Web Appendix for
The Economics of Human Development
and Social Mobility

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A Evidence on Achievement Gaps by Age for Different Socioeconomic Groups

Table A.1: Hart & Risley, 1995

Children enter school with "meaningful differences" in vocabulary knowledge.

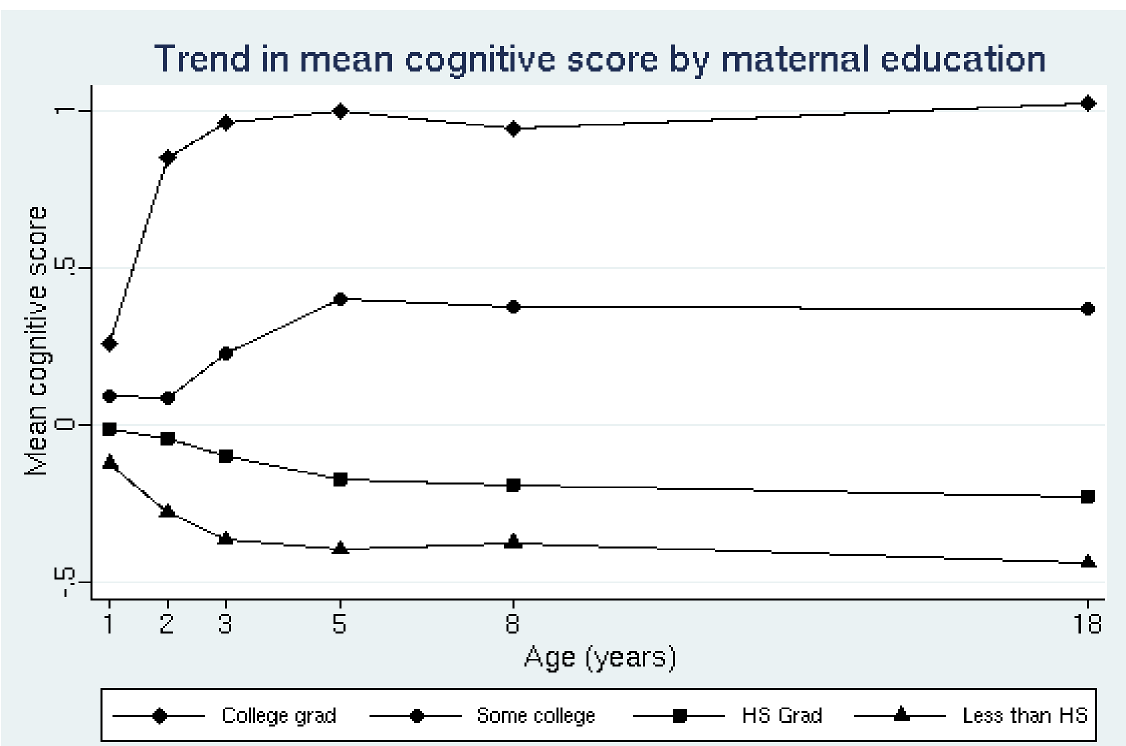
1. Emergence of the Problem
In a typical hour, the average child hears:

Family Status	Actual Differences in Quantity of Words Heard	Actual Differences in Quality of Words Heard
Welfare	616 words	5 affirmatives, 11 prohibitions
Working Class	1,251 words	12 affirmatives, 7 prohibitions
Professional	2,153 words	32 affirmatives, 5 prohibitions

2. Cumulative Vocabulary at Age 3

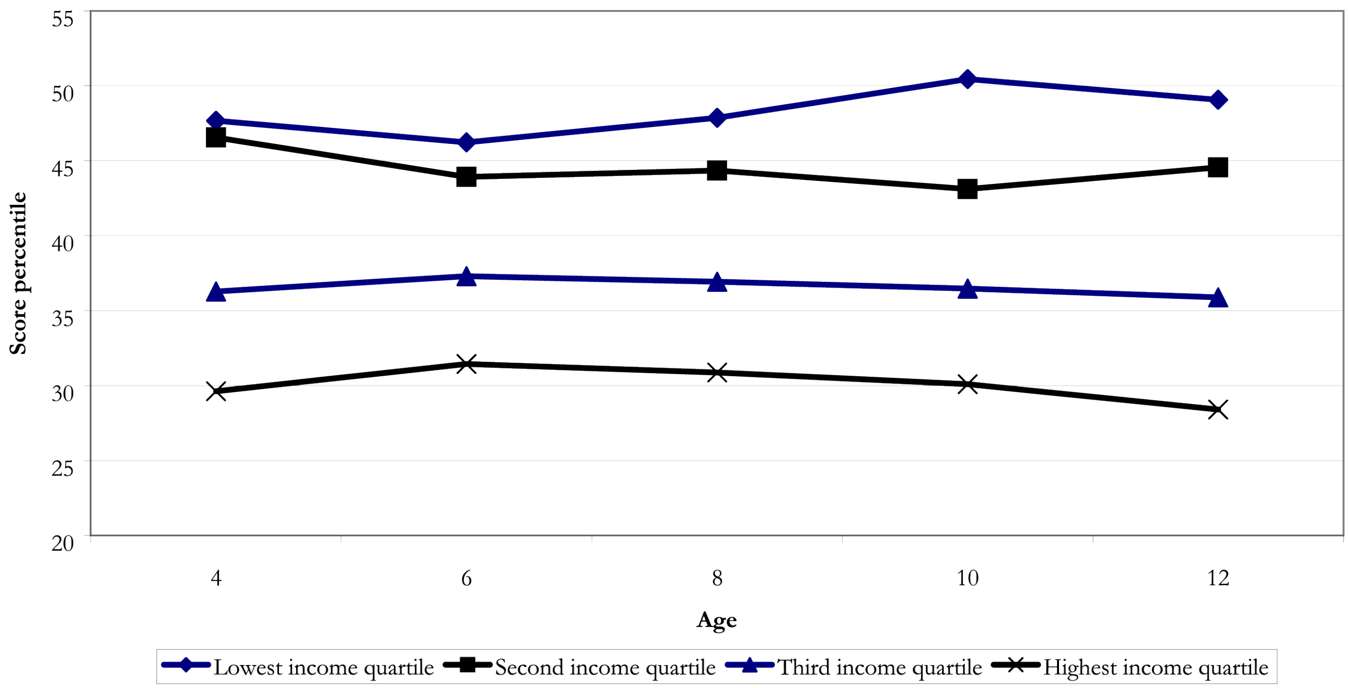
Cumulative Vocabulary at Age 3	
Children from welfare families:	500 words
Children from working class families:	700 words
Children from professional families:	1,100 words

Figure A.1: Trend in Mean Cognitive Score by Maternal Education



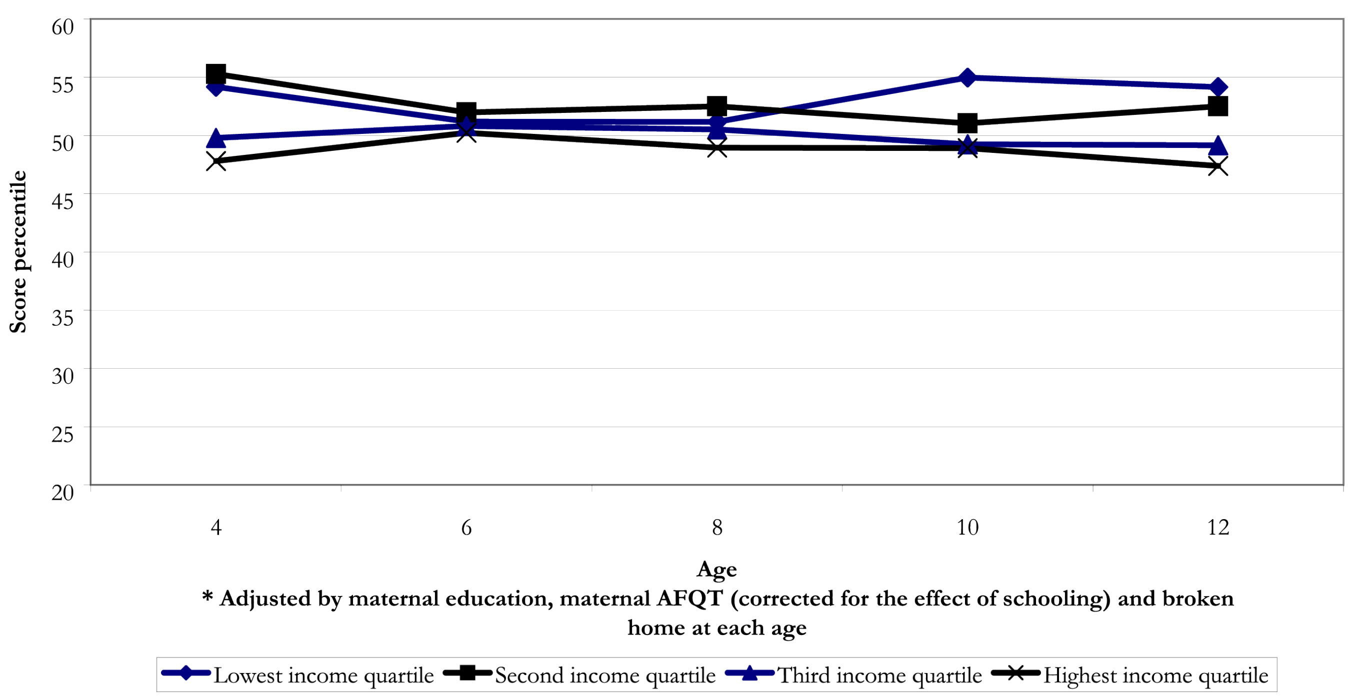
Source: Brooks-Gunn et al., (2006).

Figure A.2: Children of NLSY
Average percentile rank on anti-social behavior score, by income quartile



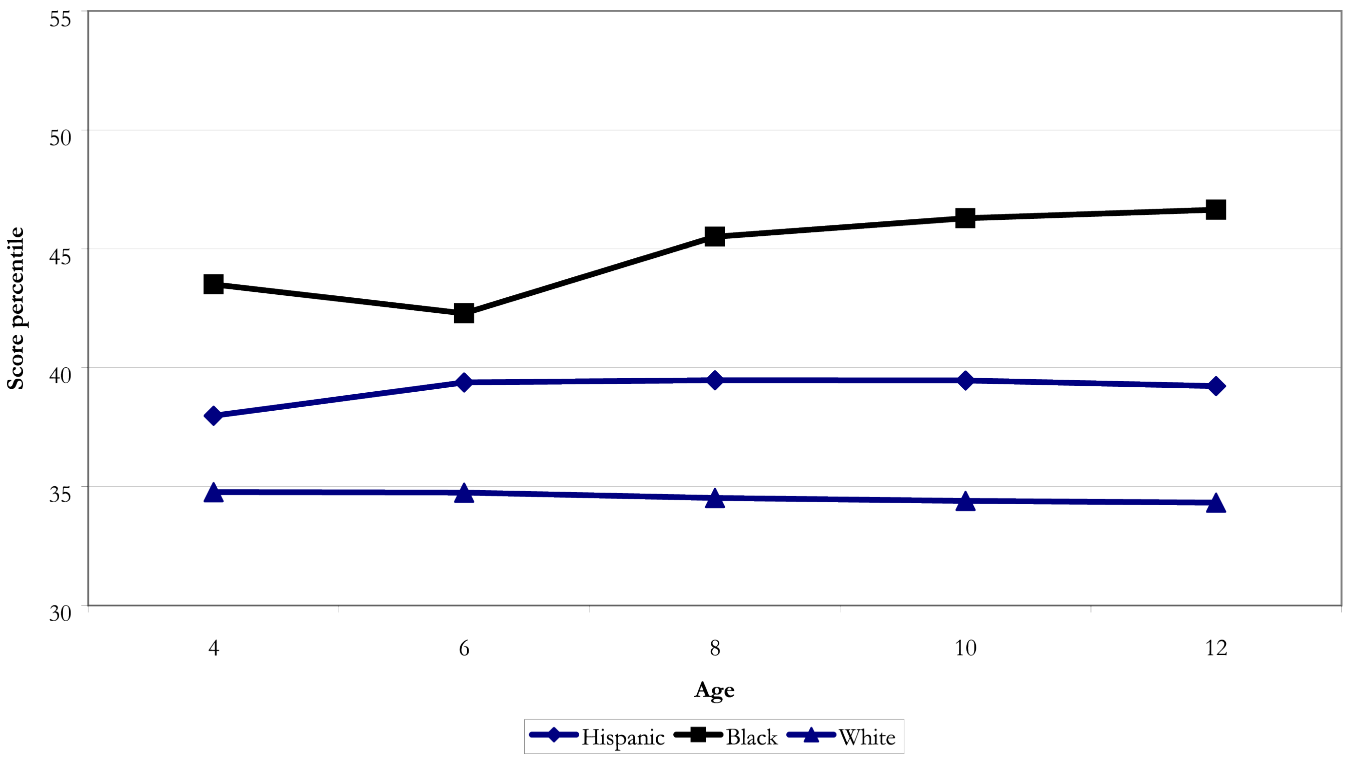
Source: Cunha et al. (2006).

Figure A.3: Children of NLSY
Adjusted average anti-social behavior score percentile by income quartile*



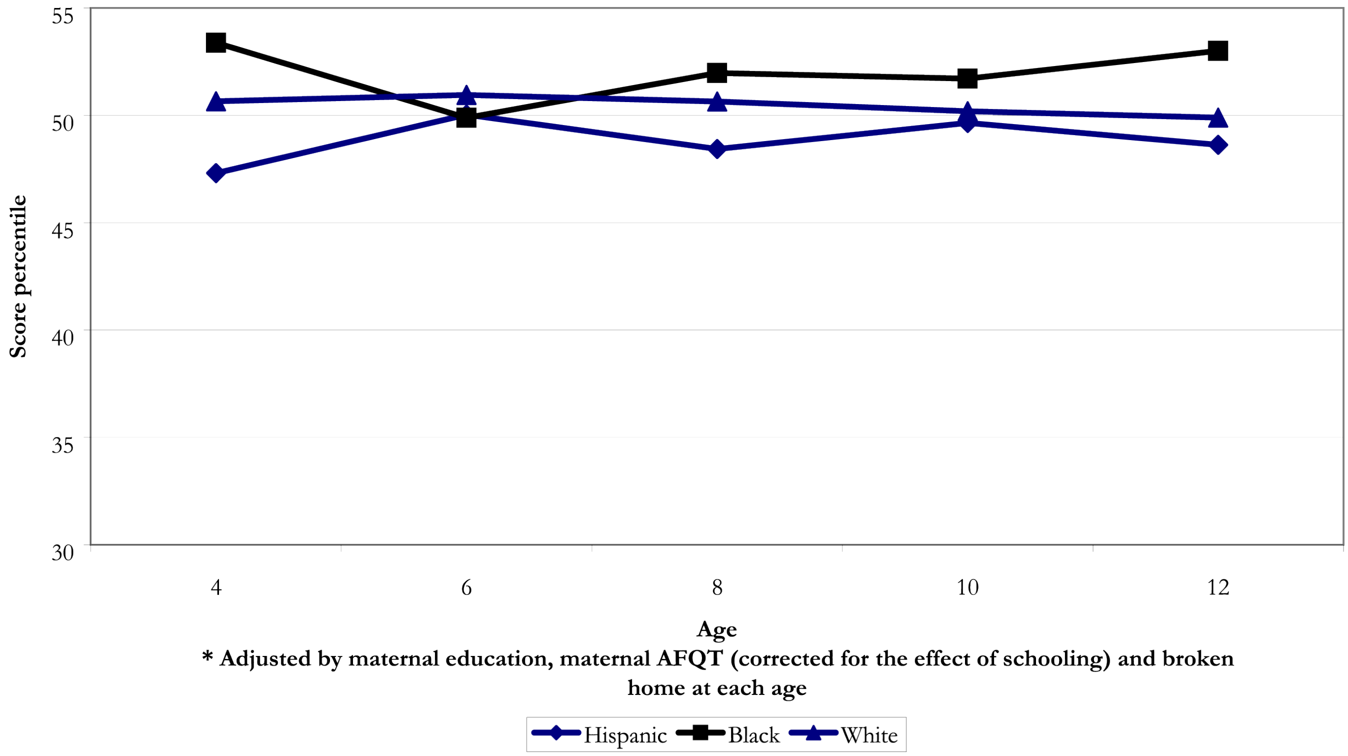
Source: Cunha et al. (2006).

Figure A.4: Children of NLSY
Average percentile rank on anti-social behavior score, by race



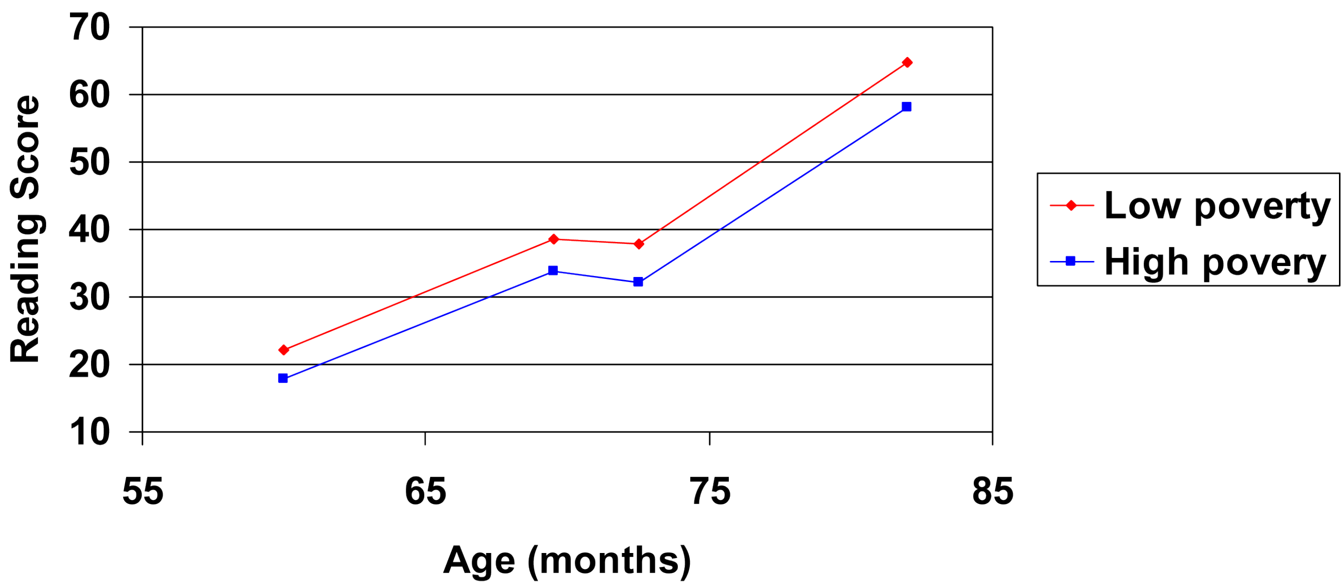
Source: Cunha et al. (2006).

Figure A.5: Adjusted average anti-social behavior score percentile by race



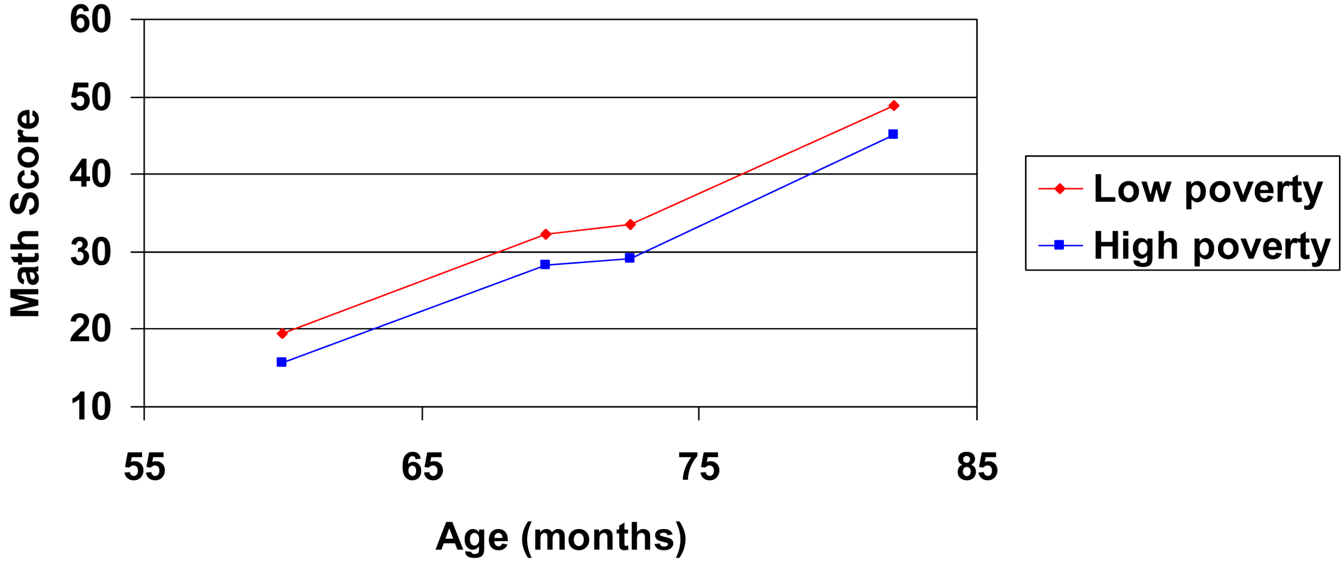
Source: Cunha et al. (2006).

