Supplementary Information
# Ranking scientific publications: the effect of nonlinearity

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## 1 Basic Properties

Here we discuss the basic properties of the NonlinearRank method, including convergence, score distribution and similarity to PageRank and degree rank. In the following study, we make use of the APS citation network data.

Since our method has nonlinearity in the iterative equation, its convergence cannot be directly proved by the Perron-Frobenius theorem [1, 2]. However, we numerically investigate the convergence of our method. We first study the dependence of the total score on the iteration steps in Fig. S1(A). One can see that the total score is kept unchanged when $\theta = 0$ (i.e. the PageRank algorithm). If $\theta > 0$, the total score first decreases with the iteration steps and then becomes stable. In Fig. S1(B), we define the quantity *error* at step $t$ as $\sum_i |s_i(t) - s_i(t-1)|$ where $s_i(t)$ is the score of node $i$ at iteration step $t$. If the *error* can finally approach a small value, then the ranking algorithm converges (as the score of nodes no longer changes). One can see in Fig. S1(B) that in the whole parameter range, our algorithm converges. We also observe that NonlinearRank converges faster when $\theta$ is bigger. Based on these results, we stop the iterations when $t = 50$ in our simulation since nodes' score already reaches stable state.

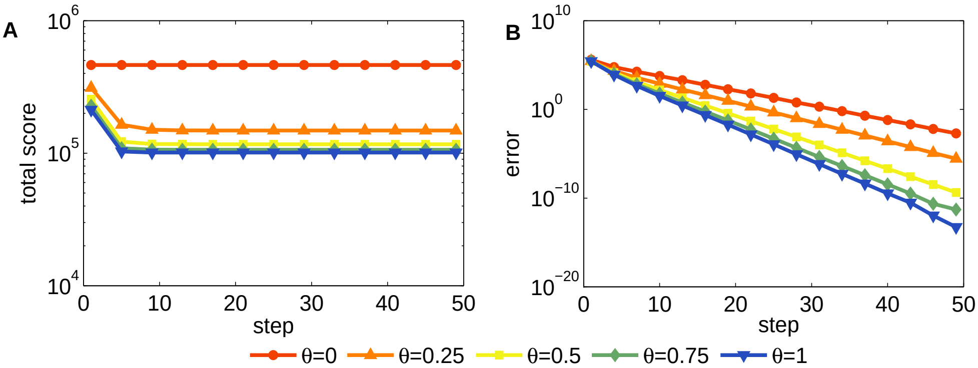


**Figure S1** (A) the dependence of the total score on iteration step $t$ in NonlinearRank, and (B) the dependence of the quantity *error* on iteration step $t$ in NonlinearRank. In this figure $c = 0.15$ but different $\theta$ values are considered. Notice that NonlinearRank reduces to Pagerank when $\theta = 0$.

We then study the distribution of final NonlinearRank score $s$ under different $\theta$ in Fig. S2. One immediate observation in this figure is that the distribution follows power-law form. The parameter $\theta$ can control the exponent of the power-law. Generally speaking, the power-law exponent increases with $\theta$. The narrow distribution doesn't necessarily mean that the nodes are not well separated. Instead, we show in the manuscript that the ranking based on the NonlinearRank score is actually outstanding with respect to awarded papers, spreading, prediction and reliability.

In order to investigate the similarity between NonlinearRank and the other two methods (i.e. PageRank and degree rank), we investigate the Pearson correlation coefficient between their scores. The results are reported in Fig. S3. In Fig. S3(A) and (B), we take all nodes into account and find that the correlation between NonlinearRank and PageRank decreases with $\theta$. The curve starts from 1 because NonlinearRank degenerates to PageRank when $\theta = 0$. The correlation between NonlinearRank and degree rank stays a low

