File S1

Supplementary Materials

S1 Expressions of the constant coefficients

The coefficients C_1 and C_2 in equation (14a) are determined by the initial conditions $\alpha_0(12) = C_1 + C_2$ and $\alpha_1(12) = \beta_0(12) = C_1\lambda_1 + C_2\lambda_2$. Thus we have

$$C_1 = \frac{\beta_0(12) - \alpha_0(12)\lambda_2}{\lambda_1 - \lambda_2}, \tag{S1.1a}$$

$$C_2 = -\frac{\beta_0(12) - \alpha_0(12)\lambda_1}{\lambda_1 - \lambda_2}. \tag{S1.1b}$$

The coefficients C_3 and C_4 in equation (14b) are determined by the initial conditions $\alpha_0(123) = C_3 + C_4$ and $\alpha_1(123) = C_3\lambda_3 + C_4\lambda_4$. According to equation (5a) it holds $\alpha_1(123) = s\alpha_0(123) + (1-2s)\beta_0(123)$, and thus we have

$$C_3 = \frac{(1-2s)\beta_0(123) + (s-\lambda_4)\alpha_0(123)}{\lambda_3 - \lambda_4}, \tag{S1.2a}$$

$$C_4 = \alpha_0(123) - C_3 \tag{S1.2b}$$

where we set $C_3 = \alpha_0(123)$ and $C_4 = 0$ for the case of $\lambda_3 = \lambda_4$.

According to the recurrence equations (10a, 10b), we have

$$J_t(1122) = (1-s)J_{t-1}(1122) + \frac{s}{2}J_{t-2}(1122) + \frac{s}{2}R_{t-2}. \tag{S1.3}$$

Substituting the equations (14c, 14d) into the above recurrence relation and noting that $\lambda_{1,2}^2 - (1-s)\lambda_{1,2} - s/2 = 0$, we have

$$C_6 = -\frac{C_1}{1 + s - 2\lambda_1 s}, \tag{S1.4}$$

$$C_8 = -\frac{C_2}{1 + s - 2\lambda_2 s}. \tag{S1.5}$$

Let $C_0 = [R_0 + C_1/(1-\lambda_1) + C_2/(1-\lambda_2)]$, from the initial condition $J_0(1122) = C_0 + C_5 + C_7$, $J_1(1122) = K_0(1122) = C_0 + (C_5 + C_6)\lambda_1 + (C_7 + C_8)\lambda_2$, the expressions for C_5 and C_7

are given by

$$C_5 = \frac{K_0(1122) - C_0 - C_6\lambda_1 - C_8\lambda_2 - \lambda_2\left[J_0(1122) - C_0\right]}{\lambda_1 - \lambda_2}, \tag{S1.6}$$

$$C_7 = J_0(1122) - C_5 - C_0. \tag{S1.7}$$

We can obtain expressions for $C_9, ...C_{12}$ similarly. According to the recurrence equations (8a, 8b), we have

$$J_t(1232) = (1-s)J_{t-1}(1232) + \frac{s}{2}J_{t-2}(1213) + \alpha_{t-1}(123). \tag{S1.8}$$

Substituting the equations (14b, 14e) into the above recurrence relation, we have

$$C_{11} = \frac{C_3\lambda_3}{\lambda_3^2 - (1-s)\lambda_3 - s/2}, \tag{S1.9}$$

$$C_{12} = \frac{C_4\lambda_4}{\lambda_4^2 - (1-s)\lambda_4 - s/2}. \tag{S1.10}$$

From the inital conditions $J_0(1232) = C_9 + C_{10} + C_{11} + C_{12}$ and $J_1(1232) = K_0(1232) + \alpha_0(123) = C_9\lambda_1 + C_{10}\lambda_2 + C_{11}\lambda_3 + C_{12}\lambda_4$, we have

$$C_9 = \frac{K_0(1232) + \alpha_0(123) - C_{11}\lambda_3 - C_{12}\lambda_4 - \lambda_2\left[J_0(1232) - C_{11} - C_{12}\right]}{\lambda_1 - \lambda_2} \tag{S1.11}$$

$$C_{10} = J_0(1232) - C_9 - C_{11} - C_{12}. \tag{S1.12}$$