Figure A.6: Early Childhood Longitudinal Study (ECLS)
(a) Reading



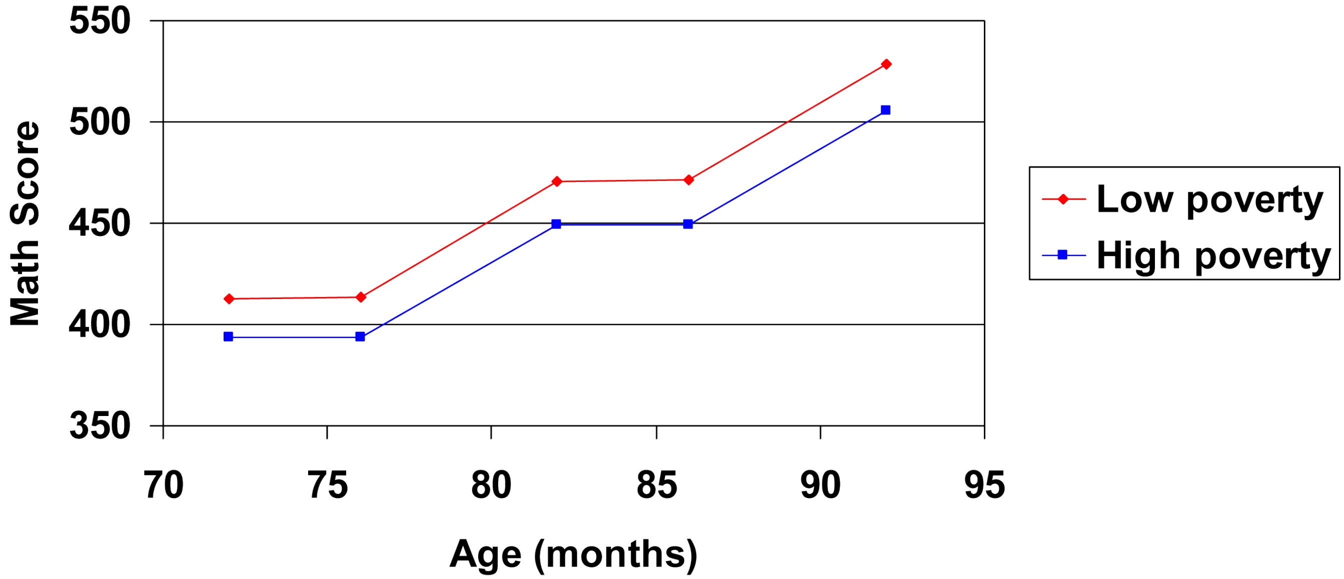
Source: Raudenbush (2006)

Figure A.7: Mean Trajectories, high and low poverty schools (ECLS)
(b) Math



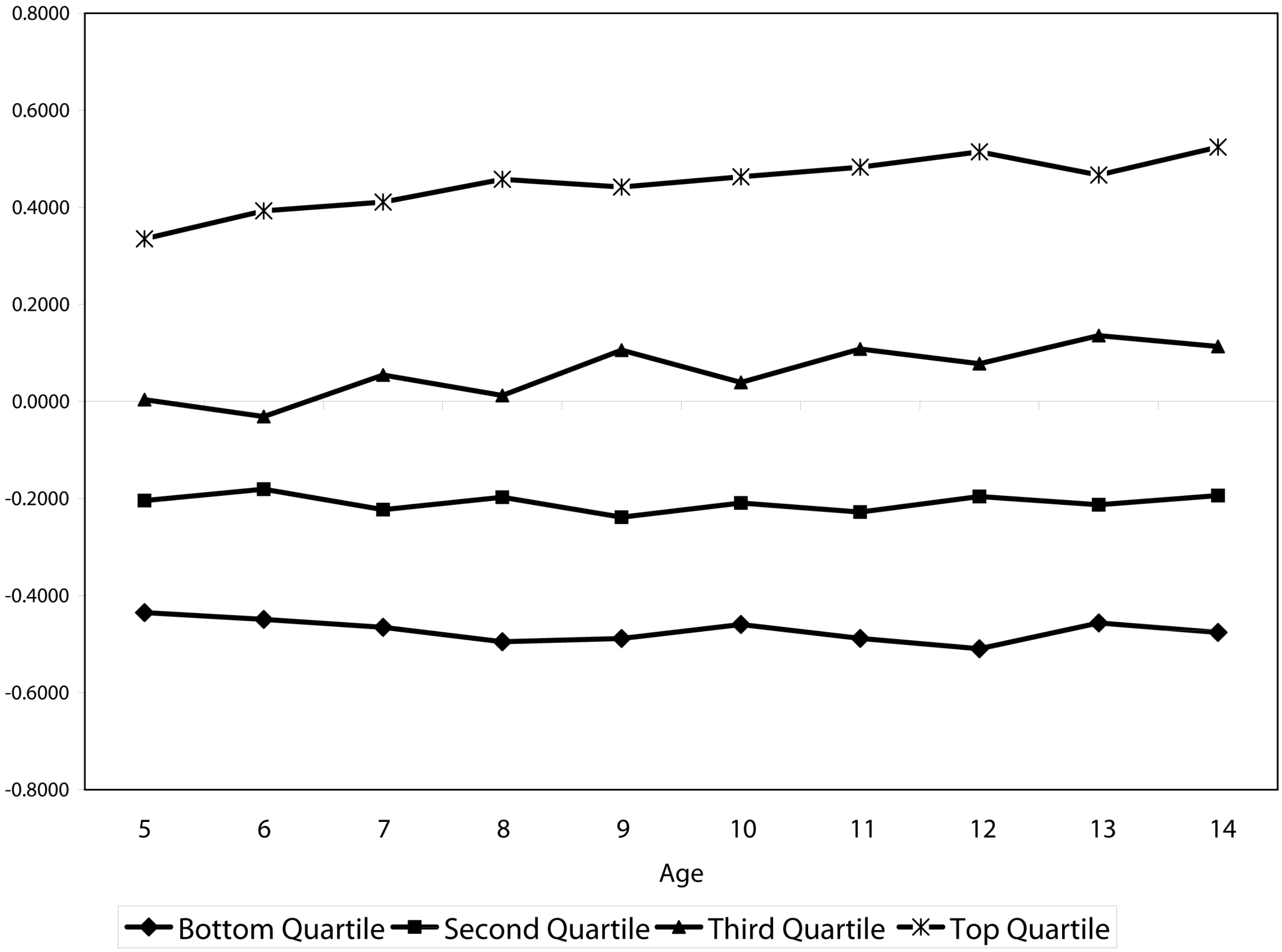
Source: Raudenbush (2006)

Figure A.8: Average trajectories, Grades 1-3, high and low poverty schools (Sustaining Effects Study)
(b) Math



Source: Raudenbush (2006)

Figure A.9: Children of the NLSY:
Average Standardized Score for PIAT Math by Permanent Income Quartile



Source: Cunha et al. (2006).

This figure shows the average standardized score in the PIAT Math test from ages 5 to 14 by quartile of family permanent income. The sample consists of all Children of NLSY/79. Family permanent income is the mean family income from age 0 to age 18 of the child. At each age, we standardize the PIAT math score so it has mean zero and variance one. That is, let $m_{i,t}$ denote the score of child i at age t. Let μ_t, σ_t^2 denote the mean and variance of the PIAT-Math score at age t. We construct the variable $z_{i,t}$ as:

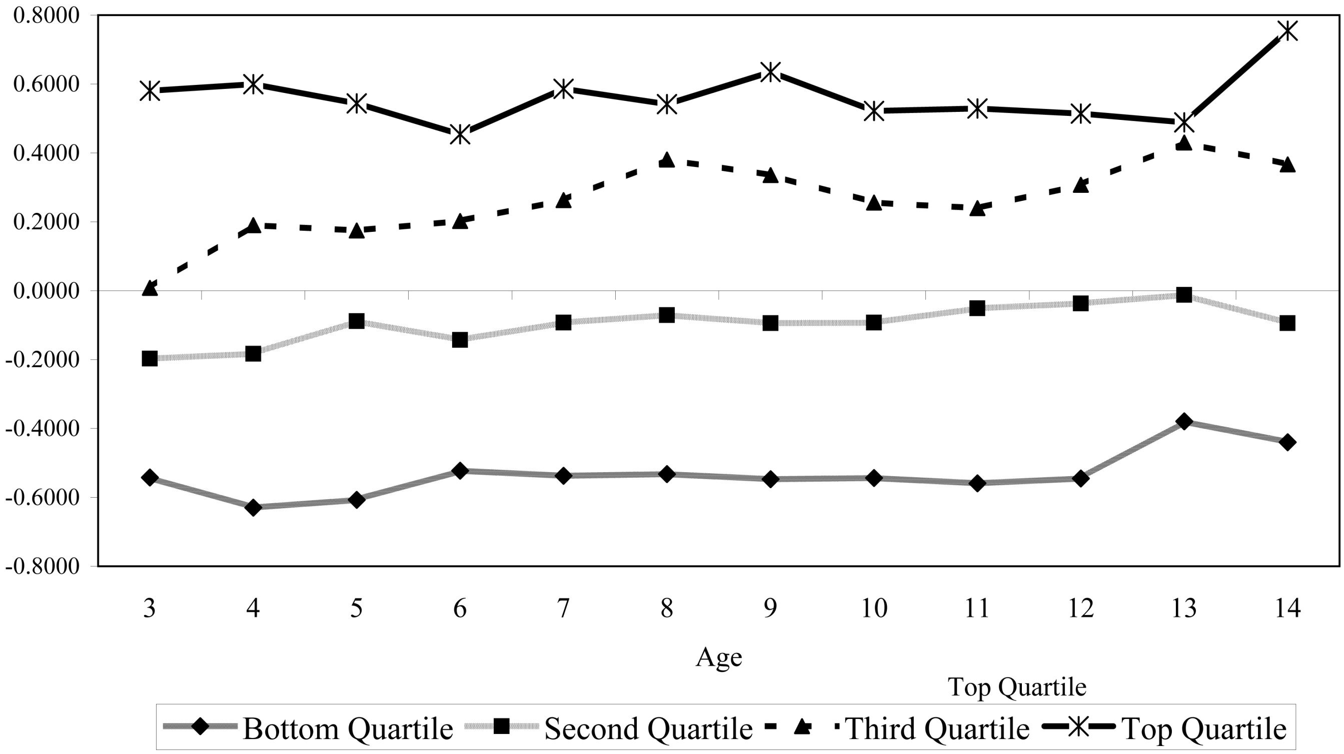
$$z_{i,t} = \frac{m_{i,t} - \mu_t}{\sigma_t}$$

We then proceed by calculating the mean $z_{i,t}$ by quartile of family income. Let $1\left(q_i = Q_j\right)$ denote the function that takes the value one if the family permanent income of child i is in quartile Q_j and zero otherwise. Let $\bar{z}_{j,t}$ denote the mean standardized score at age t of the children whose permanent income is in quartile Q_j :

$$\bar{z}_{j,t} = \frac{\sum_i z_{i,t} 1\left(q_i = Q_j\right)}{\sum_i 1\left(q_i = Q_j\right)}$$

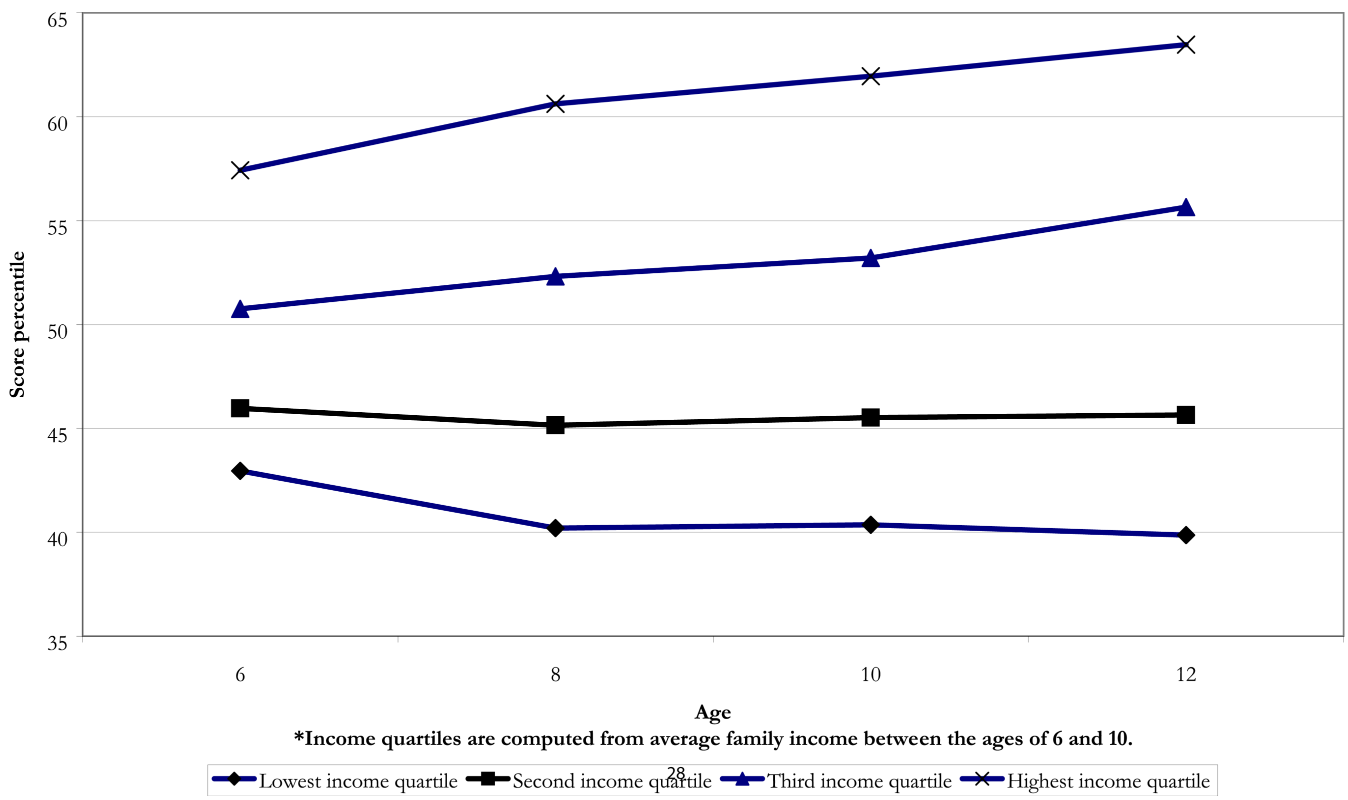
.

Figure A.10: Children of NLSY: Average Standardized Score//Peabody Picture Vocabulary Test by Permanent Income Quartile



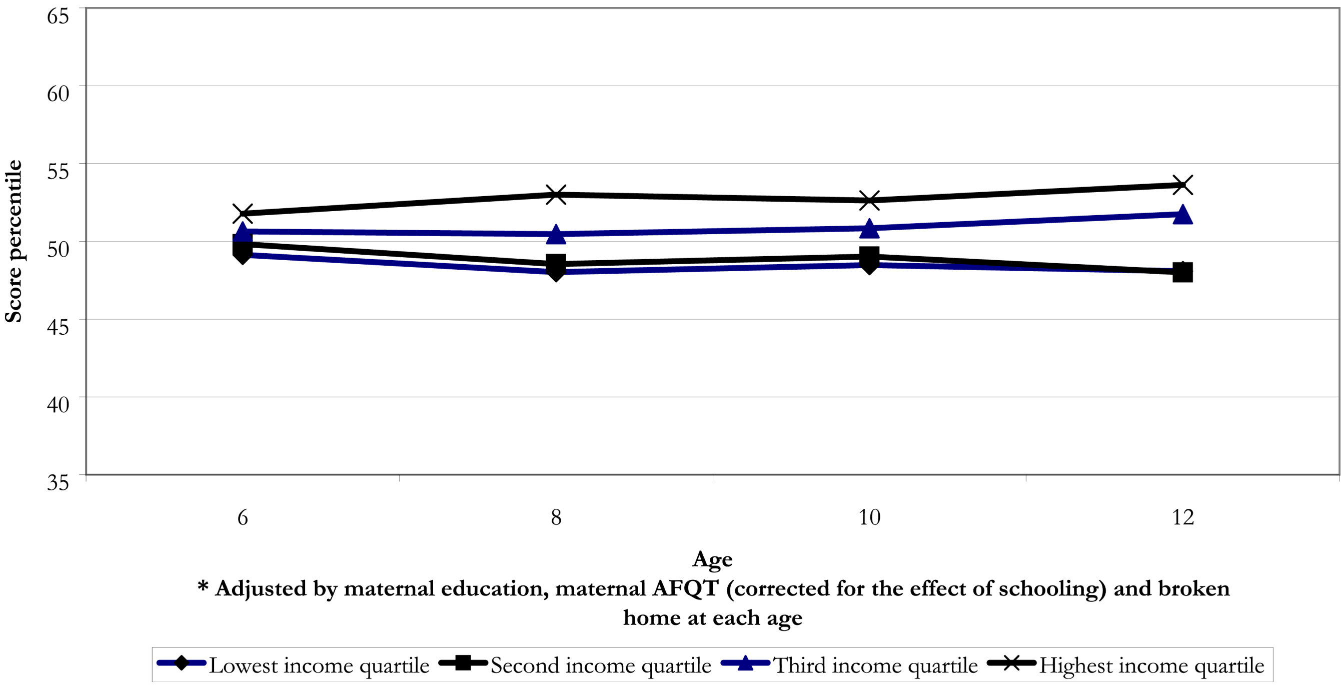
Source: Cunha et al. (2006).

Figure A.11: Children of NSLY
Average Percentile Rank on PIAT math score, by income quartile*



Source: Cunha et al. (2006).

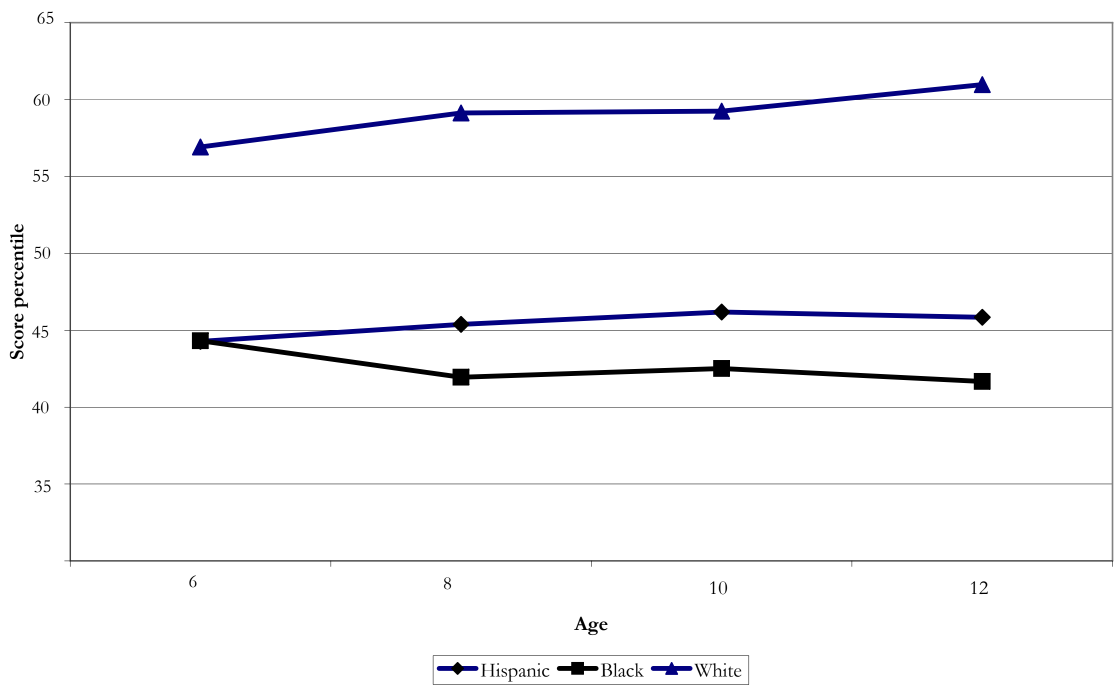
Figure A.12: Children of NSLY
Adjusted average PIAT math score percentiles, by income quartile*



*** Adjusted by maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age**

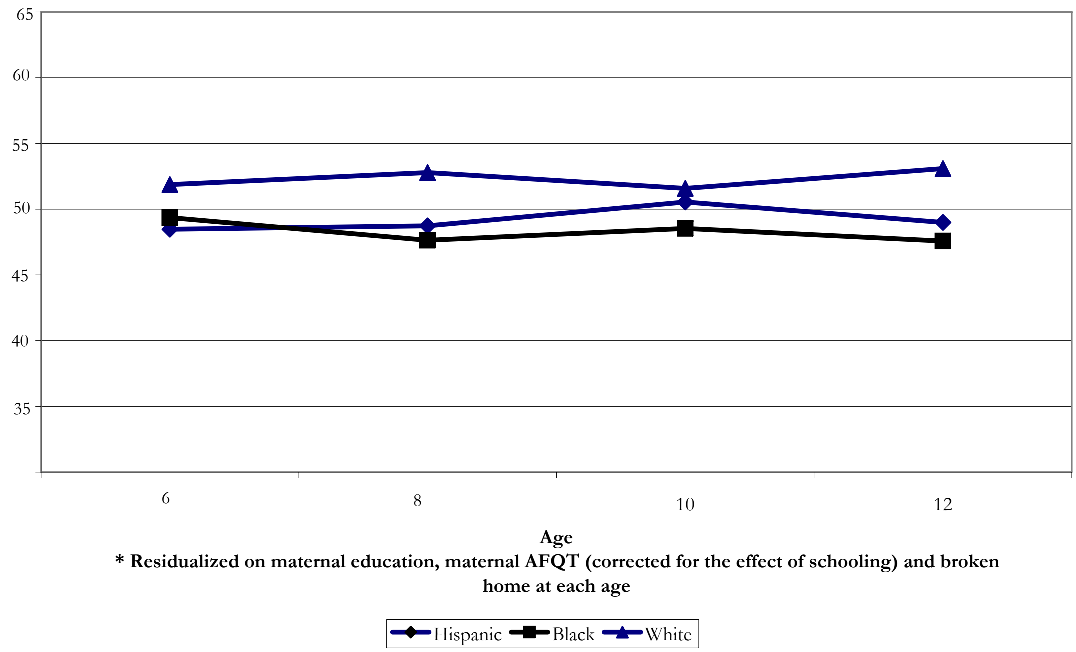
Source: Cunha et al. (2006).

Figure A.13: Average percentile rank on PIAT-Math score, by race



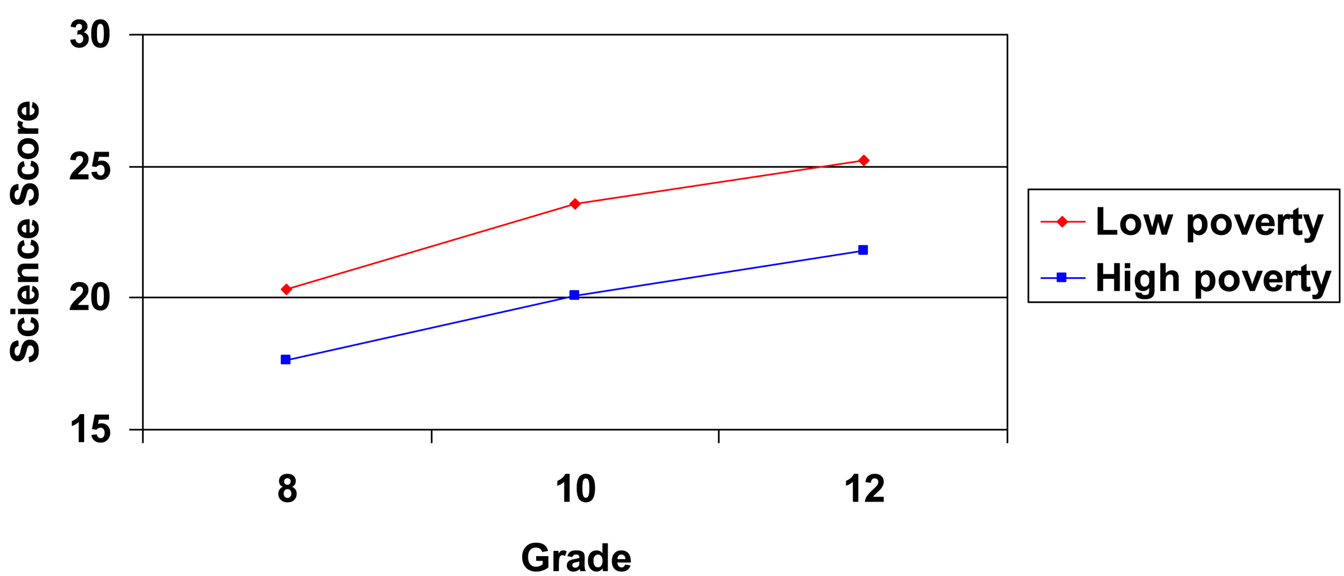
Source: Cunha et al. (2006).

