derived assuming the linear losses of the ring to be identical to the linear losses inside the MZI, i.e., bending losses are assumed not to play a role. In other words, for a small lumped element linear modulator with identical drive voltage and phase shifter configuration than the previously analyzed ring, the modulation amplitude per phase shifter length is smaller by a factor $1.3\frac{\mathcal{F}}{\pi}$ relative to the ring, this factor thus also giving the capacitance and RF power consumption reduction enabled by the RRM. For long phase shifters, the optical losses also play an important role in the linear modulators. The modulation amplitude per index change is exactly maximized for $L_{MZI} = 2/\alpha$. At that length, the electrical power efficiency of the linear modulator (in terms of electrical power consumption per optical modulation amplitude) is $1.3e\frac{\mathcal{F}}{\pi}$ smaller than for the ring modulator and the attained optical modulation amplitude $\frac{Q}{n_g}\frac{2}{e} \approx 0.74\frac{Q}{n_g}$. In other words, the optical modulation amplitude attainable by either device is essentially equivalent, the primary difference residing in the power consumption enhancement factor scaling with the finesse of the ring.

II.a Perturbative derivation of the small signal E/O response

We start with equations (1) and (2) and make the additional assumption that the applied voltage induces a small index change and thus a small perturbation in the resonator response. The amplitude a inside the resonator responds to the resonant frequency change $\delta\omega_r$ with δa. Equation (1) is rewritten as

$$\frac{\partial a}{\partial t} + \frac{\partial \delta a}{\partial t} = \left(-i\omega_r - i\delta\omega_r(t) - \frac{1}{\tau_a}\right)(a + \delta a) + i\mu E_{in}(t) \tag{15}$$

Neglecting second order terms, this equation reduces to equation (5), i.e.,

$$\frac{\partial \delta a}{\partial t} = \left(-i\omega_r - \frac{1}{\tau_a}\right)\delta a - i\delta\omega_r(t)a \tag{16}$$

In order to solve this equation, we decompose δa into the product of a slowly varying envelope $\overline{\delta a}$ and its natural time dependence in the absence of further excitation, $e^{\left(-i\omega_r - \frac{1}{\tau_a}\right)t}$, i.e.,

$$\delta a = \overline{\delta a}e^{\left(-i\omega_r - \frac{1}{\tau_a}\right)t} \tag{17}$$

which then results in

$$\frac{d\overline{\delta a}}{dt} = -i\delta\omega_r(t)ae^{\left(i\omega_r + \frac{1}{\tau_a}\right)t} \tag{18}$$

Using the same notations as in the main text, $a = \bar{a}e^{-i\omega_0 t}$, $E_{In} = \bar{E}_{In}e^{-i\omega_0 t}$, $E_{Out} = \bar{E}_{Out}e^{-i\omega_0 t}$ and assuming the time varying resonant frequency $\delta\omega_r cos(\omega_m t) = \frac{\delta\omega_r}{2}\left(e^{i\omega_m t} + e^{-i\omega_m t}\right)$ we obtain the differential equation

$$\frac{d\overline{\delta a}}{dt} = -i\,\frac{\delta\omega_r \bar{a}}{2}\left(e^{\left(i\omega_r - i\omega_0 + i\omega_m + \frac{1}{\tau_a}\right)t} + e^{\left(i\omega_r - i\omega_0 - i\omega_m + \frac{1}{\tau_a}\right)t}\right) \tag{19}$$

resulting in equation (4)

$$\delta a = -i\,\frac{\delta\omega_r \bar{a}}{2}\left(\frac{1}{i\omega_r - i\omega_0 + i\omega_m + \frac{1}{\tau_a}}e^{(-i\omega_0 + i\omega_m)t} + \frac{1}{i\omega_r - i\omega_0 - i\omega_m + \frac{1}{\tau_a}}e^{(-i\omega_0 - i\omega_m)t}\right) \tag{20}$$

and

$$\overline{E_{Out}} = \overline{E_{In}} + i\mu\bar{a} + \mu\,\frac{\delta\omega_r \bar{a}}{2}\left(\frac{1}{i\omega_r - i\omega_0 + i\omega_m + \frac{1}{\tau_a}}e^{i\omega_m t} + \frac{1}{i\omega_r - i\omega_0 - i\omega_m + \frac{1}{\tau_a}}e^{-i\omega_m t}\right) \tag{21}$$