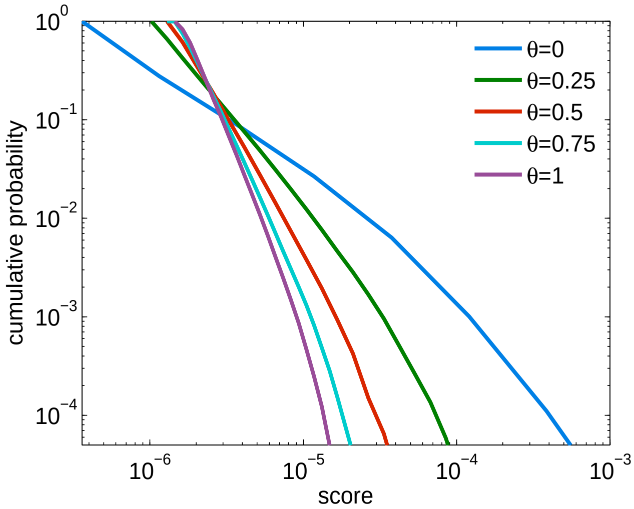


**Figure S2** The cumulative distribution of the final NonlinearRank score under $c = 0.15$ but different $\theta$.

value under all $\theta$ values. These results indicate that the NonlinearRank has completely new properties that cannot be reproduced by PageRank and degree rank. Usually, people are more interested in the most-cited papers. We then consider the ranking of the top-100 and top-1000 most popular papers in Fig. S3(C)(D) and (E)(F), respectively. One can see that the ranking of these hub papers are very different in NonlinearRank and the other two methods, which confirms again that NonlinearRank has entirely new properties.

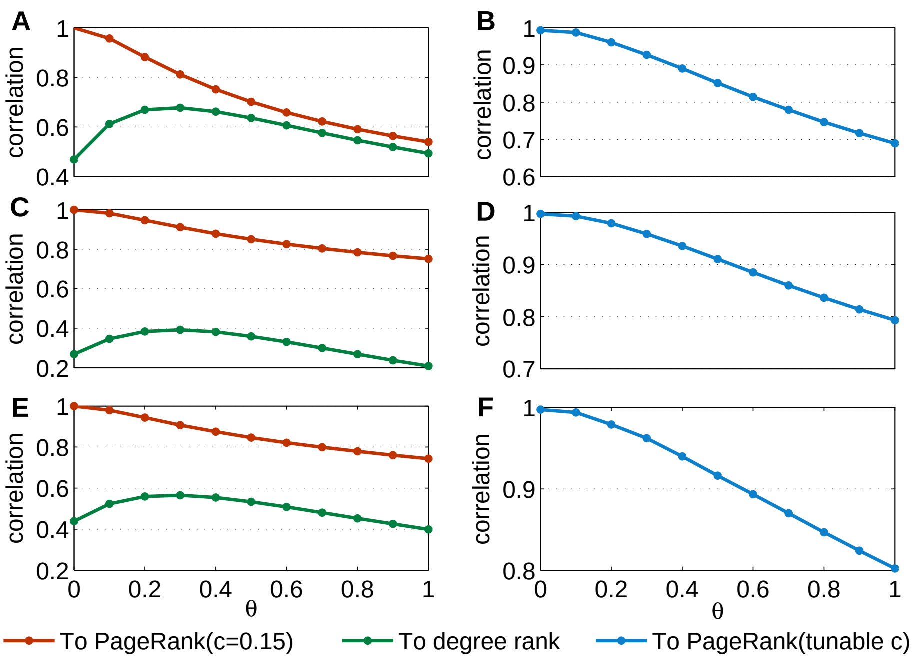


**Figure S3** The Pearson correlation of NonlinearRank score with Pagerank score and indegree, respectively. We fix $c = 0.15$ but change the value of $\theta$ to see the correlation of (A) all, (C) top-100 and (E) top-1000 papers. We make c tunable, then change the value of $\theta$ again, to see the correlation of (B) all, (D) top-100 and (F) top-1000 papers.

## 2 Nobel prize papers

In Table S1, we listed the 39 Nobel prize papers we used in the manuscript to validate our method.