Figure A.14: Adjusted average PIAT-Math score percentiles, by race



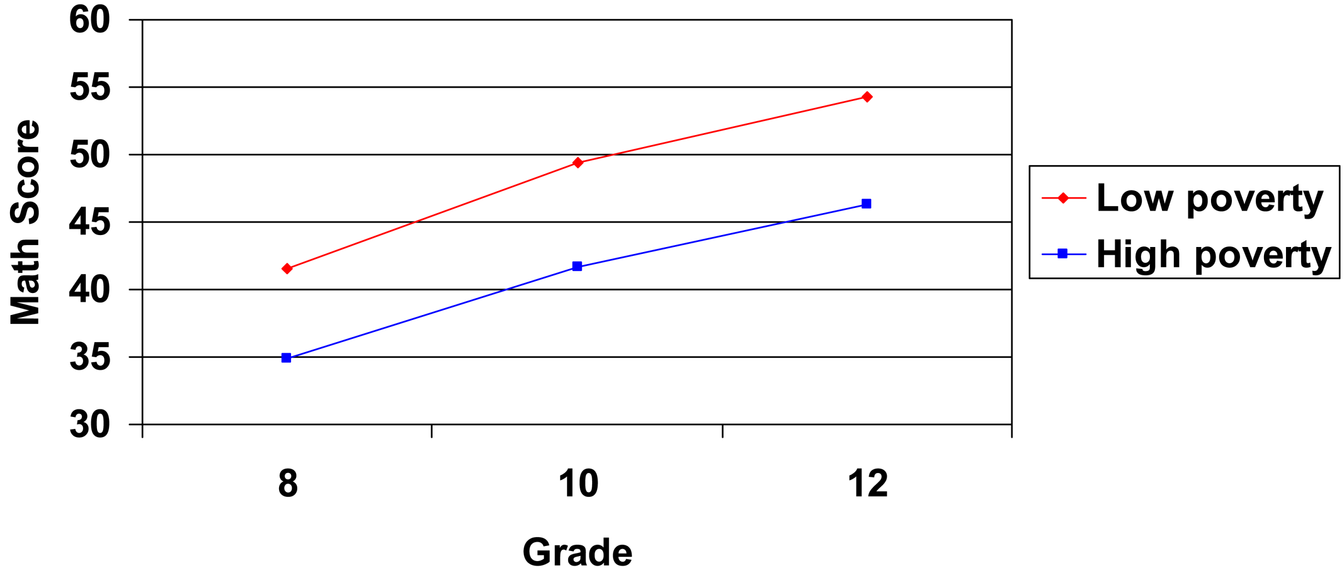
Source: Cunha et al. (2006).

Figure A.15: Average trajectories, Grades 8-12 (NELS 88).
(a) Science



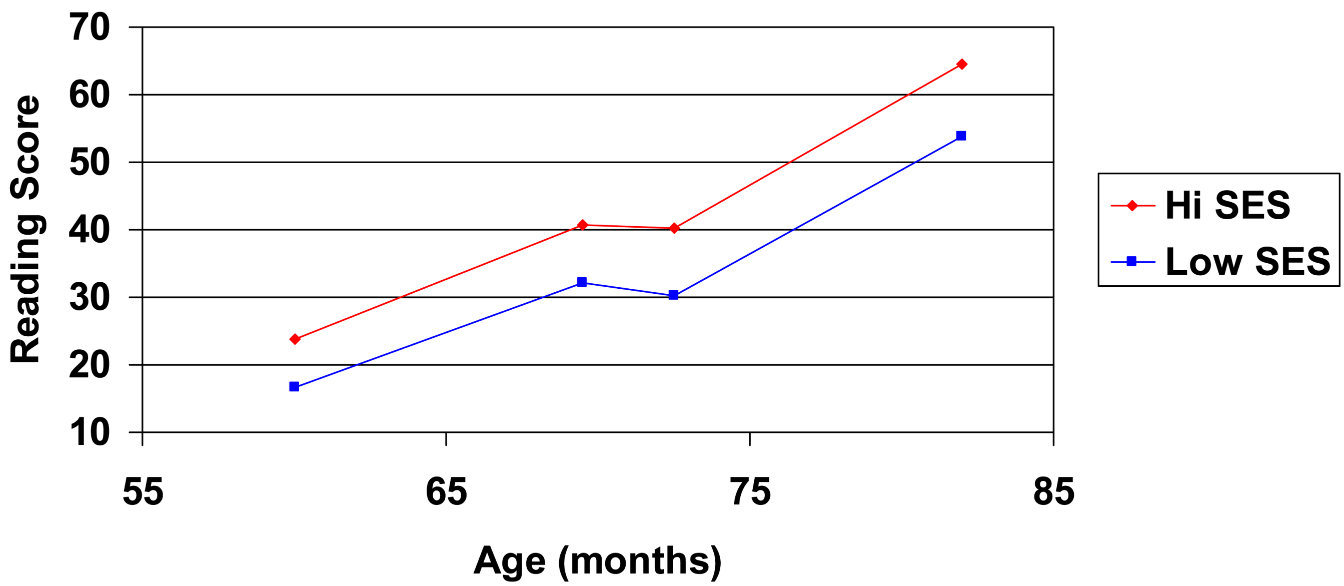
Source: Raudenbush (2006)

Figure A.16: Average trajectories, Grades 8-12 (NELS 88).
(b) Math



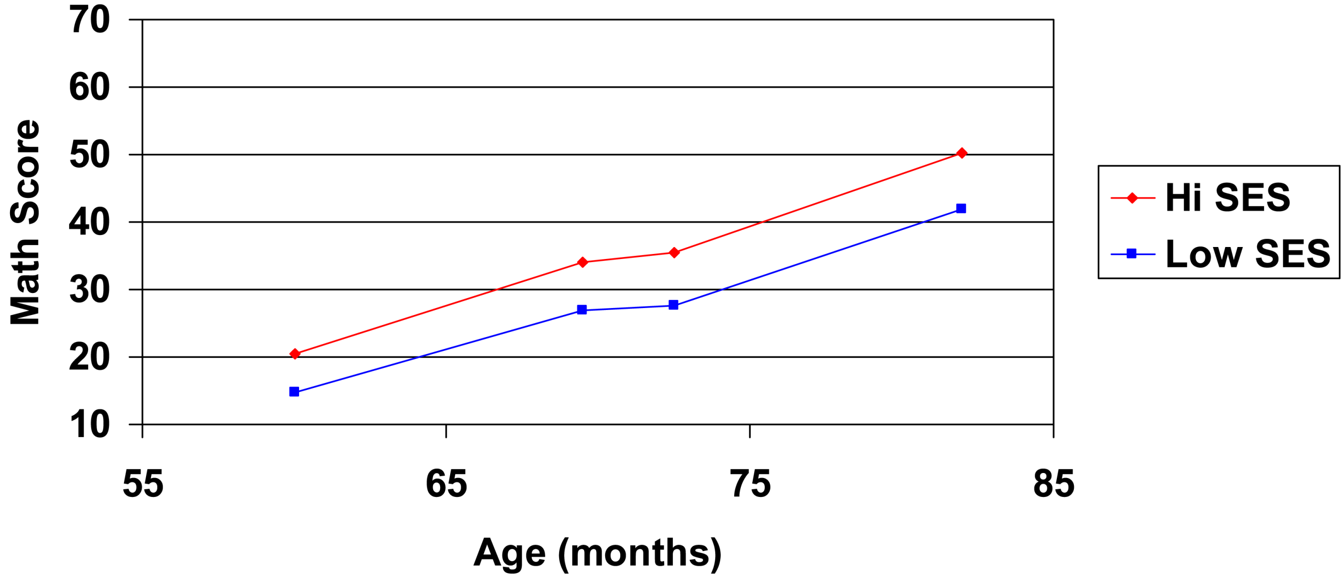
Source: Raudenbush (2006)

Figure A.17: Growth as a function of student social background: ECLS
(a) Reading



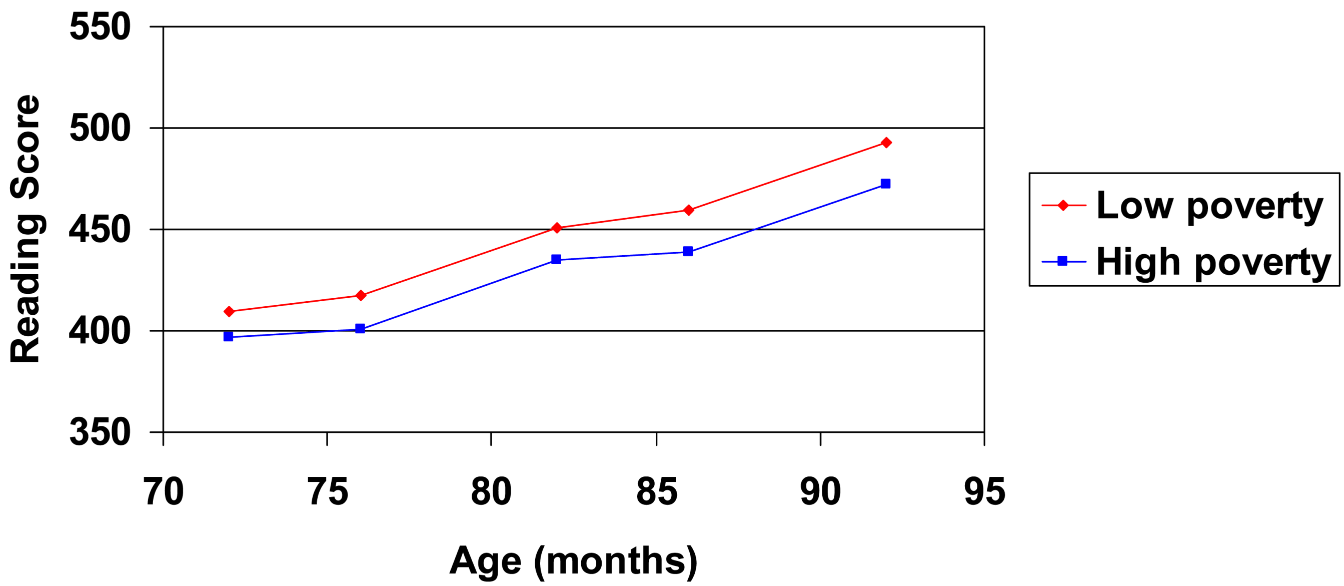
Source: Raudenbush (2006)

Figure A.18: Growth as a function of student social background: ECLS
(b) Math



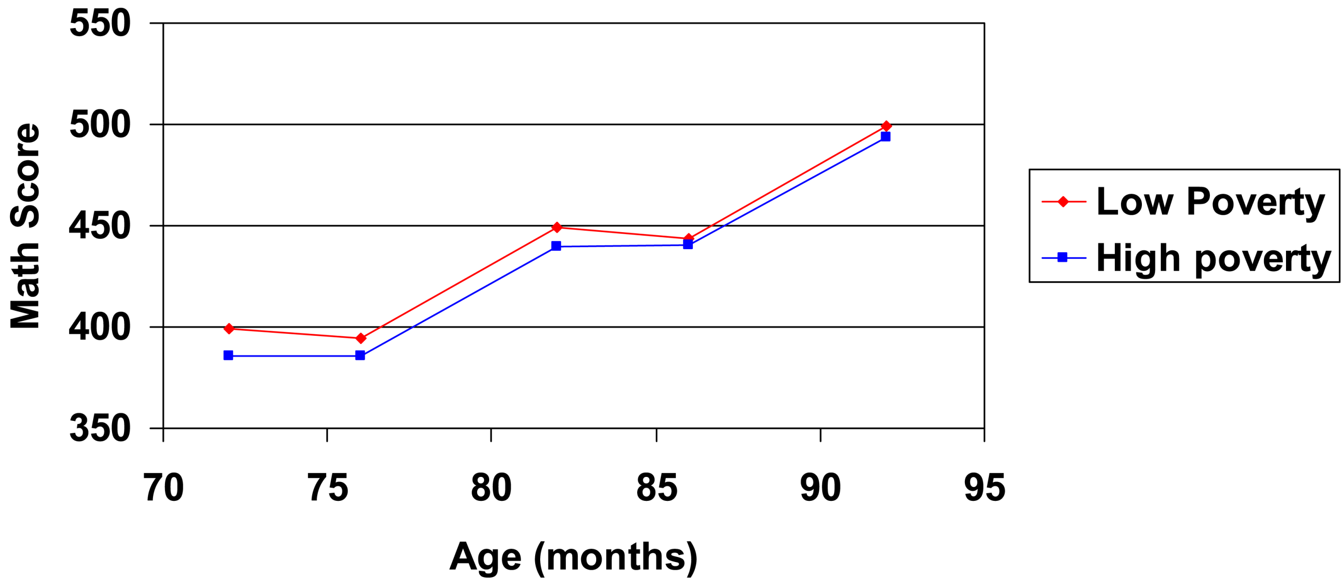
Source: Raudenbush (2006)

Figure A.19: Growth as a Function of School Poverty for Poor Children: Sustaining Effects Data
(a) Reading



Source: Raudenbush (2006)

Figure A.20: Growth as a Function of School Poverty for Poor Children: Sustaining Effects Data (b) Math



Source: Raudenbush (2006)

Figure A.21: Health and income for children and adults, U.S. National Health Interview Survey 1986–1995. From Case, A., Lubotsky, D. & Paxson, C. (2002), American Economic Review, Vol. 92, 1308-1334.

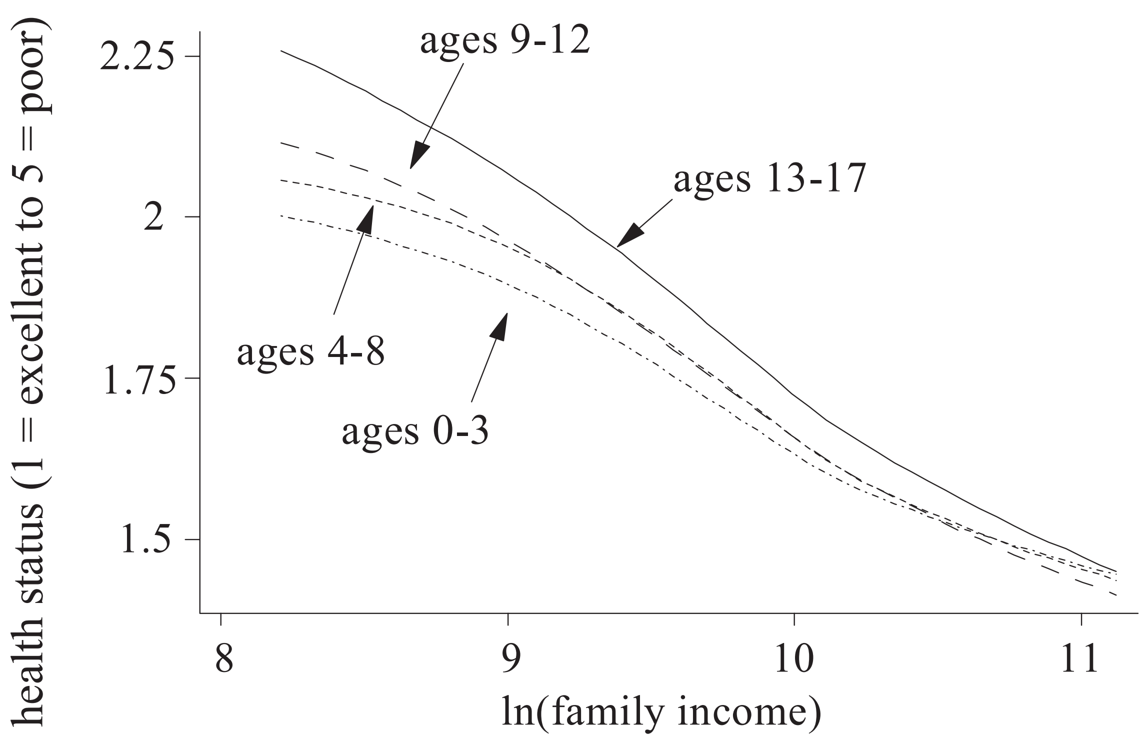


Table A.2: Gaps in HOME Scores between White and Black across Ages

(A)Females

Data	Age	Obs	Means White	Means Black	Differences(in s.d.)	p-value
CNLSY	0-3	2587	102.1	91.2	0.686	0.000
	4-7	3186	102.6	89.2	0.820	0.000
	8-11	3054	103.0	90.5	0.796	0.000
CDS 1997	0-3	276	16.1	14.3	0.769	0.000
	4-7	382	21.4	18.4	1.006	0.000
	8-11	321	22.1	19.8	0.841	0.000

(B)Males

Data	Age	Obs	Means White	Means Black	Differences(in s.d.)	p-value
CNLSY	0-3	2644	100.9	90.0	0.677	0.000
	4-7	3289	101.5	87.0	0.881	0.000
	8-11	3118	101.5	89.4	0.731	0.000
CDS 1997	0-3	250	15.5	14.5	0.415	0.002
	4-7	406	21.3	18.3	1.049	0.000
	8-11	337	22.0	20.0	0.741	0.000

Source: Moon (2014).
Notes:
(a) CNLSY is the Children of the National Longitudinal Survey of Youth
(b) CDS 1997 is the 1997 Child Development Supplement
(c) The total score of Home Observation Measurement of the Environment - Short Form (HOME-SF) is used. The standardized score and the raw score are used for CNLSY and CDS 1997, respectively.
(d) Racial gaps are divided by the standard deviation over the entire sample.
(e) P-values are obtained from t-test.

A.1 Children's Test Scores by Age And Mother's Education (CNLSY)

A.1.1 Methods

Test Score Definitions

1. **Raw Score** is the unadjusted total raw score.

2. **Sample Standardized Score** is the total raw score standardized at a particular age using the CNLSY estimation sample. This score is calculated by subtracting the mean of the score at the age and dividing by the standard deviation.

3. **Population Standardized Score** is a score that has been standardized at each age so that the mean is 100 and the standard deviation is 15 for a representative US sample. These norms are provided by the NLS and the year of the representative sample differs by test.

4. **Population Percentile** is a score that has been transformed so that it represents a percentile score at each age for a representative US sample. These norms are provided by the NLS and the year of the representative sample differs by test.

Data Notes As discussed in the National Longitudinal Survey's Topical Guide to the Data, the population norms for the Peabody Individual Achievement Test (PIAT) Reading Comprehension test are unreliable under age 7. For this reason, these ages are excluded from the analysis.

The Behavior Problems Index total score provided by NLS appears to add subscores in a way that does not appropriately account for missing values. In particular, some of the questions of the BPI only apply to children who are in school. Children not enrolled in school appeared to receive a "positive" score on this field, and this score counted towards their overall score. Therefore, children who enrolled in school at earlier ages appeared to have worse overall scores. This bias made it seem that children from well-educated mothers had worse behavioral problems at young ages. To account for this bias, the BPI total score is calculated by averaging across the questions without missing values and multiplying the average by the number of questions without missing values.

Mother's Education Due to the sparsity of the data, several of the educational categories have been collapsed to the following four categories:

1. **Dropout/GED** includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education.

2. **High School Graduate** includes high school graduates who have not attempted college.

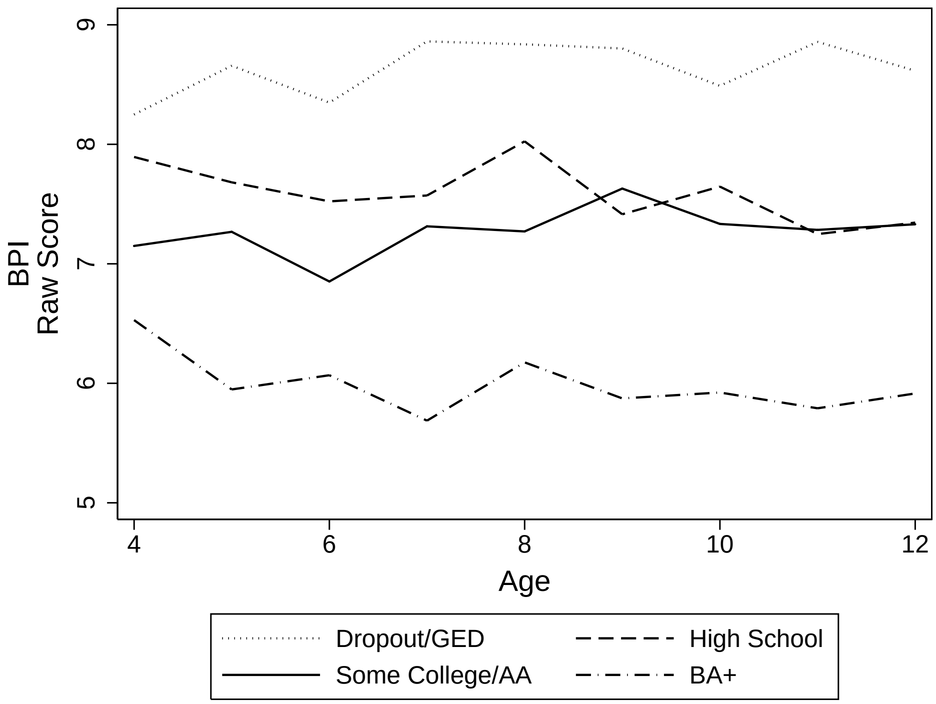
3. **Some College/AA** includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college appear in this category.

4. **BA+** includes anyone has earned a BA degree or more. GED recipients who earn BA degrees appear in this category.