The output power response of the RRM is then given by

$$|\overline{E_{Out}}|^2 - \langle|\overline{E_{Out}}|^2\rangle =$$

$$|\overline{E_{Out}}|^2 - |\overline{E_{In}} + i\mu\bar{a}|^2 = \mu Re\left(\left[\frac{\delta\omega_r \bar{a}(\overline{E_{In}} + i\mu\bar{a})^*}{\frac{1}{\tau_a} - i\omega_m - (i\omega_0 - i\omega_r)} + \frac{\delta\omega_r{}^* \bar{a}^*(\overline{E_{In}} + i\mu\bar{a})}{\frac{1}{\tau_a} - i\omega_m + (i\omega_0 - i\omega_r)}\right]e^{-i\omega_m t}\right) \tag{22}$$

resulting in equation (3). As described in the main text, the terms $\pm(i\omega_r - i\omega_0)$ in the denominator result in asymmetric side-band generation and peaking. The average amplitude inside the ring is given by

$$\bar{a} = \sqrt{\frac{L}{v_g}}\frac{i\kappa}{1 - te^{\left(i\beta - \frac{\alpha}{2}\right)L}} \tag{23}$$

where the preterm $\sqrt{L/v_g}$ is due to the normalization of a in equation (1).

Equation (1), on which this entire analysis is based, is a simplification in that it assumes the amplitude of the field inside the ring to be described by a single scalar, i.e., the dependence of the field along the ring's circumference is neglected. This is obviously not exact since light coupled into the ring requires at least a time delay equal to the round trip time before it can further interact with the modulator output. A theoretical treatment taking this into account can be found in [37]. The round trip time of the modulator analyzed here is 0.8 ps and is thus sufficiently small to be ignored compared to the period T of the modulating signal ($T/4$ as low as 5 ps for 50 GHz modulation, the fastest modulation speed measured here), a typical situation for compact RRMs.

II.b Calculation of ϑ

As mentioned in the main text, the response of the ring resonance to the drive voltage, $\delta\omega_r$, also comprises an imaginary component describing the modulation of the photon lifetime. The ratio of the imaginary to the real part of $\delta\omega_r$ can be simply derived from the free carrier induced absorption and refractive index change published in [39,42]

$$\delta n = -8.8\times10^{-22}N_e - 8.5\times10^{-18}N_h^{0.8} \tag{24}$$

$$\delta\hat{\alpha} = 8.5\times10^{-18}N_e + 6.0\times10^{-18}N_h \tag{25}$$

where δn is the refractive index change, $\delta\hat{\alpha}$ is the change of the absorption coefficient of the material (as opposed to the waveguide losses α) given in cm^{-1}, N_e and N_h are respectively the electron and hole density in cm^{-3}. Since the optical overlap with the carriers causes both absorption and index shift, the overlap drops from the ratio and geometric factors can be fully ignored assuming the carrier density modulation to be uniform in the active region – an assumption that holds true on either side of the step profile diode assumed here.

The resonance shift of the cavity is given by

$$\frac{Re(\delta\omega_r)}{\omega_r} = -\frac{\delta n_{eff}}{n_{eff}} \tag{26}$$

where n_{eff} is the effective index of the waveguide. The effective index shift can be calculated as follows

$$\delta n_{eff} = \frac{\partial n_{eff}}{\partial n}\delta n + \frac{\partial n_{eff}}{\partial \omega_r}\delta\omega_r = \frac{\partial n_{eff}}{\partial n}\delta n - \frac{\partial n_{eff}}{\partial \omega_r}\omega_r\frac{\delta n_{eff}}{n_{eff}} \tag{27}$$

$$\frac{\delta n_{eff}}{n_{eff}}\left(n_{eff} + \omega_r\frac{\partial n_{eff}}{\partial \omega_r}\right) = \frac{\delta n_{eff}}{n_{eff}}n_g = \frac{\partial n_{eff}}{\partial n}\delta n \tag{28}$$

$$\delta n_{eff} = \frac{\partial n_{eff}}{\partial n}\frac{n_{eff}}{n_g}\delta n = O_v\frac{n_{eff}}{n_g}\delta n \tag{29}$$

where O_v is the overlap of the waveguide mode with the region in which the refractive index n is modified. Thus we obtain

$$\frac{Re(\delta\omega_r)}{\omega_r} = -O_v\frac{\delta n}{n_g} \tag{30}$$

The *intrinsic* 1/e time constant τ_i of the *E-field* decay inside the cavity (i.e., ignoring coupling losses to the waveguide) is given by

