File S1

Supplementary Materials

S1 Expressions of the constant coefficients

The coefficients C_1 and C_2 in equation (14a) are determined by the initial conditions $\alpha_0(12) = C_1 + C_2$ and $\alpha_1(12) = \beta_0(12) = C_1\lambda_1 + C_2\lambda_2$. Thus we have

$$C_1 = \frac{\beta_0(12) - \alpha_0(12)\lambda_2}{\lambda_1 - \lambda_2}, \tag{S1.1a}$$

$$C_2 = -\frac{\beta_0(12) - \alpha_0(12)\lambda_1}{\lambda_1 - \lambda_2}. \tag{S1.1b}$$

The coefficients C_3 and C_4 in equation (14b) are determined by the initial conditions $\alpha_0(123) = C_3 + C_4$ and $\alpha_1(123) = C_3\lambda_3 + C_4\lambda_4$. According to equation (5a) it holds $\alpha_1(123) = s\alpha_0(123) + (1-2s)\beta_0(123)$, and thus we have

$$C_3 = \frac{(1-2s)\beta_0(123) + (s - \lambda_4)\alpha_0(123)}{\lambda_3 - \lambda_4}, \tag{S1.2a}$$

$$C_4 = \alpha_0(123) - C_3 \tag{S1.2b}$$

where we set $C_3 = \alpha_0(123)$ and $C_4 = 0$ for the case of $\lambda_3 = \lambda_4$.

According to the recurrence equations (10a, 10b), we have

$$J_t(1122) = (1-s)J_{t-1}(1122) + \frac{s}{2}J_{t-2}(1122) + \frac{s}{2}R_{t-2}. \tag{S1.3}$$

Substituting the equations (14c, 14d) into the above recurrence relation and noting that $\lambda_{1,2}^2 - (1-s)\lambda_{1,2} - s/2 = 0$, we have

$$C_6 = -\frac{C_1}{1 + s - 2\lambda_1 s}, \tag{S1.4}$$

$$C_8 = -\frac{C_2}{1 + s - 2\lambda_2 s}. \tag{S1.5}$$

Let $C_0 = [R_0 + C_1/(1-\lambda_1) + C_2/(1-\lambda_2)]$, from the initial condition $J_0(1122) = C_0 + C_5 + C_7$, $J_1(1122) = K_0(1122) = C_0 + (C_5 + C_6)\lambda_1 + (C_7 + C_8)\lambda_2$, the expressions for C_5 and C_7

are given by

$$C_5 = \frac{K_0(1122) - C_0 - C_6\lambda_1 - C_8\lambda_2 - \lambda_2\left[J_0(1122) - C_0\right]}{\lambda_1 - \lambda_2}, \quad \text{(S1.6)}$$

$$C_7 = J_0(1122) - C_5 - C_0. \quad \text{(S1.7)}$$

We can obtain expressions for $C_9, ...C_{12}$ similarly. According to the recurrence equations (8a, 8b), we have

$$J_t(1232) = (1-s)J_{t-1}(1232) + \frac{s}{2}J_{t-2}(1213) + \alpha_{t-1}(123). \quad \text{(S1.8)}$$

Substituting the equations (14b, 14e) into the above recurrence relation, we have

$$C_{11} = \frac{C_3\lambda_3}{\lambda_3^2 - (1-s)\lambda_3 - s/2}, \quad \text{(S1.9)}$$

$$C_{12} = \frac{C_4\lambda_4}{\lambda_4^2 - (1-s)\lambda_4 - s/2}. \quad \text{(S1.10)}$$

From the inital conditions $J_0(1232) = C_9 + C_{10} + C_{11} + C_{12}$ and $J_1(1232) = K_0(1232) + \alpha_0(123) = C_9\lambda_1 + C_{10}\lambda_2 + C_{11}\lambda_3 + C_{12}\lambda_4$, we have

$$C_9 = \frac{K_0(1232) + \alpha_0(123) - C_{11}\lambda_3 - C_{12}\lambda_4 - \lambda_2\left[J_0(1232) - C_{11} - C_{12}\right]}{\lambda_1 - \lambda_2} \quad \text{(S1.11)}$$

$$C_{10} = J_0(1232) - C_9 - C_{11} - C_{12}. \quad \text{(S1.12)}$$

Table S1 List of symbols and their brief explanations.