A.1.2 Results

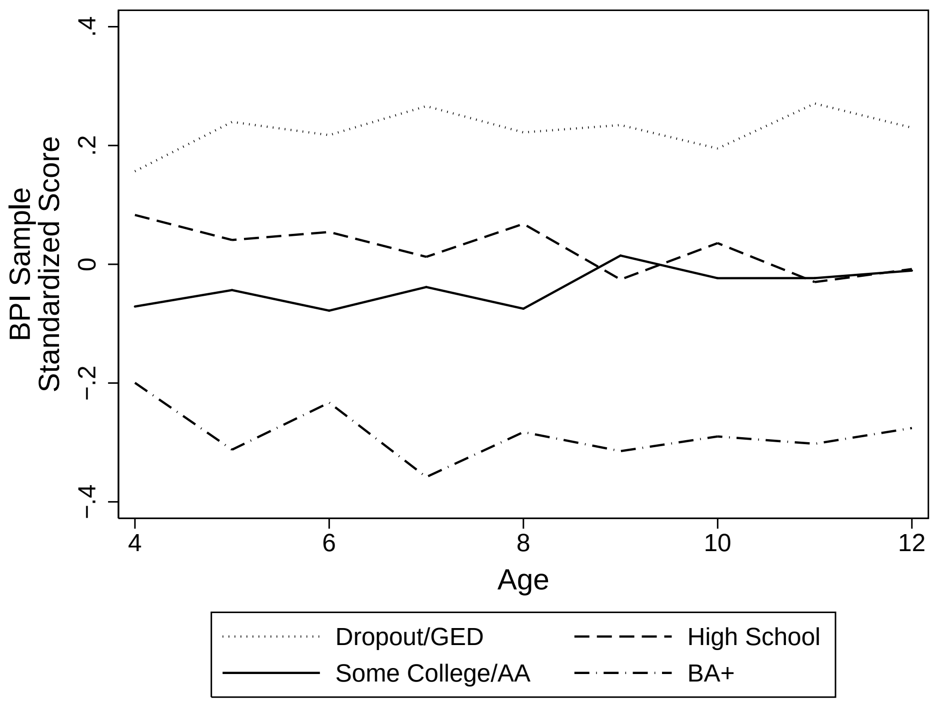
BPI

Figure A.22: Raw Behavioral Problems Index (BPI) Scores by Age and Mother's Education at Birth



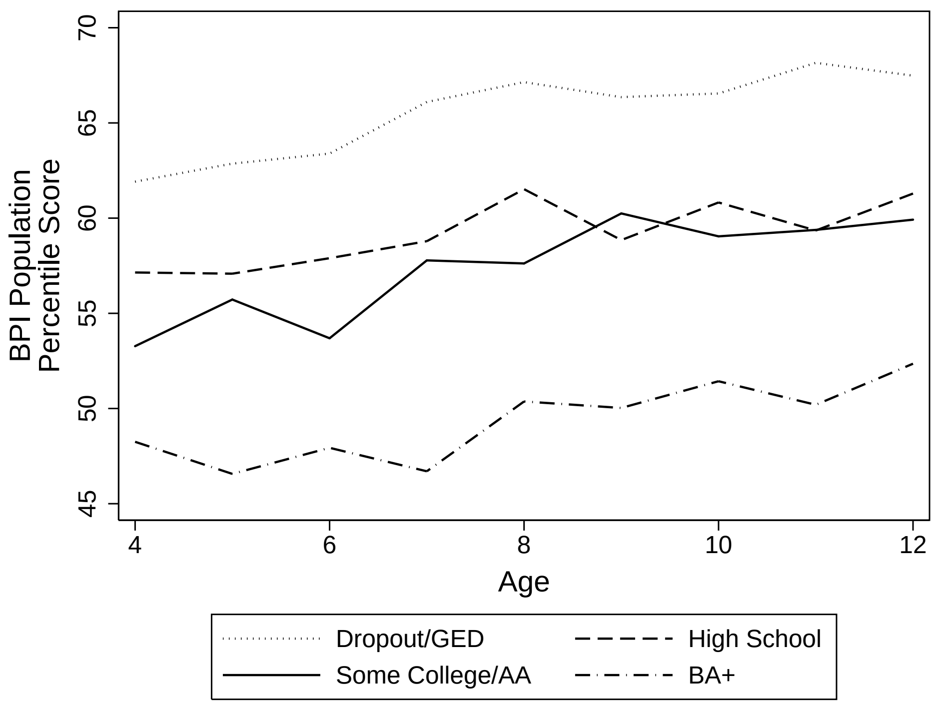
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. Higher scores on the BPI indicate more behavior problems.

Figure A.23: Sample Standardized Behavior Problems Index (BPI) Scores by Age and Mother's Education at Birth



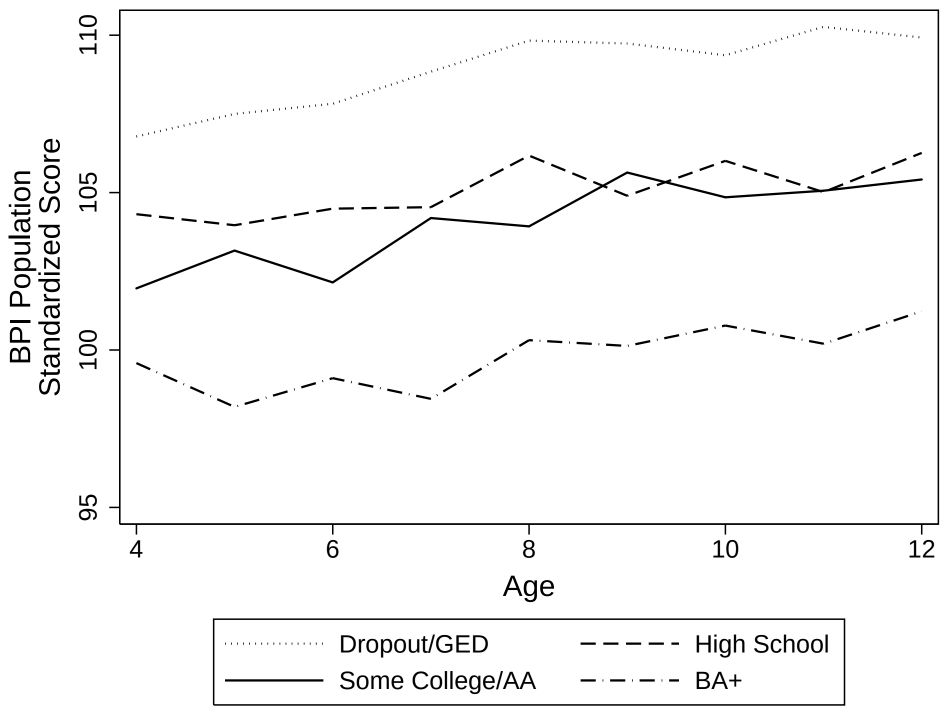
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. Higher scores on the BPI indicate more behavior problems.

Figure A.24: Population Percentile Behavior Problems Index (BPI) Scores by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. Higher scores on the BPI indicate more behavior problems. The scores are normed based on a representative sample of the US in 1981.

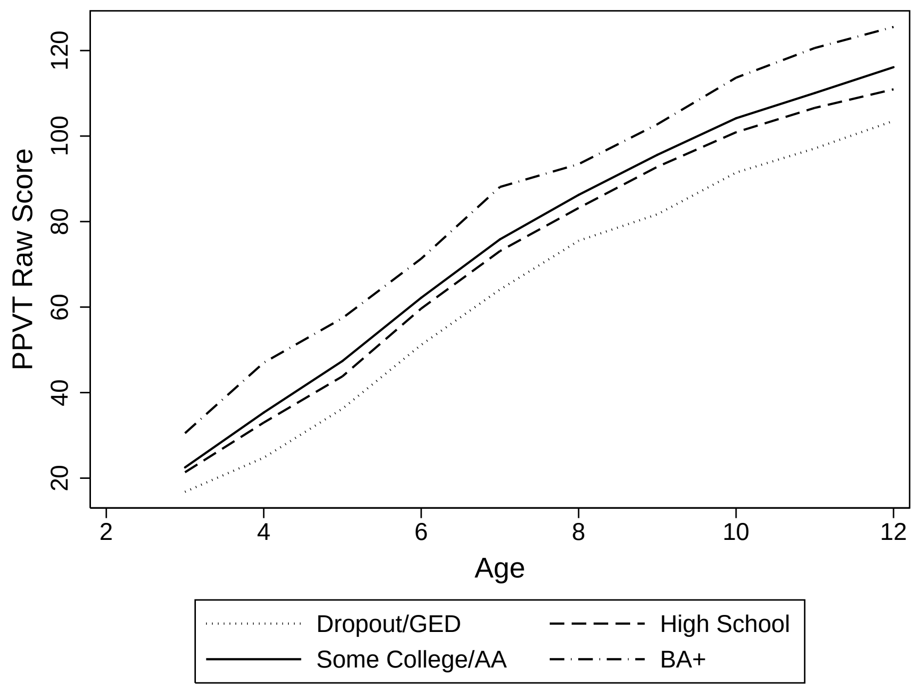
Figure A.25: Population Standardized Behavior Problems Index (BPI) Scores by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. Higher scores on the BPI indicate more behavior problems. The scores are normed based on a representative sample of the US in 1981.

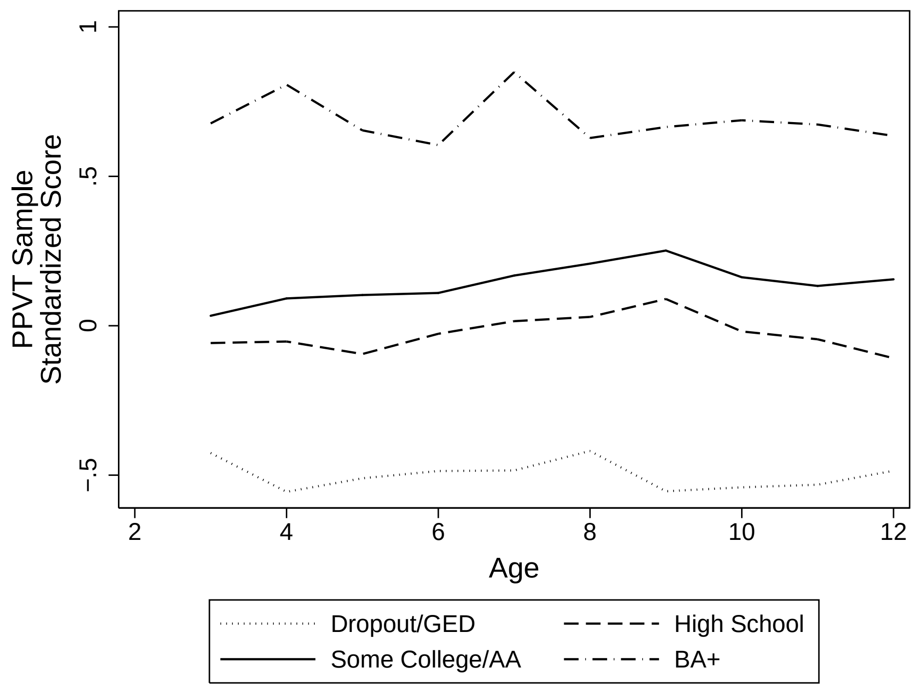
A.1.3 PPVT

Figure A.26: Raw Peabody Picture Vocabulary Test (PPVT) Scores by Age and Mother's Education at Birth



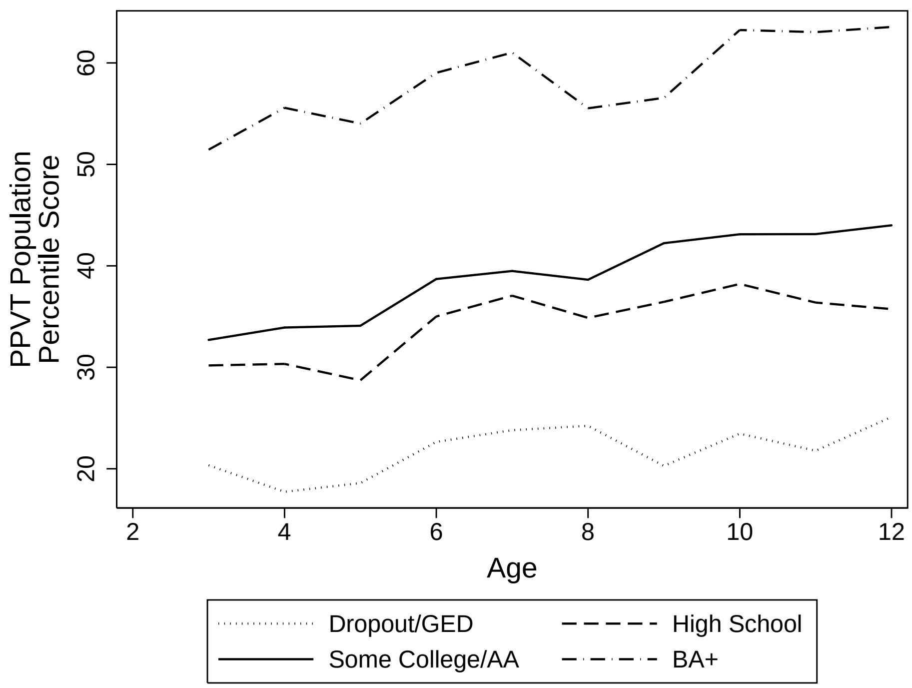
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.27: Sample Standardized Peabody Picture Vocabulary Test (PPVT) Scores by Age and Mother's Education at Birth



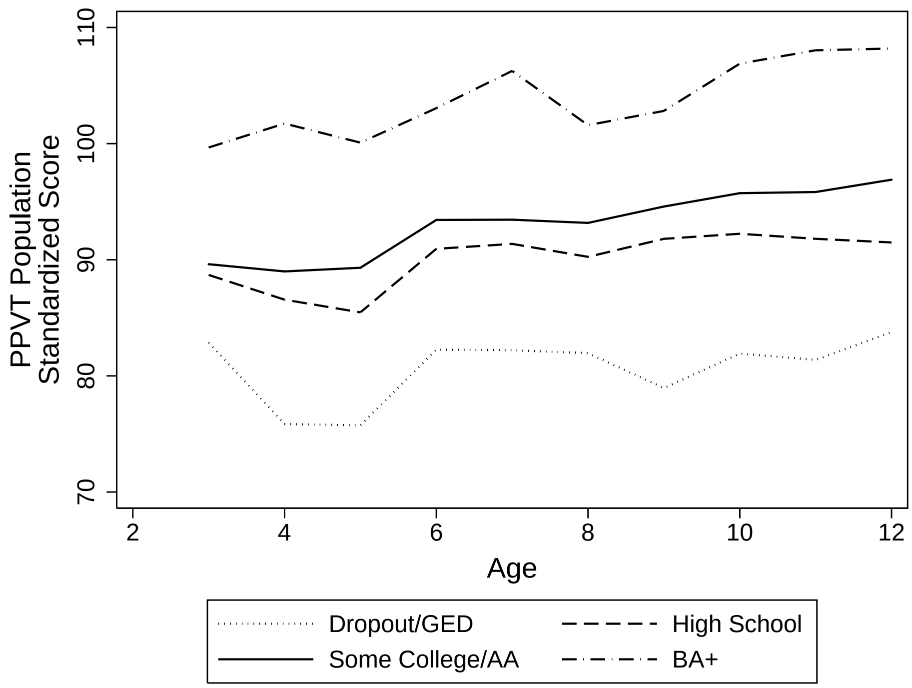
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.28: Population Percentile Peabody Picture Vocabulary Test (PPVT) Scores by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. The scores are normed based on a representative sample of the US in 1979.

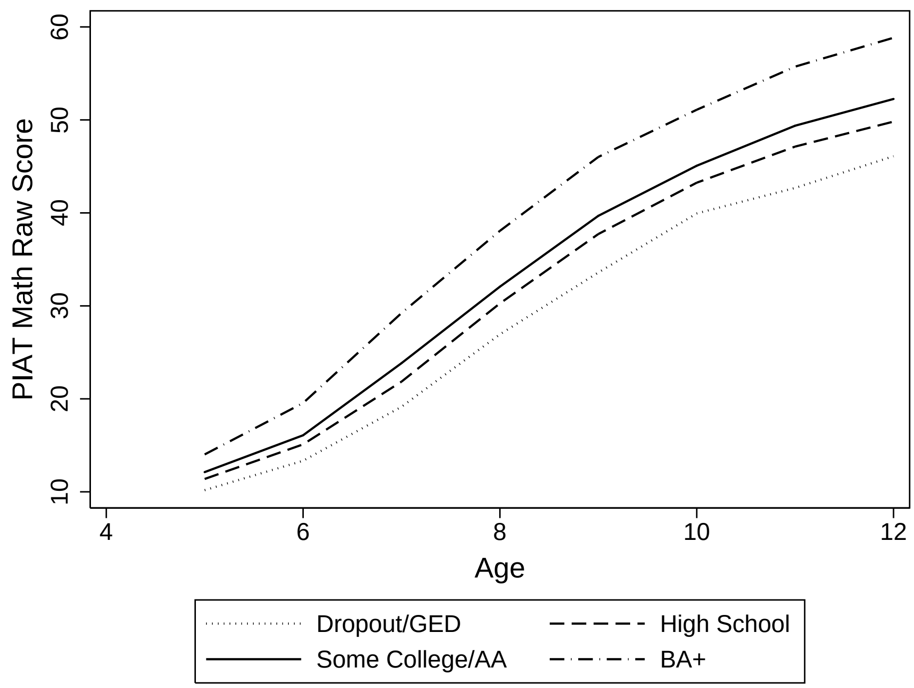
Figure A.29: Population Standardized Peabody Picture Vocabulary Test (PPVT) Scores by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. Higher scores on the BPI indicate more behavior problems. The scores are normed based on a representative sample of the US in 1979.

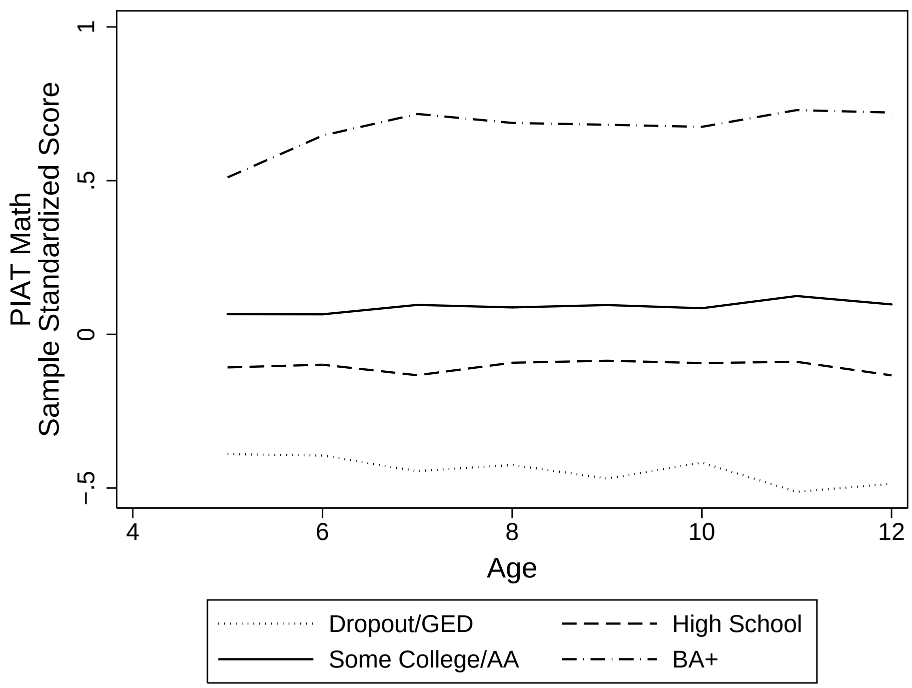
A.1.4 PIAT Math

Figure A.30: Raw Peabody Individual Achievement Test (PIAT) Math Scores by Age and Mother's Education at Birth



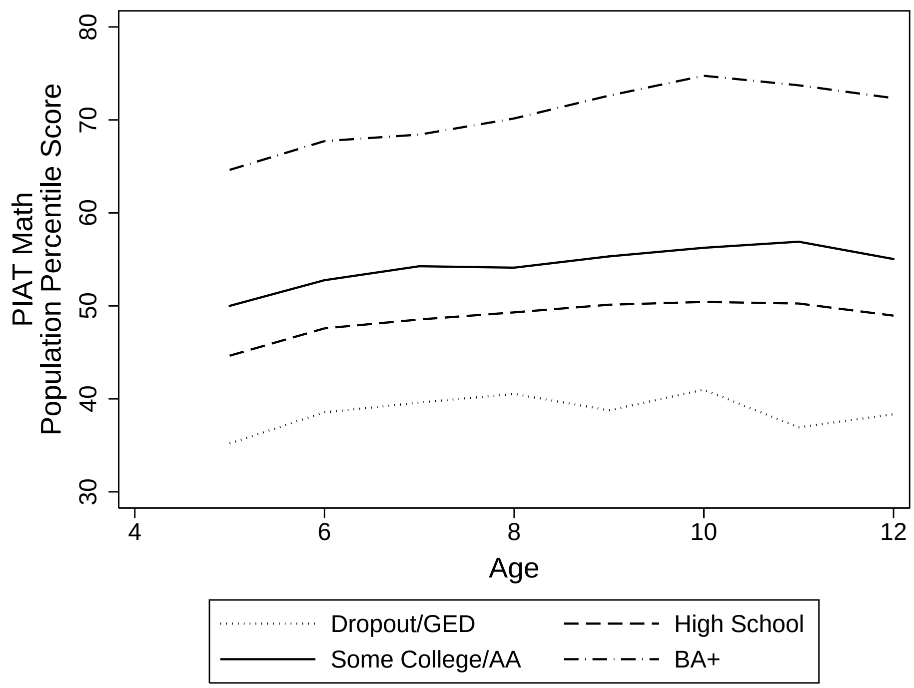
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.31: Sample Standardized Peabody Individual Achievement Test (PIAT) Math Scores by Age and Mother's Education at Birth



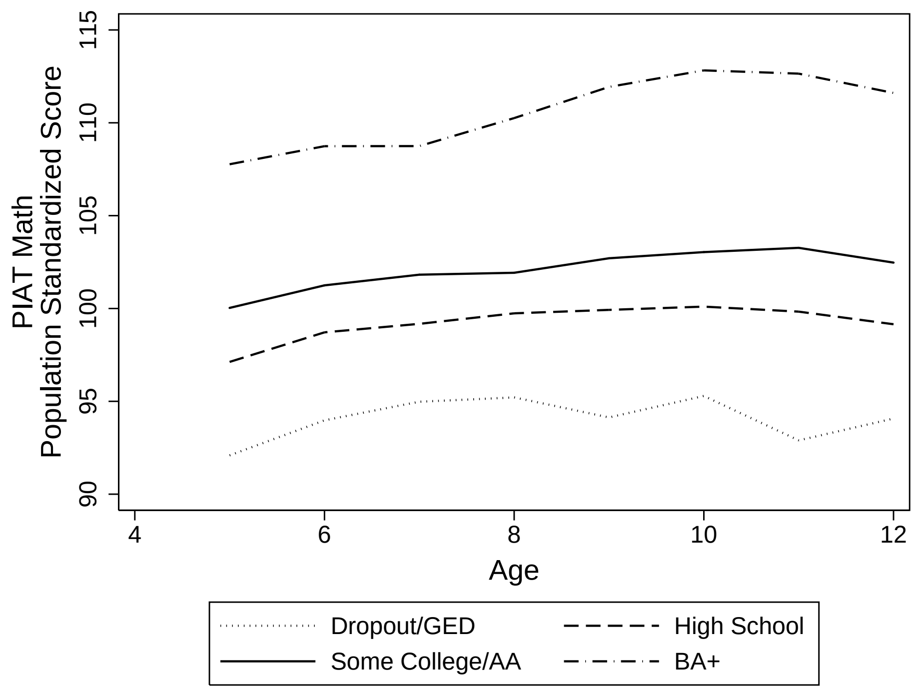
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.32: Population Percentile Peabody Individual Achievement Test (PIAT) Math by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. The scores are normed based on a representative sample of the US in 1968.