$$\frac{1}{\tau_i} = \frac{\alpha c_0}{2n_g} \tag{31}$$

We can then derive the dynamic change of the photon lifetime as

$$\frac{\delta\frac{1}{\tau_a}}{\frac{1}{\tau_i}} = \frac{\delta\frac{1}{\tau_i}}{\frac{1}{\tau_i}} = \frac{\delta\alpha}{\alpha} - \frac{\delta n_g}{n_g} \cong \frac{\delta\alpha}{\alpha} = \frac{O_v\delta\hat{\alpha}}{\alpha} \tag{32}$$

where the external coupling losses are ignored since they are not modified by the free carrier density modulation. $\hat{\alpha}$ is the material absorption (as opposed to the waveguide absorption α). These equations result in a very compact expression for the imaginary part of $\delta\omega_r$

$$\vartheta = -\frac{Im(\delta\omega_r)}{Re(\delta\omega_r)} = \frac{\delta\frac{1}{\tau_a}}{Re(\delta\omega_r)} = -\frac{c_0}{2\omega_r}\frac{\delta\hat{\alpha}}{\delta n} = -\frac{\lambda_r}{4\pi}\frac{\delta\hat{\alpha}}{\delta n} \tag{33}$$

where λ_r is the resonant wavelength in vacuum. This equation can be weighted based on the respective overlaps of the space charge region on the n and p sides of the diode with the optical field. The unweighted values are ϑ = 0.12 on the n-side for n^-=1e17 cm^{-3} and ϑ = 0.028 on the p-side for p^-=3e17 cm^{-3}. Since p^-=3n^- in the investigated RRM, the variation of the space charge region on the n-side is 3 times larger. Assuming the modal E-field to be constant across the space charge region and taking these overlaps into account this results in ϑ = 0.051 evaluated as

$$\vartheta = -\frac{\lambda_r}{4\pi}\frac{n^-\delta\hat{\alpha}_{p^-} + p^-\delta\hat{\alpha}_{n^-}}{n^-\delta n_{p^-} + p^-\delta n_{n^-}} = \frac{\lambda_r}{4\pi}\frac{8.5\times10^{-18} + 6.0\times10^{-18}}{8.8\times10^{-22} + 8.5\times10^{-18}(p^-)^{-0.2}} \tag{34}$$

where the formula also has to be evaluated with λ_r in cm and p^- in cm^{-3} due to the units of the coefficients given in equations (24) and (25). Since δn and $\delta\hat{\alpha}$ depend linearly on the dopant concentrations, with the exception of the dependence of δn on the hole concentration that has an exponent slightly different from 1, ϑ only depends weakly on the free carrier concentration in the modulation region.

ϑ can also be directly determined experimentally by applying a DC voltage to the RRM and recording both the induced resonant frequency shift and the modification of the Q-factor. Since the Q-factor did not change sufficiently in reverse bias for this small modification to be reliably extracted, we operated the RRM in forward injection to obtain a stronger effect (fig. 11(a)). The resulting modification of waveguide losses is shown in fig. 11(b). The extracted ϑ is 0.074, while the calculated ϑ based on the simulated free carrier distribution at 1V forward bias is 0.060, in reasonably close agreement. This strictly corresponds to a different situation than the reverse biased diode since the modulated hole and electron concentrations take different values, it provides however a good validation of the calculations.

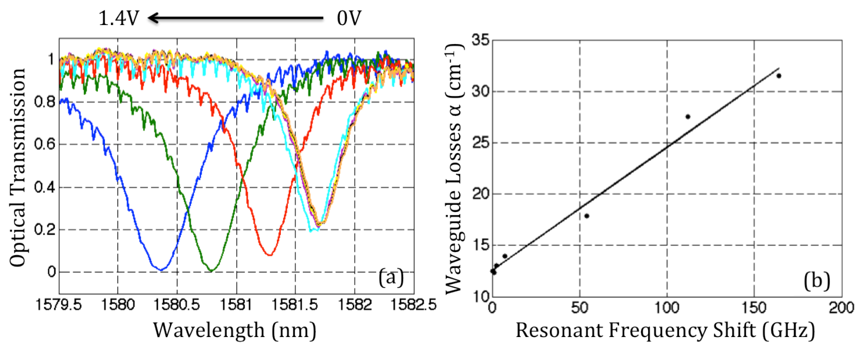

Figure 11|Experimental determination of ϑ by applying a DC voltage bias to the RRM. (a) shows the optical transmission of the RRM for different forward biases between 0V and 1.4V (applied in 0.2V increments). (b) summarizes the extracted waveguide losses as a function of the induced resonant frequency shift. The slope of (b) is used to extract ϑ.