Category	Symbol	Explanation
Two-gene	(ab)	Two-gene IBD configurations include (11) and (12)
	$\alpha_t(ab)$	Within-individual probability of configuration (ab) in generation *t*
	$\beta_t(ab)$	Between-individual probability of configuration (ab) in generation *t*
	$\alpha_t(11)$	Within-individual two-gene IBD probability in generation *t*
	$\alpha_t(12)$	Within-individual two-gene non-IBD probability in generation *t*
	s_t	Two-gene coalescence probability that both come from a single individual of the previous generation $t-1$
Three-gene	(abc)	Three-gene IBD configurations include (111), (112), (121), (122), (123)
	$\alpha_t(abc)$	Probability of configuration (abc) in generation *t,* given that genes a and c are in a single individual and gene b in another
	$\beta_t(abc)$	Probability of configuration (abc) in generation *t*, given that the three genes are in three distinct individuals
	$\alpha_t(123)$	Non-IBD probability of the three genes
	$\alpha_t(122)$	Probability that the genes a and b are non-IBD and genes b and c are IBD
	$\alpha_t(1\_2)$	Marginal non-IBD probability between genes a and c
	q_t	Three-gene coalescence probability that one particular gene comes from one individual and other two genes come from another individual of the previous generation $t-1$.
Four-gene	$(abcd)$	Four-gene IBD configurations include the 15 configurations shown in Table 1
	$D(abcd)$	Two-locus probability of configuration $(abcd)$
	$J_t(abcd)$	Within-individual expected junction density of type $(abcd)$ in generation *t*. The seven junction types are shown in Table 1
	$K_t(abcd)$	Between-individual expected junction density of type $(abcd)$ in generation *t*
Breeding design	L	Number of distinct founder genome labels (FGL)
	U	Number of intercross generations
	V	Number of inbreeding generations
	N_t	Population size in generation *t*
	N_F	Constant size of founder population, and N_F=*L* if founders are fully inbred
	N_I	Constant size of intercross populations
	N_{II}	Constant size of inbred populations. $N_{II}=1$ if $\mathcal{M}_{II}=$ Selfing, and $N_{II}=2$ if $\mathcal{M}_{II}=$ Sibling
	$\mathcal{M}_t$	Mating scheme from the generation *t* to the next generation.
	$\mathcal{M}_F$	Constant mating scheme from the founder population to the F_1 population, $\mathcal{M}_F=\mathcal{M}_0$
	$\mathcal{M}_I$	Constant mating scheme in the intercross stage, $\mathcal{M}_I=\mathcal{M}_1=\cdots=\mathcal{M}_U$
	$\mathcal{M}_{II}$	Constant mating scheme in the inbreeding stage, $\mathcal{M}_{II}=\mathcal{M}_{U+1}=\cdots=\mathcal{M}_{U+V}$
Map resolution	R	Map expansion, the expected junction density (per Morgan) on one chromosome
	ρ	Overall expected junction density, the expected junction density (per Morgan) on two homologous chromosomes

Table S2 The three largest overall expected junction densities ρ for 2^n-way RIL on autosomes at the last generation $g = U + V + 1$.

Scheme	n	U	(ρ, V)		
Selfing	1	0	(2.5, 2)	(2.5, 3)	(2.375, 4)
	2	1	(3.75, 2)	(3.625, 3)	(3.5, 1)
	3	2	(5, 1)	(5, 2)	(4.75, 3)
	4	3	(6.5, 1)	(6.25, 2)	(6, 0)
	5	4	(8, 0)	(8, 1)	(7.5, 2)
	6	5	(10, 0)	(9.5, 1)	(8.75, 2)
Sibling mating	1	0	(4.875, 8)	(4.863, 9)	(4.844, 7)
	2	0	(7.301, 8)	(7.297, 7)	(7.256, 9)
	3	1	(8.512, 7)	(8.484, 6)	(8.475, 8)
	4	2	(9.75, 6)	(9.727, 7)	(9.688, 5)
	5	3	(11.016, 5)	(11.016, 6)	(10. 941, 7)
	6	4	(12.344, 5)	(12.312, 4)	(12.281, 6)