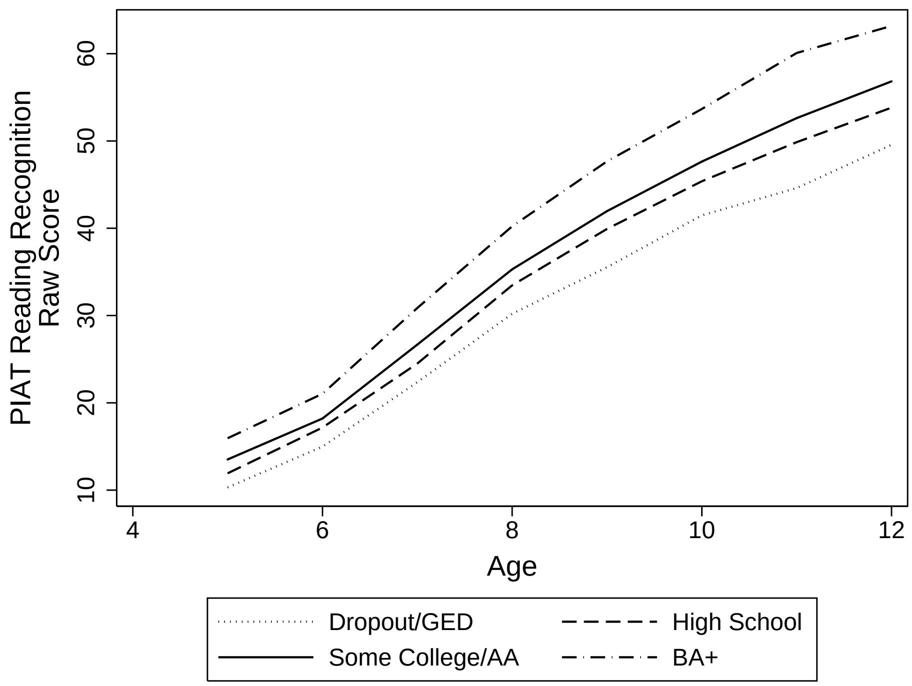
Figure A.33: Population Standardized Peabody Individual Achievement Test (PIAT) Math Scores by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. Higher scores on the BPI indicate more behavior problems. The scores are normed based on a representative sample of the US in 1968.

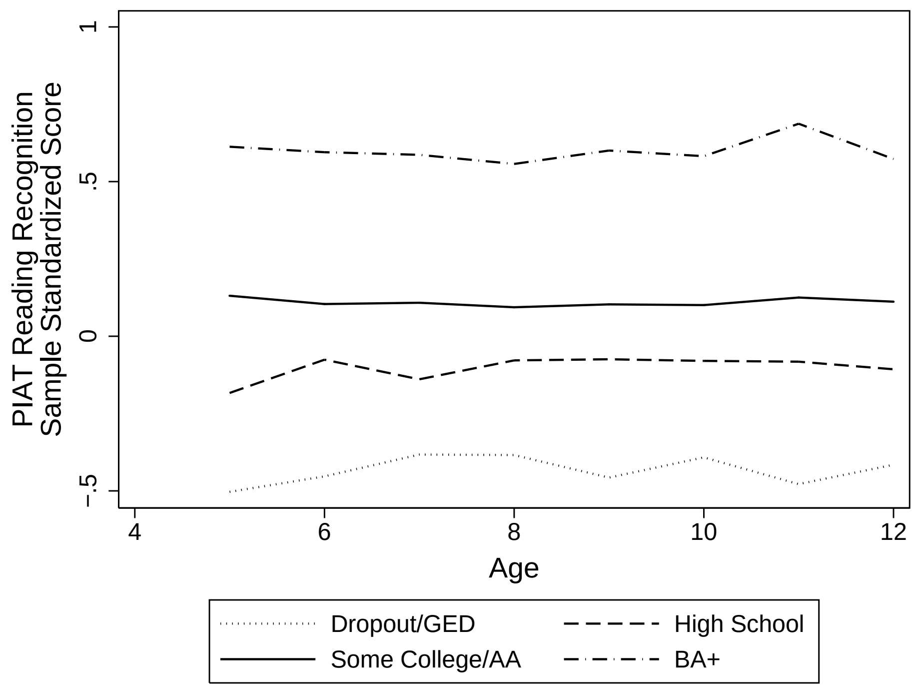
A.1.5 PIAT Reading Recognition

Figure A.34: Raw Peabody Individual Achievement Test (PIAT) Reading Recognition Scores by Age and Mother's Education at Birth



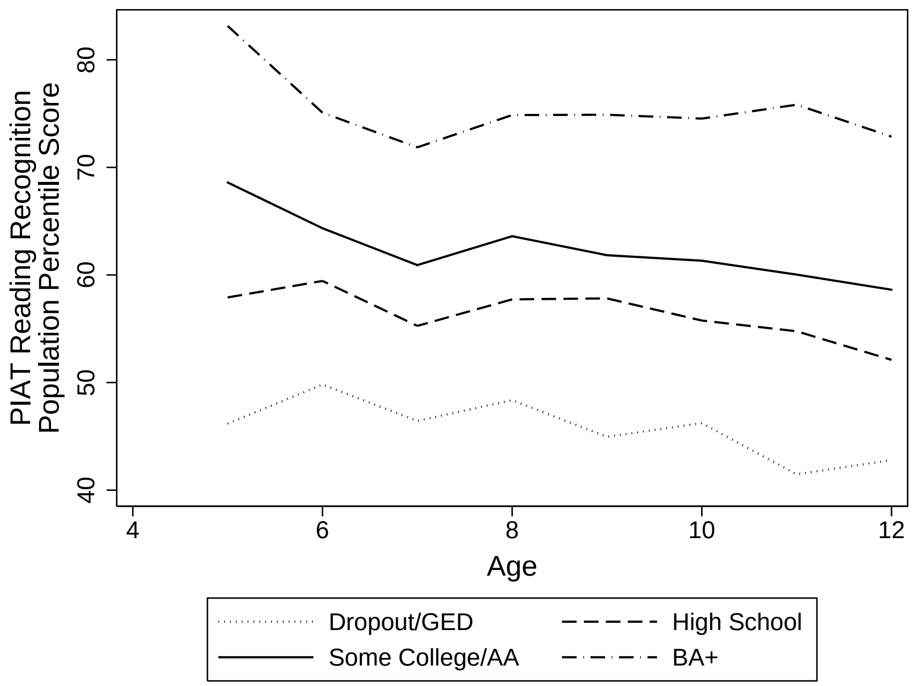
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.35: Sample Standardized Peabody Individual Achievement Test (PIAT) Reading Recognition Scores by Age and Mother's Education at Birth



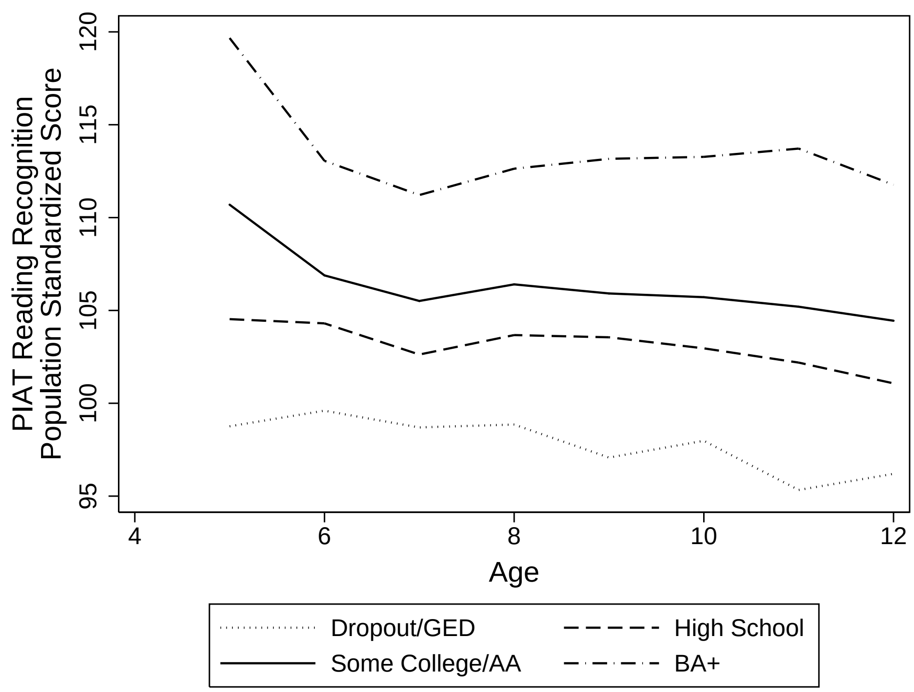
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.36: Population Percentile Peabody Individual Achievement Test (PIAT) Reading Recognition by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. The scores are normed based on a representative sample of the US in 1968.

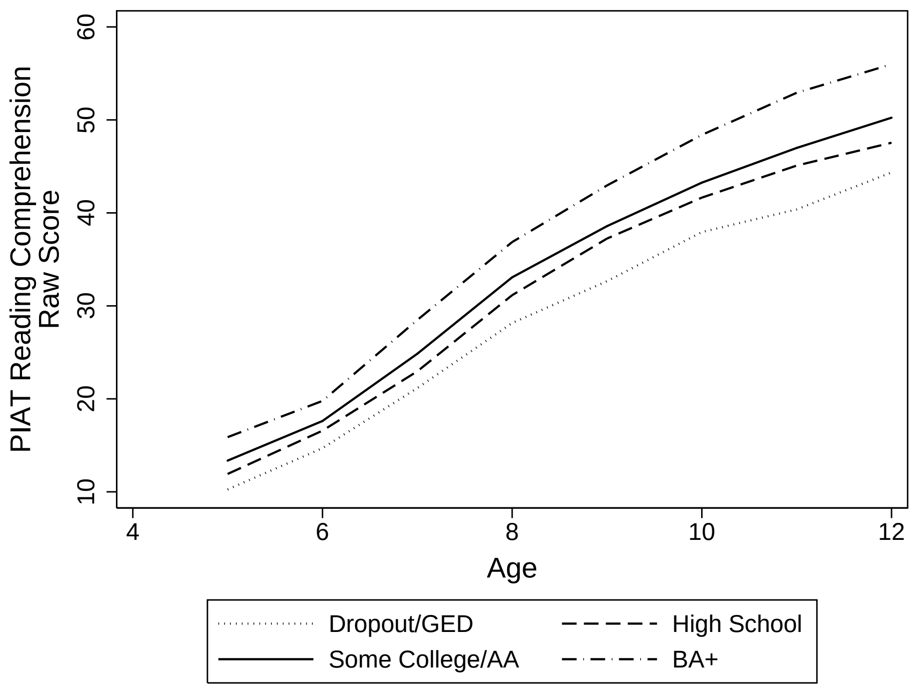
Figure A.37: Population Standardized Peabody Individual Achievement Test (PIAT) Reading Recognition Scores by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. The scores are normed based on a representative sample of the US in 1968.

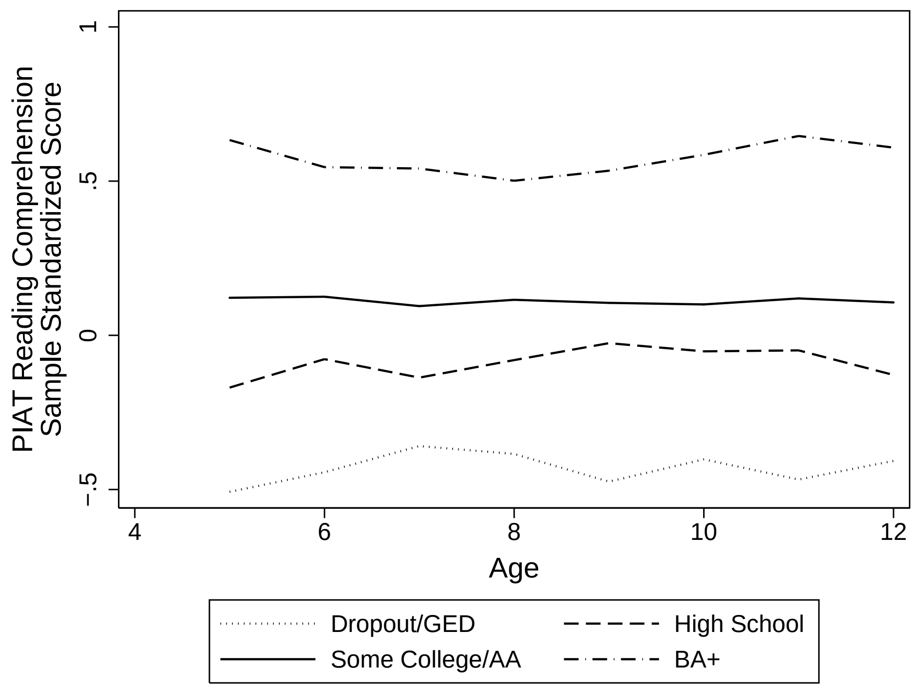
A.1.6 PIAT Reading Comprehension

Figure A.38: Raw Peabody Individual Achievement Test (PIAT) Reading Comprehension Scores by Age and Mother's Education at Birth



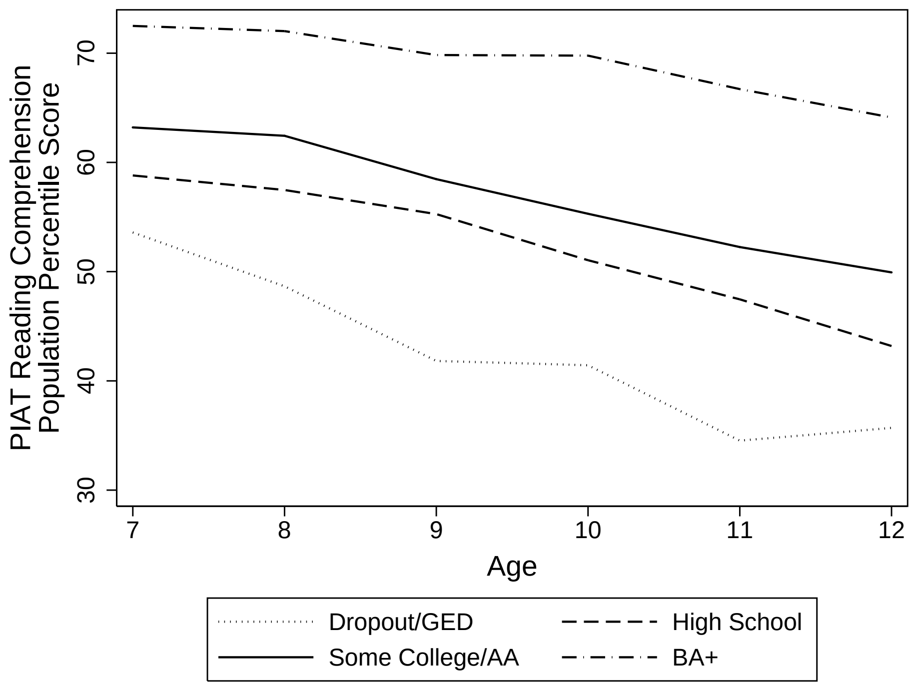
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.39: Sample Standardized Peabody Individual Achievement Test (PIAT) Reading Comprehension Scores by Age and Mother's Education at Birth



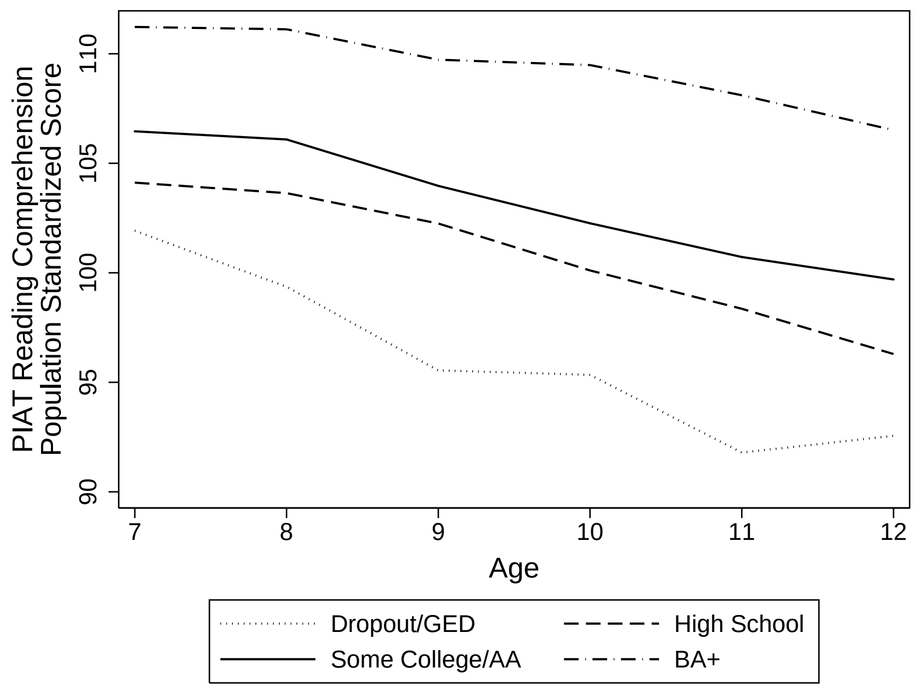
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category.

Figure A.40: Population Percentile Peabody Individual Achievement Test (PIAT) Reading Comprehension by Age and Mother's Education at Birth



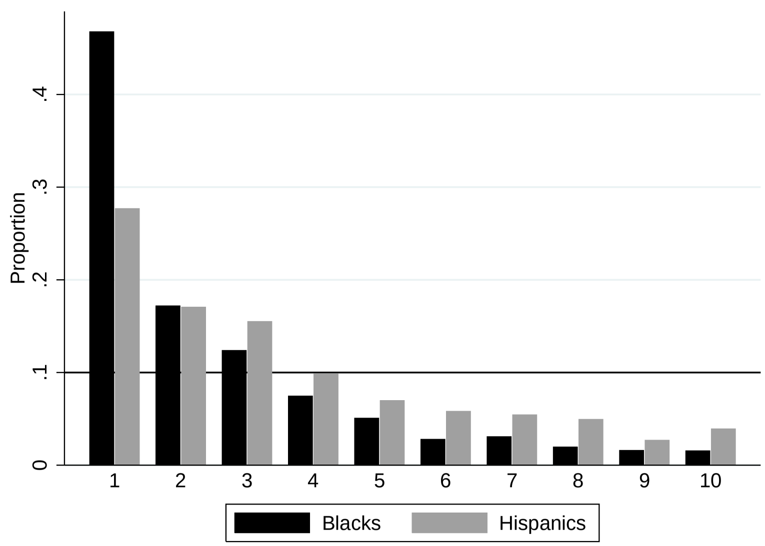
Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. The scores are normed based on a representative sample of the US in 1968.

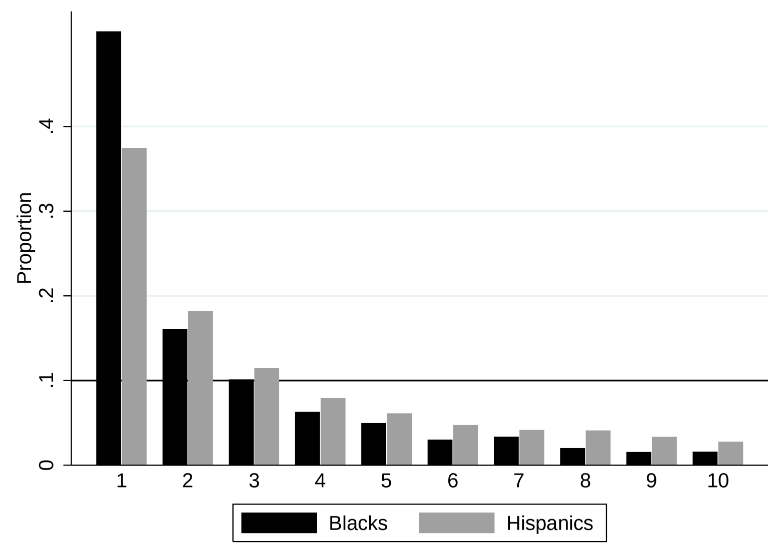
Figure A.41: Population Standardized Peabody Individual Achievement Test (PIAT) Reading Comprehension Scores by Age and Mother's Education at Birth



Source: Children of the National Longitudinal Survey of Youth (CNLSY). Notes: Mother's education is measured at the time of the child's birth. "Dropout/GED" includes anyone who has dropped out of high school or earned a GED but has not attempted further post-secondary education. "High School" includes high school graduates who have not attempted college. "Some College/AA" includes anyone who has ever attended a 2- or 4-year college or earned an associate's degree (AA) but has not earned a bachelor's degree or more. GED recipients who attempt college are placed in this category. "BA+" includes anyone has earned a BA degree or more. GED recipients who earn BA degrees are in this category. The scores are normed based on a representative sample of the US in 1968.

Figure A.42: Minority AFQT Scores Placed in the White Distribution—Males (left) and Females (right)



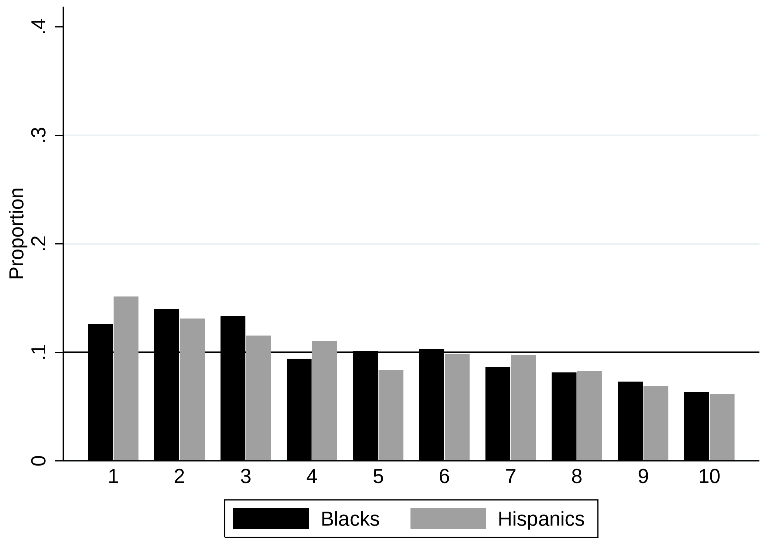


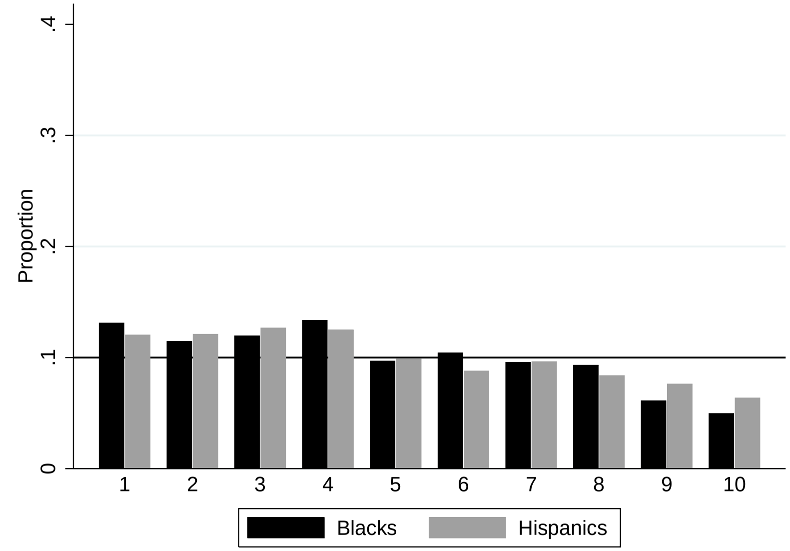
Heckman (2011, Web Appendix).