III. RRM equivalent circuit with a peaking amplifier model

We introduce the change of variables

$$\bar{R}_2 = \frac{1}{A_v+1}\frac{R_2}{R_1} \quad (35)$$

$$\bar{L} = \frac{1}{A_v+1}\frac{L}{R_1} \quad (36)$$

$$\bar{C} = CR_1 \quad (37)$$

Equation (7) than simplifies to

$$\frac{V_{Out}}{V_{In}} = -Av\frac{\bar{R}_2 - i\bar{L}\omega}{1+\bar{R}_2 - i\bar{L}\omega - i\bar{C}\bar{R}_2\omega - \bar{C}\bar{L}\omega^2} \quad (38)$$

and can also be described by the equivalent circuit shown in fig. 12:

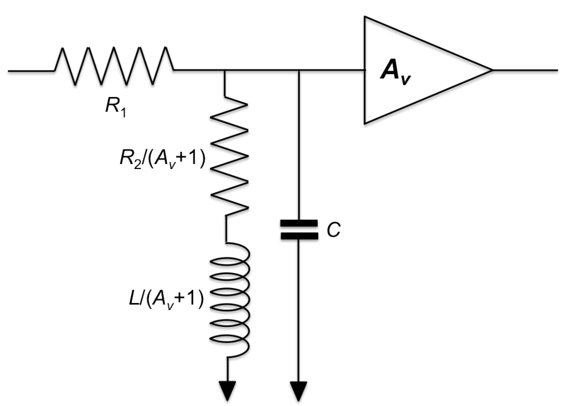

Figure 12| Simplified equivalent circuit for the RRM response.

Please note that here too the transfer function is complex conjugate compared to the convention usually used in electronics in order to maintain consistency with the implicit time dependence as $exp(-i\omega t)$ used elsewhere in this paper. Figure 13 summarizes the electrical component values emulating the RMM S21 shown in fig. 2 assuming R_1 = 10Ω. The resulting S21 parameters are undistinguishable from those generated by equation (3). The loaded Q-factor of the RRM at the measured wavelength is 3500.

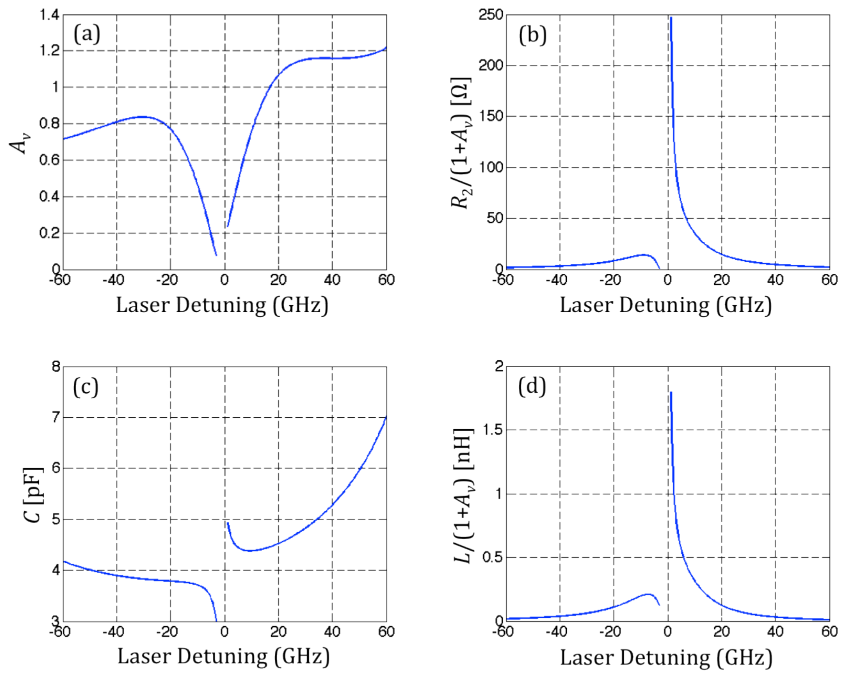

Figure 13| Values of the electrical components for the equivalent circuit model shown in figs. 6 and 10 emulating the S21-parameters of the RRM shown in fig. 2 as a function of laser detuning. R1 is assumed to be 10Ω.