**Table S1.** 39 Nobel prize papers that published in APS from 1950-2009

| ID | Title | Authors | Year |
|---|---|---|---|
| 1 | Dynamical model of elementary particles based on... (I) | Y. Nambu | 1961 |
| 2 | Dynamical model of elementary particles based on... (II) | Y. Nambu | 1961 |
| 3 | Giant magnetoresistance of (001) Fe/(001) Cr magnetic... | A. Fert | 1988 |
| 4 | Enhanced magnetoresistance in layered magnetic... | P. Grnberg | 1989 |
| 5 | Coherent and incoherent states of the radiation field | R. J. Glauber | 1963 |
| 6 | Ultraviolet behavior of non-abelian gauge theories | D. J. Gross, F. Wilczek | 1973 |
| 7 | Reliable perturbative results for strong interactions? | H. D. Politzer | 1973 |
| 8 | A theoretical description of the new phases of $liquid^3$ He | A. J. Leggett | 1975 |
| 9 | Evidence for X-rays from sources outside the solar system | R. Giacconi | 1962 |
| 10 | Bose-Einstein Condensation in a Gas of Sodium Atoms | E. A. Cornell et al. | 1995 |
| 11 | Trapping of neutral sodium atoms with radiation pressure | S Chu | 1987 |
| 12 | Evidence for a New Phase of Solid $He^3$ | D. M. Lee et al. | 1972 |
| 13 | Evidence for Anomalous Lepton Production... | M. L. Perl | 1975 |
| 14 | Neutron diffraction by paramagnetic and... | C. G. Shull | 1951 |
| 15 | Inelastic electron-proton scattering at large... | J. I. Friedman et al. | 1972 |
| 16 | Electron coupled interactions between... | N. F. Ramsey | 1953 |
| 17 | New high-precision comparison of electron and... | H. G. Dehmelt | 1987 |
| 18 | New method for high-accuracy determination of... | K. Klitzing | 1980 |
| 19 | Renormalization Group and Critical Phenomena... (I) | K. G. Wilson | 1971 |
| 20 | Renormalization Group and Critical Phenomena... (II) | K. G. Wilson | 1971 |
| 21 | Relaxation effects in nuclear magnetic resonance absorption | N, Bloembergen | 1948 |
| 22 | Evidence for the 2 Decay of the $K_2^0$ Meson | J. W. Cronin, V. L. Fitch | 1964 |
| 23 | Localized magnetic states in metals | P. W. Anderson | 1961 |
| 24 | Evidence for Anomalous Lepton Production in... | B. Richter | 1975 |
| 25 | Experimental observation of a heavy particle J | S. C. Ting | 1974 |
| 26 | New phenomenon in narrow germanium pn junctions | L. Esaki | 1958 |
| 27 | Energy gap in superconductors measured by... | I. Giaever | 1960 |
| 28 | Theory of superconductivity | J. Bardeen | 1957 |
| 29 | Evidence for at= 0 three-pion resonance | L. W. Alvarez | 1961 |
| 30 | On gauge invariance and vacuum polarization | J. Schwinger | 1951 |
| 31 | Space-time approach to non-relativistic quantum mechanics | R. P. Feynman | 1948 |
| 32 | Infrared and optical masers | C. H. Townes | 1958 |
| 33 | Electron scattering and nuclear structure | R Hofstadter | 1956 |
| 34 | Some effects of ionizing radiation on the formation of... | D. A Glaser | 1952 |
| 35 | Observation of antiprotons | E. G. Segr | 1955 |
| 36 | Question of Parity Conservation in Weak Interactions | C. N. Yang | 1956 |
| 37 | Fine structure of the hydrogen atom by... | W. E. Lamb | 1947 |
| 38 | Nuclear induction | F. Bloch | 1946 |
| 39 | Resonance absorption by nuclear magnetic moments in... | E. M. Purcell | 1946 |

# References


[1] Perron, O., Zur Theorie der Matrices, *Mathematische Annalen* **64**, 248 (1907).

[2] Frobenius, G., Ueber Matrizen aus nicht negativen Elementen, *Sitzungsber. Konigl. Preuss. Akad. Wiss.*, 456 (1912).