Notes: Because individuals are at different ages when given the AFQT, the scores have been adjusted to reflect an estimated value at the time just prior to high school using the method described in Heckman et al. (2011).

Comparison of Rotter Locus of Control Distributions

Figure A.1: Minority Rotter Scores Placed in the White Distribution - Males (left) and Females (right)



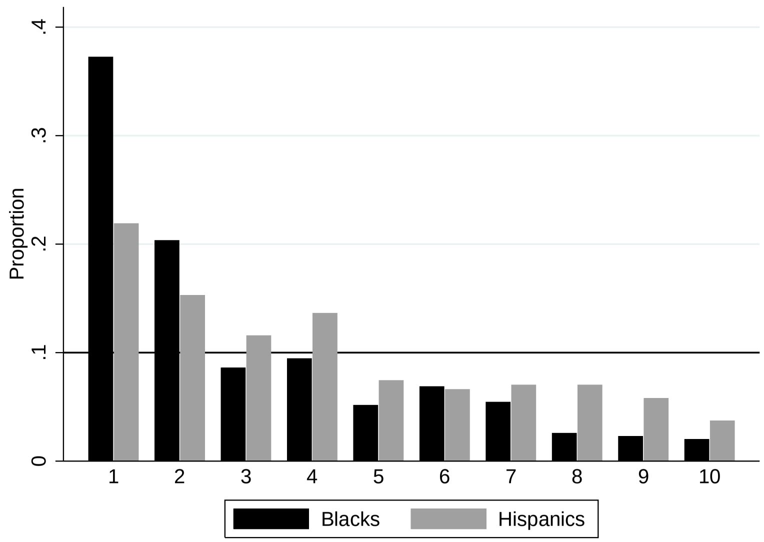


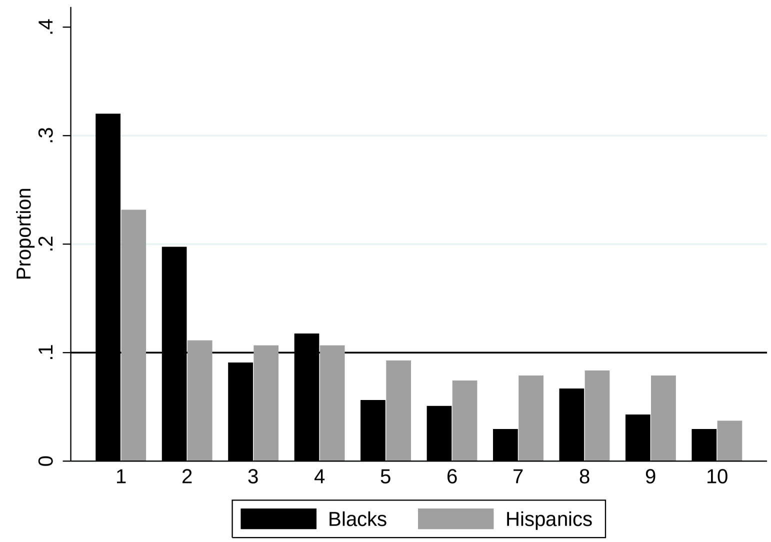
Heckman (2011, Web Appendix).

Notes: Because individuals are at different ages when given the Rotter Locus of Control assessment, the scores have been adjusted to reflect an estimated value at the time just prior to high school using the method described in Heckman et al. (2011).

Comparison of PIAT Distributions

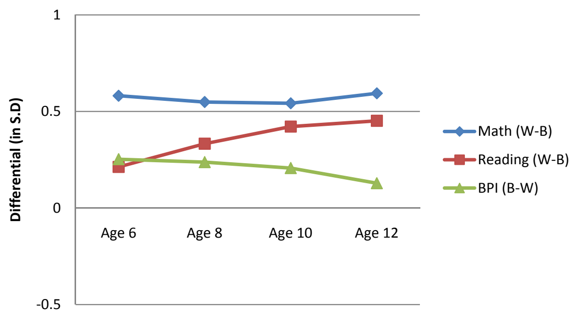
Figure A.2: Minority PIAT Scores Placed in the White Distribution - Males (left) and Females (right)



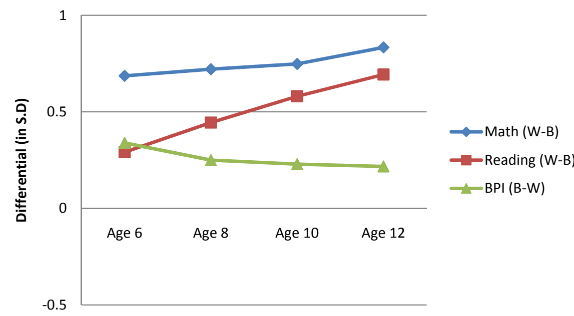


Heckman (2011, Web Appendix).

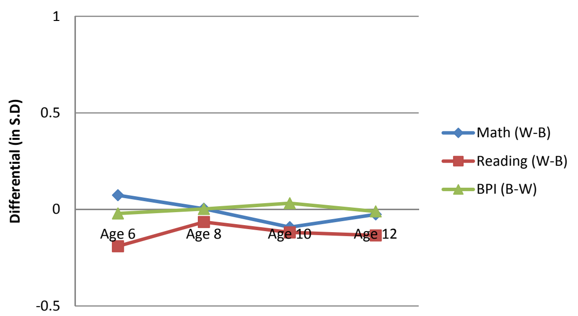
Figure A.3: Black-White Gaps in Skill Measures over Ages



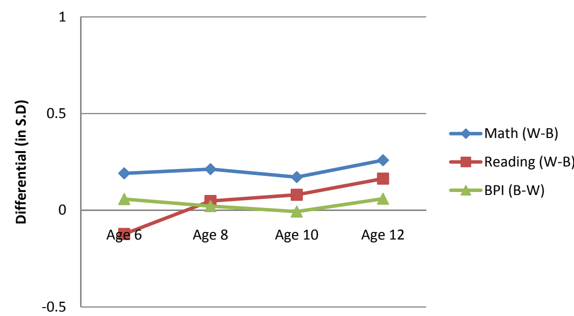
(a) Girls: Scores



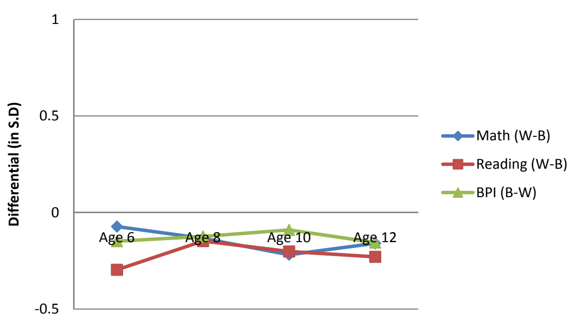
(b) Boys: Scores



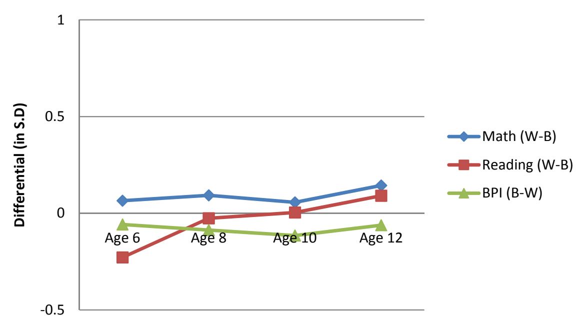
(c) Girls: Residuals (1) - w/o HOME



(d) Boys: Residuals (1) - w/o HOME



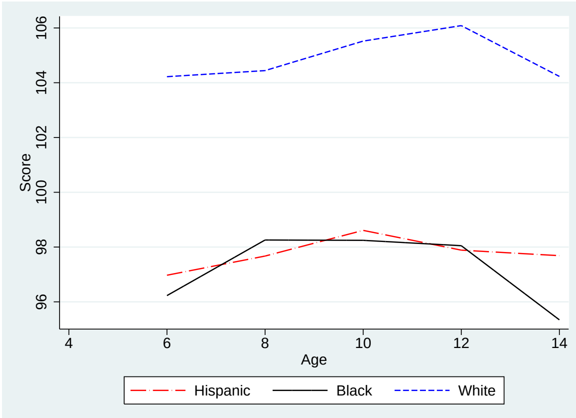
(e) Girls: Residuals (2) - w/ HOME



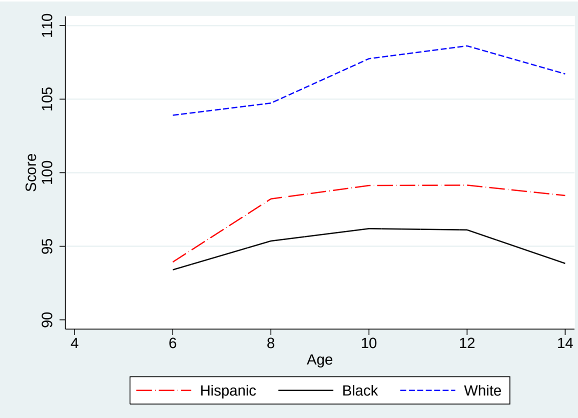
(f) Boys: Residuals (2) - w/ HOME

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Note: (a) Skill measures are standardized scores of PIAT Math and Reading, and Behavior Problem Index (BPI);
(b) Residuals (1) are taken from a regression of skill measures on mother's AFQT, mother's highest grade completed, family income averaged over the whole childhood (from birth to age 15), and a dummy indicator for whether a child was born to an "intact" family. An "intact" family is defined as a family headed by a couple in wedlock who both are the kid's biological parents.
(c) Residuals (2) are taken from another regression with three types of parental investment (material resource, cognitive stimulation, and emotional support) in the kid's early childhood (from birth to age 8) estimated by a factor analysis using all individual indicators in HOME-SF Inventory.
Source: Moon (2014).

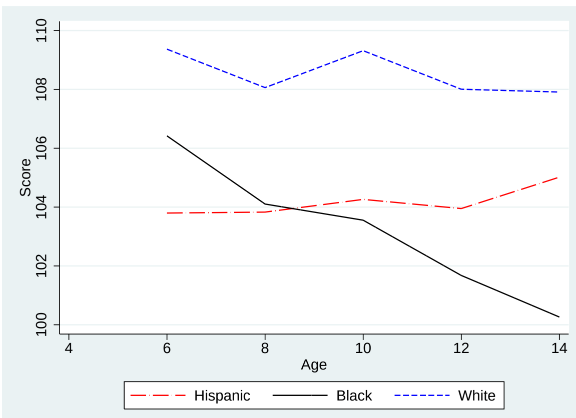
Figure A.4: Skill Measures over Childhood across Ethnic Groups



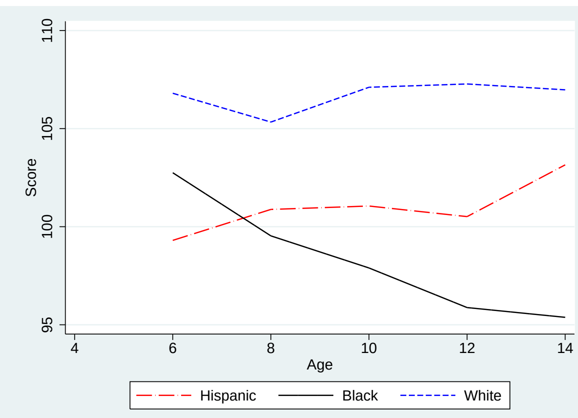
(a) Girls: Math Score (standardized)



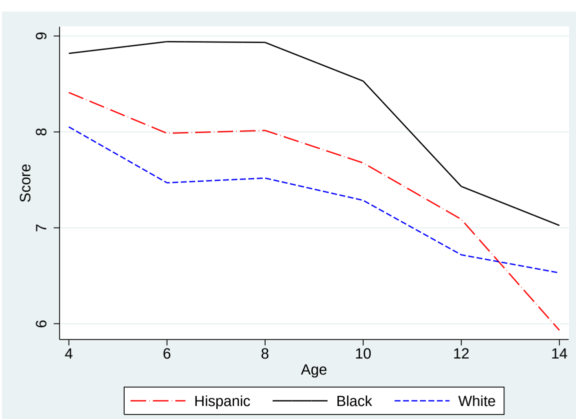
(b) Boys: Math Score (standardized)



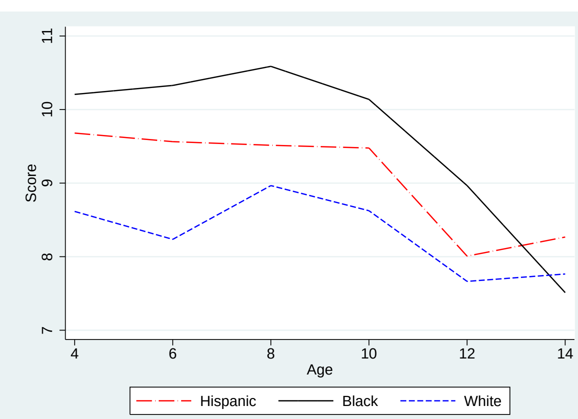
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



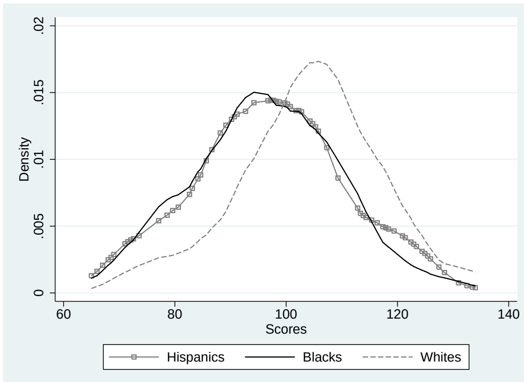
(e) Girls: BPI (Raw score)



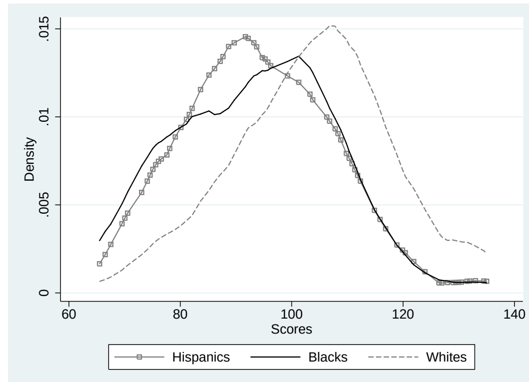
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

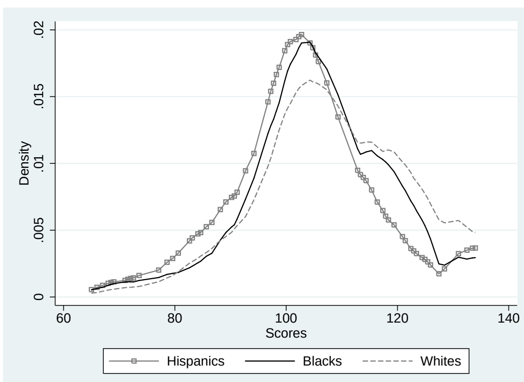
Figure A.5: Distribution of Skill Measures across Ethnic Groups: Age 6



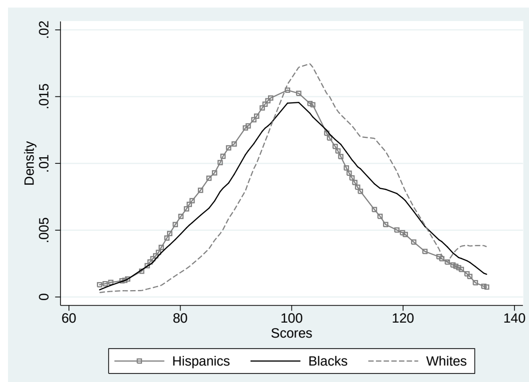
(a) Girls: Math Score (standardized)



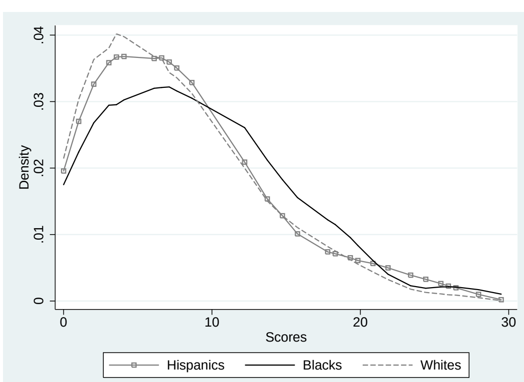
(b) Boys: Math Score (standardized)



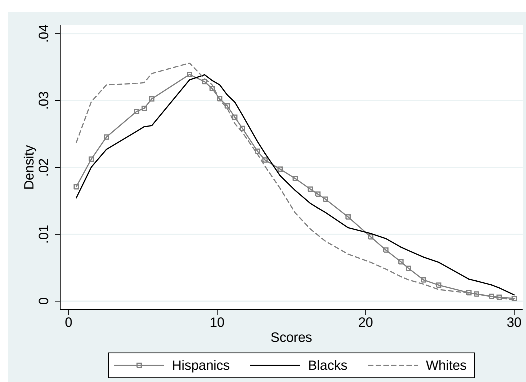
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



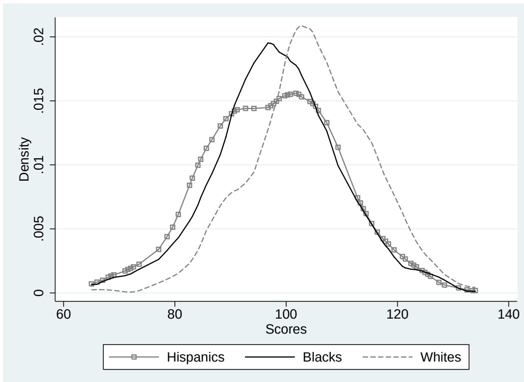
(e) Girls: BPI (Raw score)



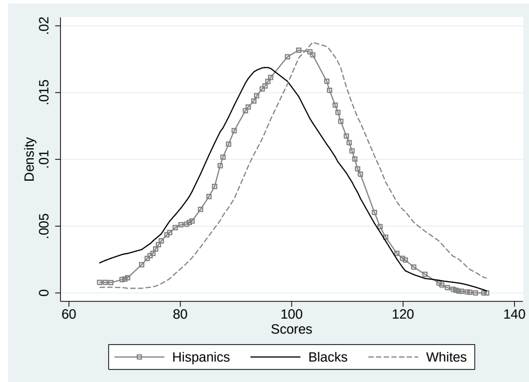
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

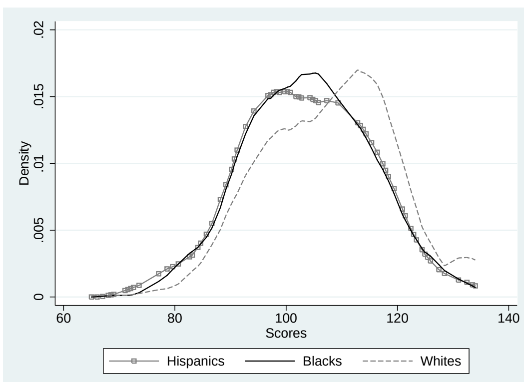
Figure A.6: Distribution of Skill Measures across Ethnic Groups: Age 8



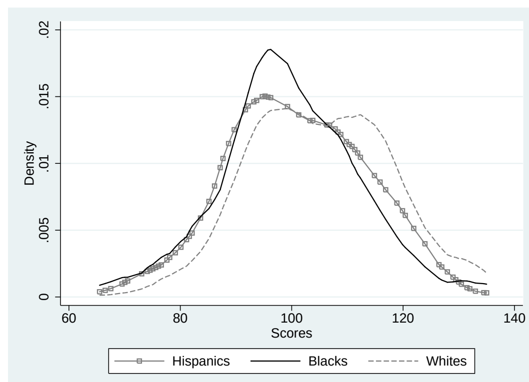
(a) Girls: Math Score (standardized)



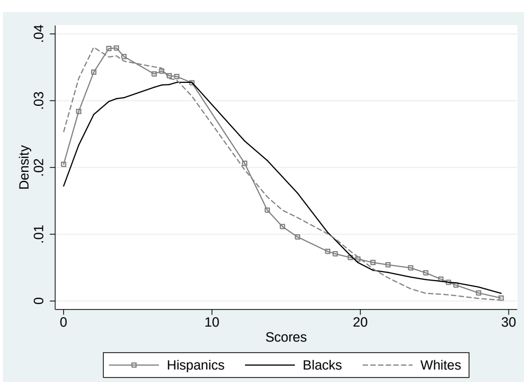
(b) Boys: Math Score (standardized)



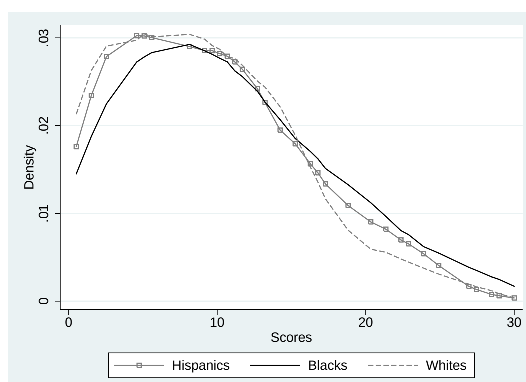
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



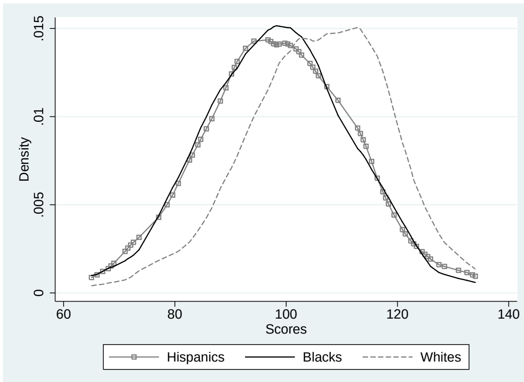
(e) Girls: BPI (Raw score)



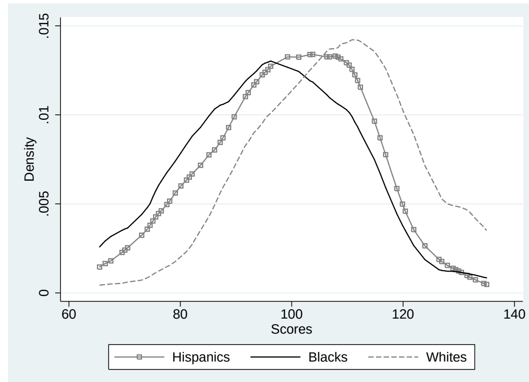
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

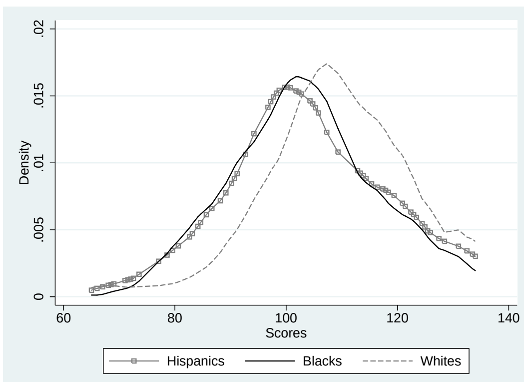
Figure A.7: Distribution of Skill Measures across Ethnic Groups: Age 10



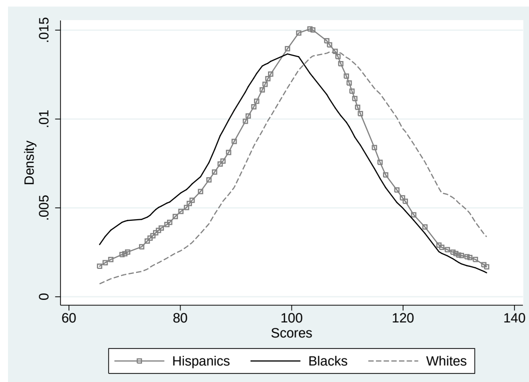
(a) Girls: Math Score (standardized)



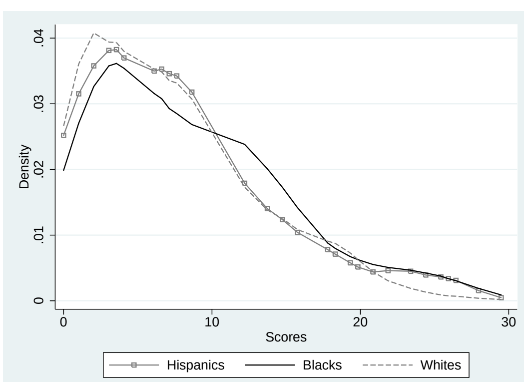
(b) Boys: Math Score (standardized)



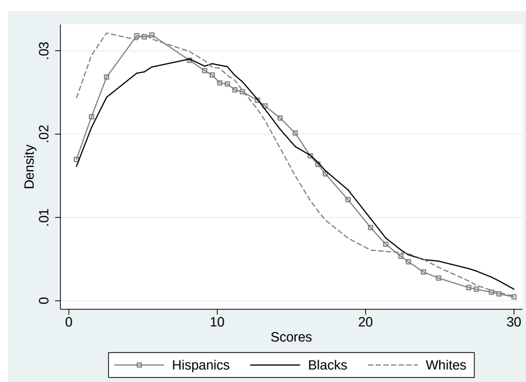
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



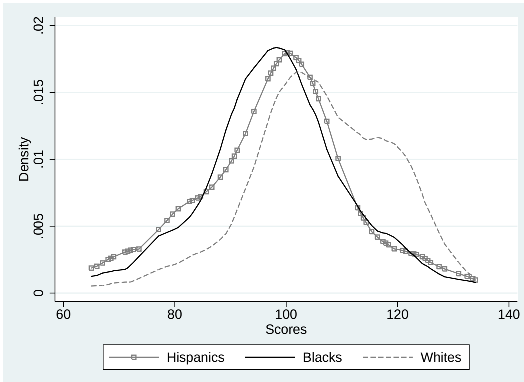
(e) Girls: BPI (Raw score)



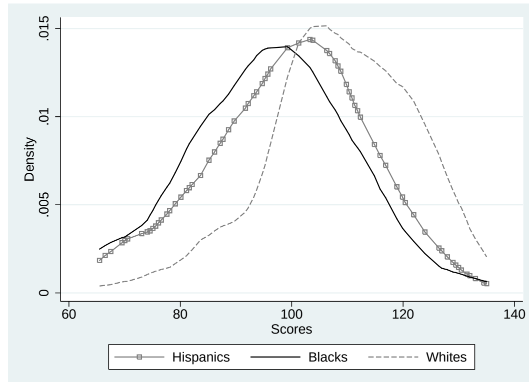
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

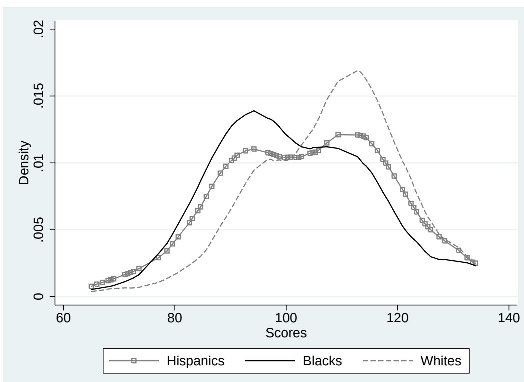
Figure A.8: Distribution of Skill Measures across Ethnic Groups: Age 12



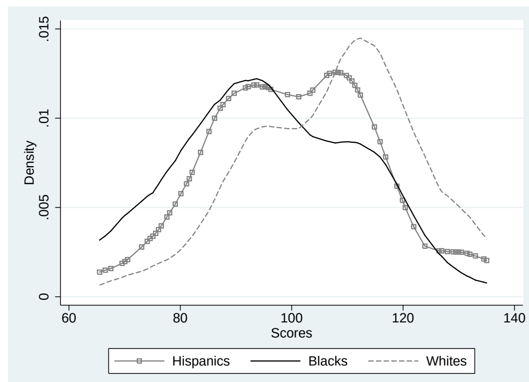
(a) Girls: Math Score (standardized)



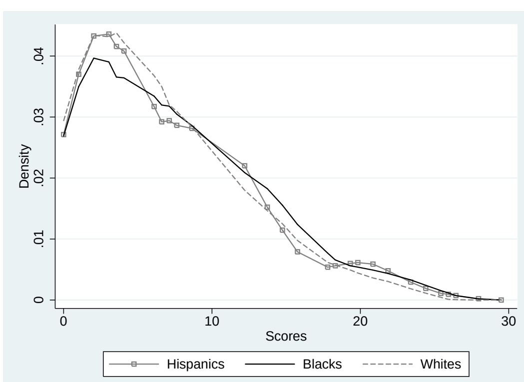
(b) Boys: Math Score (standardized)



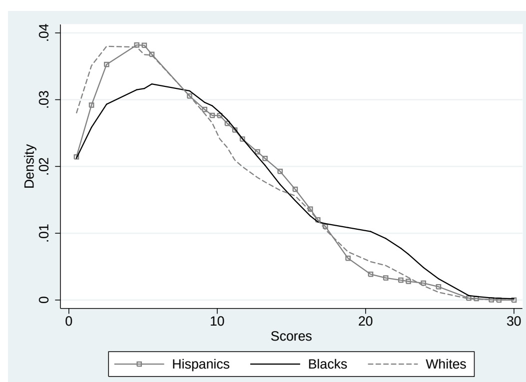
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



(e) Girls: BPI (Raw score)



(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

A.2 Ability Comparisons by Parent Characteristics and Investments

Differences in Academic Ability by Race and Socioeconomic Status - NLSY79 and CNLSY

Table A.3: Comparison of Within-Race AFQT Gaps Across Socioeconomic Status - NLSY79 - Males and Females

	Average AFQT Score						Across-Race Difference			
	Whites		Blacks		Hispanics		W-B Gap		W-H Gap	
	Avg	SE	Avg	SE	Avg	SE	Diff	SE	Diff	SE
Unconditional AFQT Averages	0.52	(0.88)	-0.55	(0.87)	-0.16	(0.92)	1.07	(0.04)	0.68	(0.05)
Mother's Educational Status										
Mother is a dropout	0.11	(0.92)	-0.70	(0.75)	-0.33	(0.88)	0.81	(0.05)	0.44	(0.06)
Mother is a high school graduate	0.60	(0.81)	-0.36	(0.89)	0.22	(0.94)	0.96	(0.06)	0.38	(0.12)
Mother is a college graduate or more	0.91	(0.77)	0.01	(0.98)	0.70	(0.68)	0.90	(0.19)	0.21	(0.16)
Difference: college graduate - dropout	0.80	(1.20)	0.71	(1.24)	1.03	(1.12)	0.09	(0.19)	-0.23	(0.17)
Family Income										
Family income from 1979 in bottom tercile	0.28	(0.93)	-0.66	(0.82)	-0.38	(0.90)	0.94	(0.05)	0.66	(0.07)
Family income from 1979 in middle tercile	0.50	(0.85)	-0.40	(0.88)	-0.02	(0.90)	0.90	(0.08)	0.52	(0.11)
Family income from 1979 in top tercile	0.72	(0.82)	-0.16	(0.86)	0.36	(0.83)	0.88	(0.11)	0.36	(0.12)
Difference: top - bottom tercile	0.44	(1.24)	0.50	(1.19)	0.74	(1.22)	-0.06	(0.12)	-0.30	(0.14)
Family Structure										
Child raised in broken home	0.29	(0.91)	-0.54	(0.89)	-0.24	(0.88)	0.83	(0.06)	0.53	(0.09)
Child raised in intact home	0.58	(0.86)	-0.56	(0.84)	-0.12	(0.95)	1.14	(0.05)	0.70	(0.06)
Difference: intact - broken	0.29	(1.26)	-0.02	(1.23)	0.12	(1.29)	0.31	(0.08)	0.17	(0.11)

Source: Heckman (2011, Web Appendix).

Notes: AFQT is measured in 1979 when individuals are aged 14-21. To account for the differences in AFQT due to schooling and other growth due to aging, AFQT measures are the "post-school" constructions are calculated as described in Heckman et al. (2011). "SE" columns show both standard deviations of ability, and calculations of the standard error of the difference of sample means.

Table A.4: Comparison of Within-Race PIAT Gaps Across Socioeconomic Status - CNLSY - Males and Females

	Average PIAT Score						Across-Race Difference			
	Whites		Blacks		Hispanics		W-B Gap		W-H Gap	
	Avg	SE	Avg	SE	Avg	SE	Diff	SE	Diff	SE
Unconditional PIAT Averages	0.30	(0.93)	-0.45	(0.96)	-0.11	(0.94)	0.75	(0.04)	0.41	(0.05)
Mother's Educational Status										
Mother is a dropout	-0.28	(0.94)	-0.97	(0.83)	-0.50	(0.94)	0.69	(0.11)	0.22	(0.12)
Mother is a high school graduate	0.16	(0.89)	-0.46	(0.97)	-0.14	(0.89)	0.62	(0.07)	0.30	(0.08)
Mother is a college graduate	0.81	(0.83)	0.07	(0.87)	0.34	(0.80)	0.74	(0.10)	0.47	(0.12)
Difference: College Graduate - Dropout	1.09	(1.25)	1.04	(1.20)	0.84	(1.24)	0.05	(0.15)	0.25	(0.17)
Mother's AFQT										
Mother's AFQT is in the bottom tercile	-0.39	(0.92)	-0.76	(0.86)	-0.40	(0.91)	0.37	(0.09)	0.01	(0.10)
Mother's AFQT is in the middle tercile	0.07	(0.84)	-0.07	(0.91)	0.03	(0.84)	0.14	(0.07)	0.04	(0.08)
Mother's AFQT is in the top tercile	0.59	(0.87)	0.44	(0.93)	0.58	(0.83)	0.15	(0.14)	0.01	(0.11)
Difference: Top - Bottom Tercile	0.98	(1.26)	1.20	(1.26)	0.98	(1.23)	-0.22	(0.16)	0.00	(0.14)
Family Income										
Average family income in 1st quartile	-0.26	(1.10)	-0.77	(0.88)	-0.44	(1.00)	0.51	(0.11)	0.18	(0.13)
Average family income in 2nd quartile	0.10	(0.86)	-0.36	(0.89)	-0.14	(0.89)	0.46	(0.08)	0.24	(0.09)
Average family income in 3rd quartile	0.27	(0.87)	-0.07	(0.94)	-0.04	(0.84)	0.34	(0.10)	0.31	(0.09)
Average family income in 4th quartile	0.64	(0.84)	0.23	(1.03)	0.39	(0.82)	0.41	(0.14)	0.25	(0.10)
Difference: Top - Bottom Quartile	0.90	(1.39)	1.00	(1.36)	0.83	(1.29)	-0.10	(0.17)	0.07	(0.16)
Family Structure										
Single parent, never married	-0.06	(0.94)	-0.59	(0.94)	-0.20	(0.93)	0.53	(0.09)	0.14	(0.12)
Broken or blended family	0.14	(0.89)	-0.43	(0.95)	-0.35	(0.94)	0.57	(0.12)	0.49	(0.14)
Intact family	0.38	(0.92)	-0.21	(0.98)	0.00	(0.93)	0.59	(0.07)	0.38	(0.06)
Difference: Intact - Single Parent	0.44	(1.32)	0.38	(1.36)	0.20	(1.31)	0.06	(0.12)	0.24	(0.13)

Source: Heckman (2011, Web Appendix).

Notes: The Armed Forces Qualifying Test (AFQT) is assessed of mothers in 1979. Individuals in the CNLSY are given the PIAT assessment every 2 years from ages 6 to 14. The measure shown here is a sum of child z-score measures of PIAT math and PIAT reading performance at age 14, which is then normalized to population mean 0, standard deviation 1. "SE" columns show both standard deviations of ability, and calculations of the standard error of the difference of sample means. Average family income is averaged from child's birth to age fifteen.

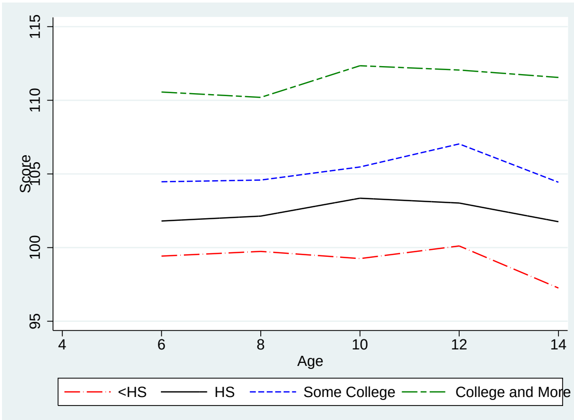
Table A.5: Comparison of Within-Race AFQT Gaps Across Socioeconomic Status—NLSY97—Males and Females

	Average AFQT Score						Across-Race Difference			
	Whites		Blacks		Hispanics		W-B Gap		W-H Gap	
	Avg	SE	Avg	SE	Avg	SE	Diff	SE	Diff	SE
Unconditional AFQT Averages	0.09	(1.00)	-0.19	(0.98)	-0.08	(1.01)	0.28	(0.05)	0.17	(0.06)
Mother's Educational Status										
Mother is a dropout	-0.08	(0.92)	-0.14	(0.96)	-0.21	(0.99)	0.06	(0.11)	0.13	(0.11)
Mother is a high school graduate	0.02	(0.99)	-0.21	(1.08)	-0.01	(1.01)	0.23	(0.09)	0.03	(0.10)
Mother is a college graduate	0.28	(1.07)	-0.07	(0.91)	0.21	(1.32)	0.35	(0.12)	0.07	(0.22)
Difference: College Graduate - Dropout	0.36	(1.41)	0.07	(1.32)	0.42	(1.65)	0.29	(0.17)	-0.06	(0.25)
Family Income										
Family income from 1997 in 1st quartile	0.05	(0.99)	-0.18	(0.91)	-0.01	(1.07)	0.23	(0.10)	0.06	(0.12)
Family income from 1997 in 2nd quartile	0.14	(1.03)	-0.22	(1.05)	-0.07	(0.88)	0.36	(0.11)	0.21	(0.12)
Family income from 1997 in 3rd quartile	0.10	(1.01)	-0.27	(0.92)	-0.11	(0.99)	0.37	(0.14)	0.21	(0.13)
Family income from 1997 in 4th quartile	0.09	(1.00)	-0.15	(1.05)	0.11	(1.20)	0.24	(0.17)	-0.02	(0.16)
Difference: Top - Bottom Quartile	0.04	(1.43)	0.03	(1.48)	0.12	(1.49)	0.01	(0.19)	-0.08	(0.20)

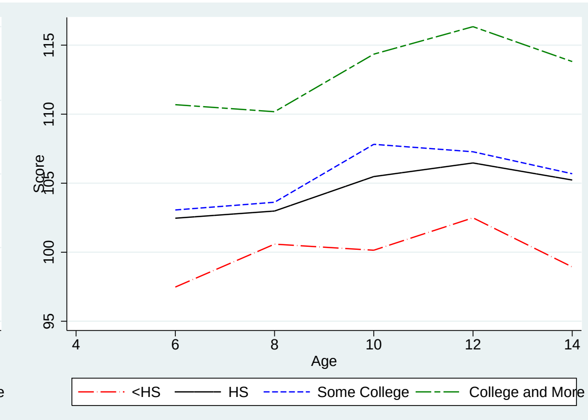
Source: Heckman (2011, Web Appendix).

Notes: AFQT is measured in 1997 when individuals are aged 12-16. To account for the differences in AFQT due to schooling and other growth due to aging, AFQT measures are the "post-school" constructions are calculated as described in Heckman et al. (2011). "SE" columns show both standard deviations of ability, and calculations of the standard error of the difference of sample means

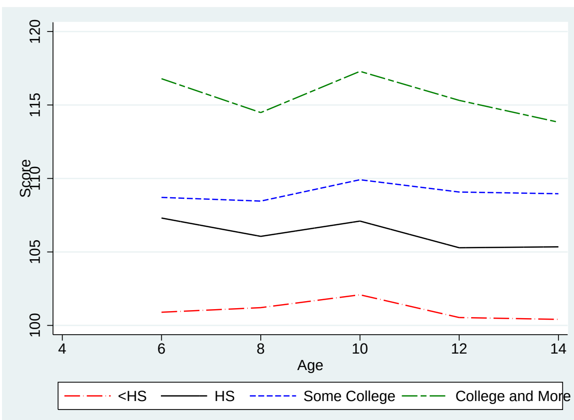
Figure A.9: Skill Measures over Childhood by Mother's Education: White



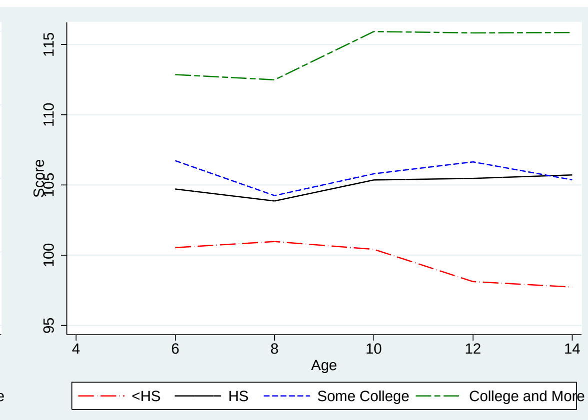
(a) Girls: Math Score (standardized)



(b) Boys: Math Score (standardized)



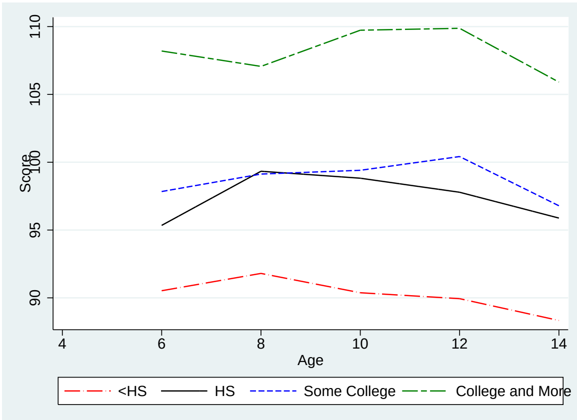
(c) Girls: Reading Score (standardized)



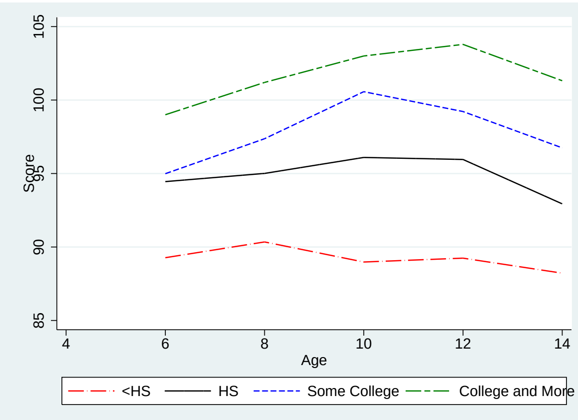
(d) Boys: Reading Score (standardized)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

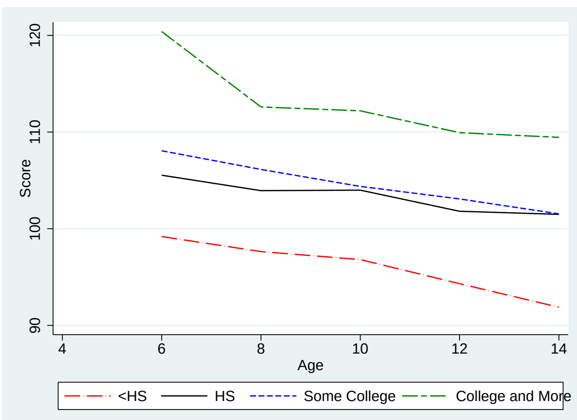
Figure A.10: Skill Measures over Childhood by Mother's Education : Black



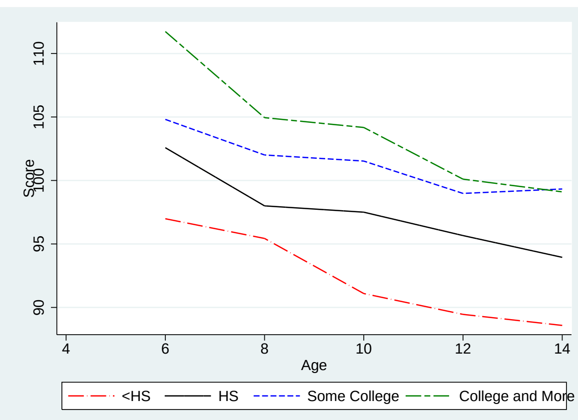
(a) Girls: Math Score (standardized)



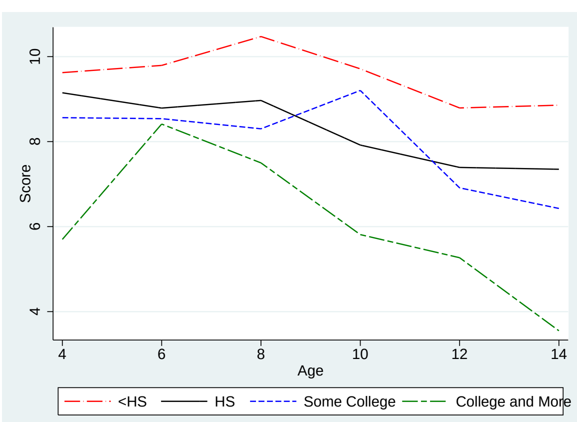
(b) Boys: Math Score (standardized)



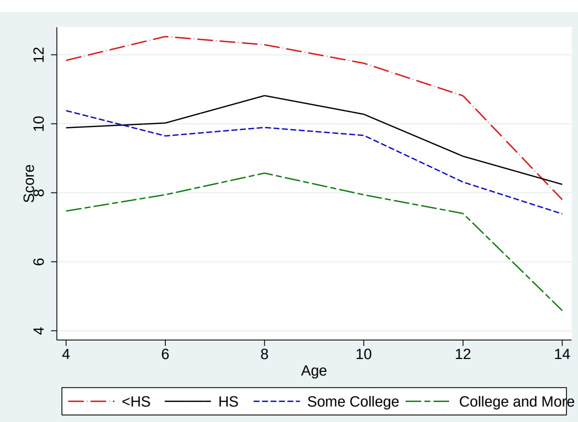
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



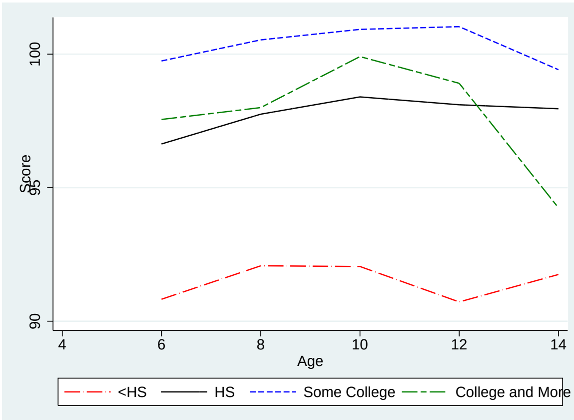
(e) Girls: BPI (Raw score)



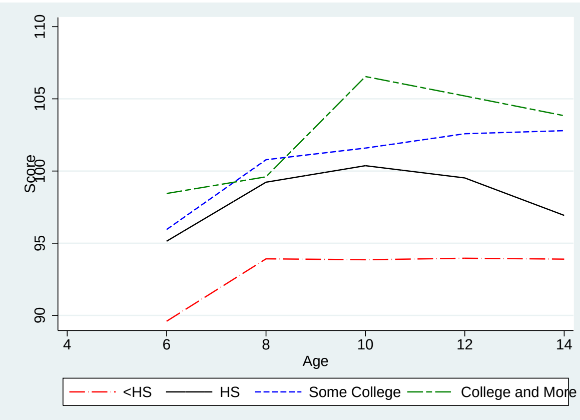
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

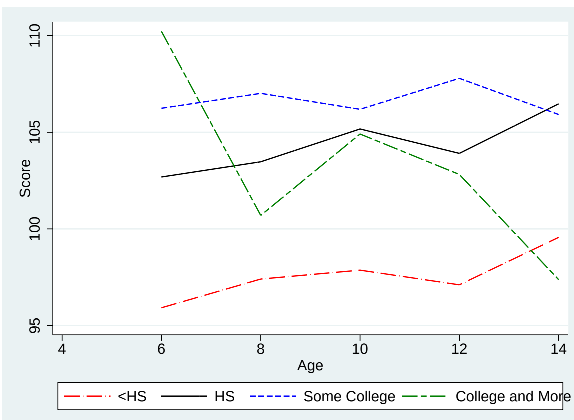
Figure A.11: Skill Measures over Childhood by Mother's Education : Hispanic



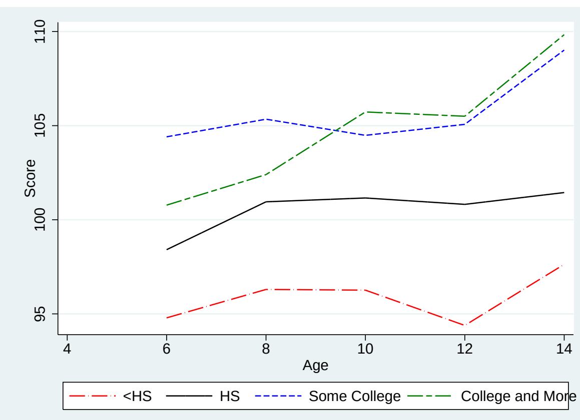
(a) Girls: Math Score (standardized)



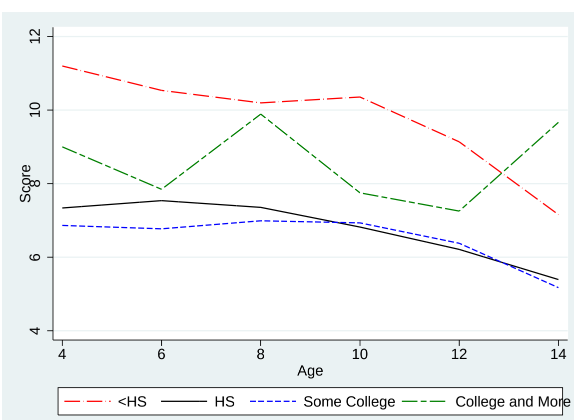
(b) Boys: Math Score (standardized)



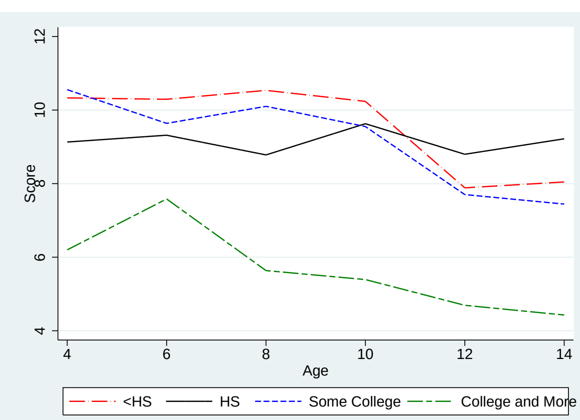
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



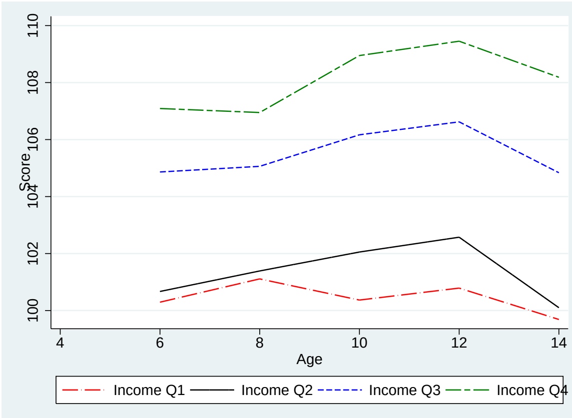
(e) Girls: BPI (Raw score)



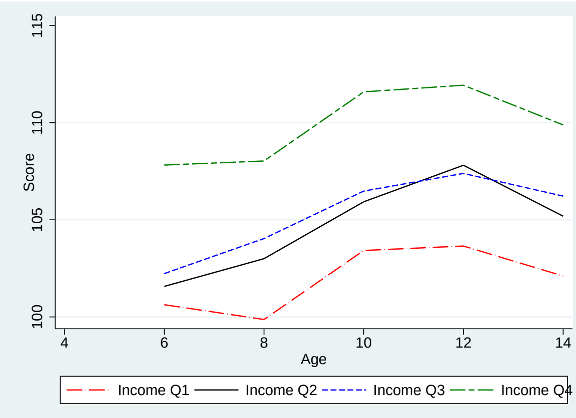
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

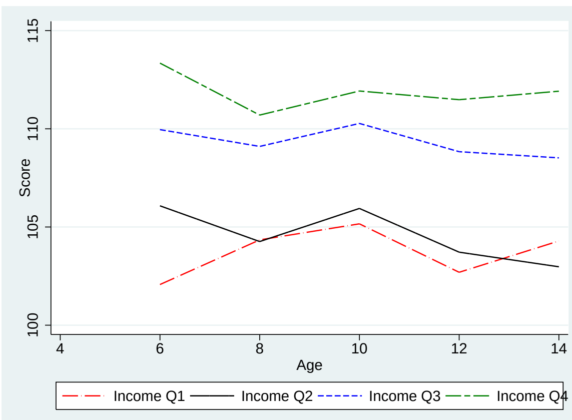
Figure A.12: Skill Measures over Childhood among Whites by Family Income Quartile



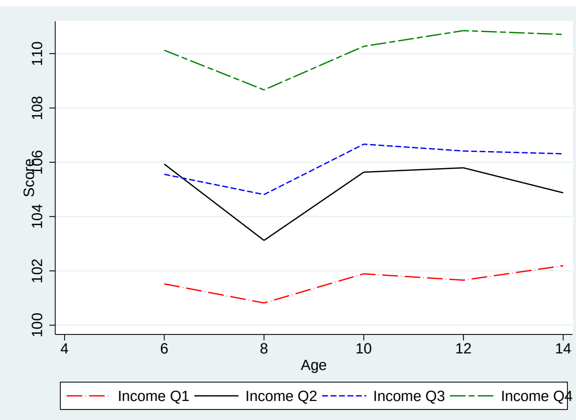
(a) Girls: Math Score (standardized)



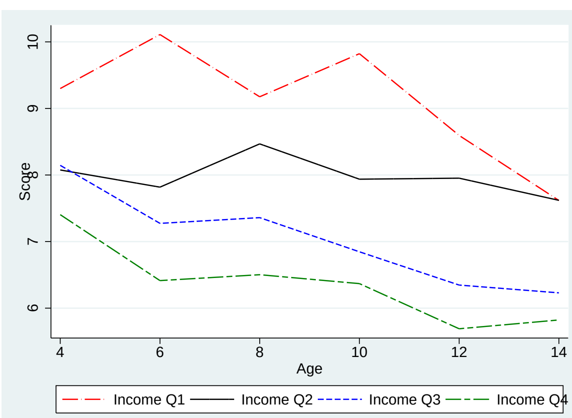
(b) Boys: Math Score (standardized)



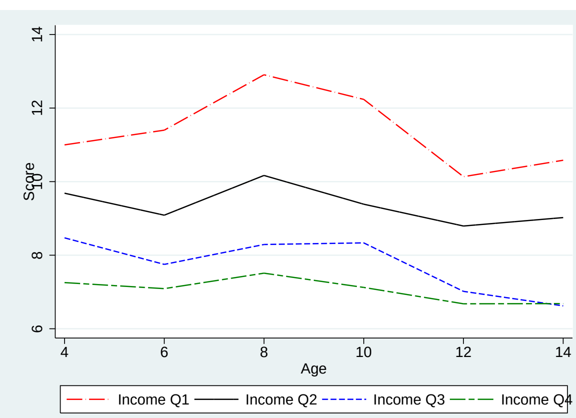
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



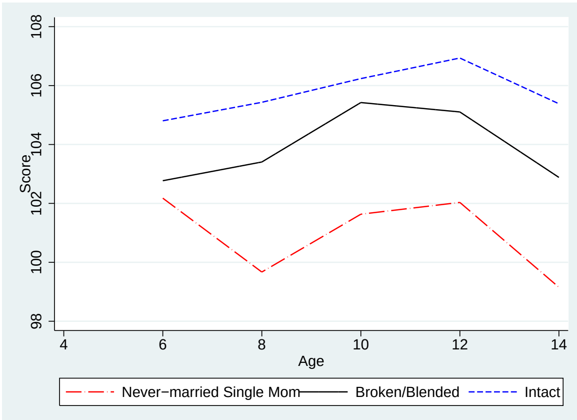
(e) Girls: BPI (Raw score)



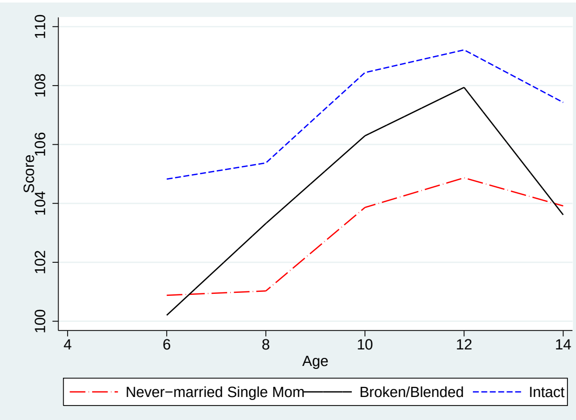
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

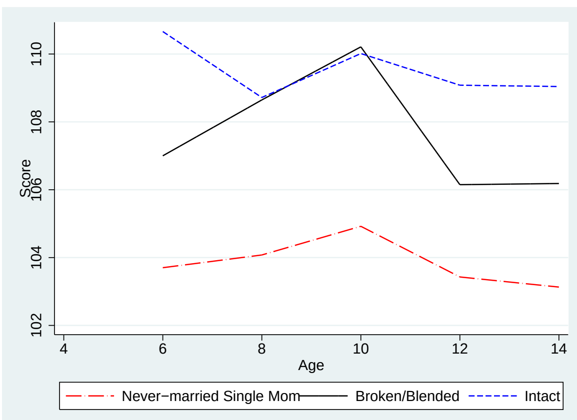
Figure A.13: Skill Measures over Childhood among Whites by Family Type



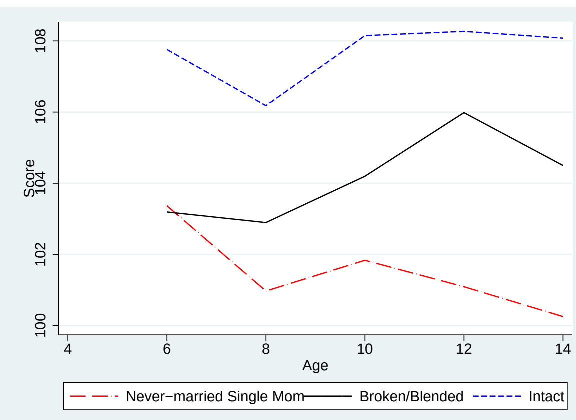
(a) Girls: Math Score (standardized)



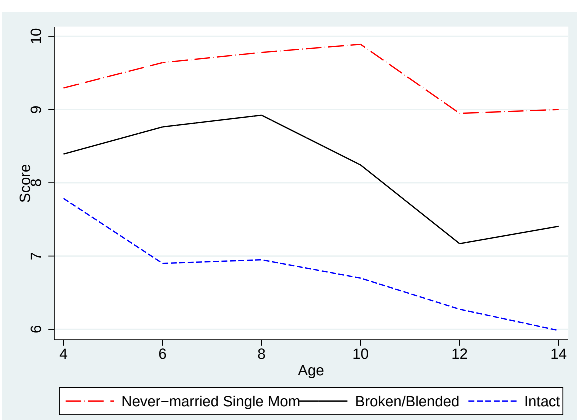
(b) Boys: Math Score (standardized)



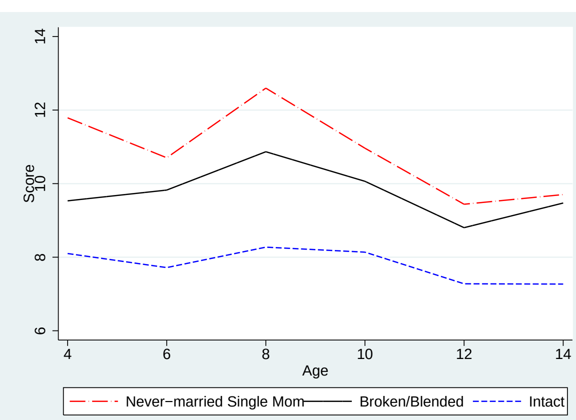
(c) Girls: Reading Score (standardized)



(d) Boys: Reading Score (standardized)



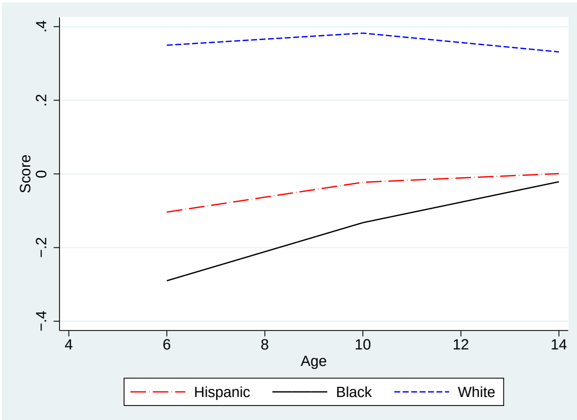
(e) Girls: BPI (Raw score)



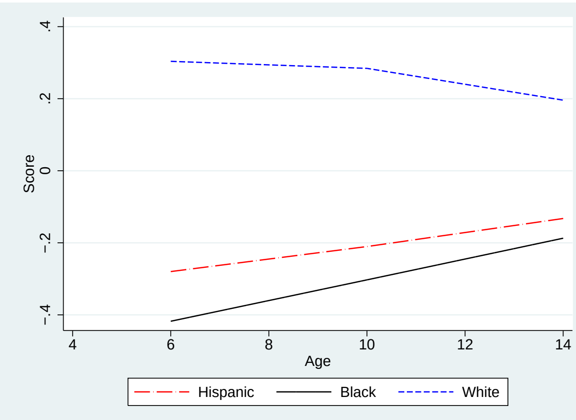
(f) Boys: BPI (Raw score)

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

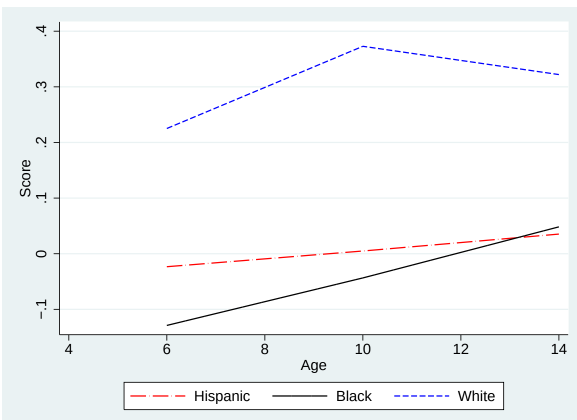
Figure A.14: Parental Investment over Childhood across Ethnic Groups



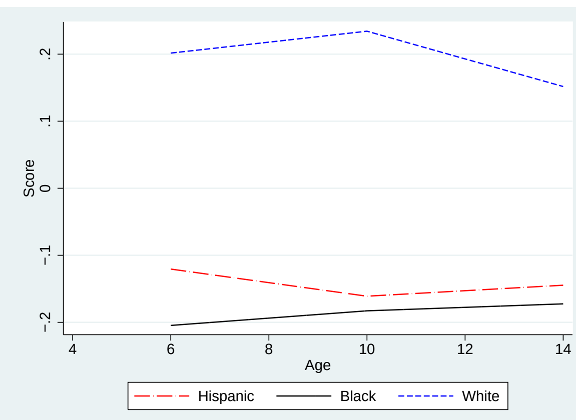
(a) Girls: Material Resource



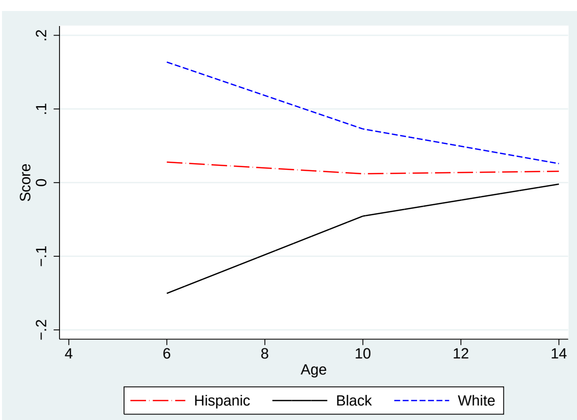
(b) Boys: Material Resource



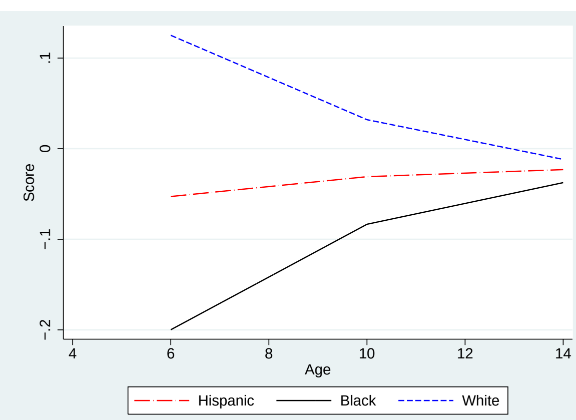
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



(e) Girls: Emotional Support



(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

Table A.6: Contributions by Components to Racial Skill Gaps at age 6: Static Decomposition, Raw Scores

	Age 6	Math				Reading			
		Mean	s.e.		%Changes	Mean	s.e.		%Changes
Girls	**Actual Gap (=W-B)**	3.0980	0.4870	***		1.2755	0.5055	***	
	Contribution by								
	Mother's Skill	3.3742	0.4675	***	**108.9%**	2.4673	0.3636	***	**193.4%**
	Mother's Cog.	3.1711	0.4366	***	**102.4%**	2.1490	0.3204	***	**168.5%**
	Mother's Non-cog.	0.1583	0.1027		5.1%	0.3776	0.0930	***	**29.6%**
	Parental Investment	1.1734	0.1667	***	**37.9%**	1.3495	0.2367	***	**105.8%**
	Material Resource	-0.1799	0.1312	**	**-5.8%**	0.5737	0.1539	***	**45.0%**
	Cognitive Stimulation	-0.4004	0.1099	***	**-12.9%**	0.7155	0.1607	***	**56.1%**
	Emotional Support	-0.4009	0.1101	***	**-12.9%**	0.7151	0.1565	***	**56.1%**
	Intact Family	0.2097	0.1901		6.8%	0.9881	0.1877	***	**77.5%**
	Family Income	-0.5796	0.1102	***	**-18.7%**	0.6688	0.1515	***	**52.4%**
	All Together Jointly	5.2503	0.4542	***	**169.5%**	4.1330	0.4446	***	**324.0%**
Boys	**Actual Gap (=W-B)**	4.1329	0.5130	***		1.7658	0.5244	***	
	Contribution by								
	Mother's Skill	-0.1985	0.6500		-4.8%	1.0583	0.2884	***	**59.9%**
	Mother's Cog.	0.2108	0.4260		5.1%	1.2406	0.2973	***	**70.3%**
	Mother's Non-cog.	-0.2191	0.1176		-5.3%	-0.1451	0.1060		-8.2%
	Parental Investment	1.6323	0.2001	***	**39.5%**	1.1938	0.1986	***	**67.6%**
	Material Resource	-0.2783	0.0802	***	**-6.7%**	0.0188	0.1257		1.1%
	Cognitive Stimulation	-0.3657	0.0851	***	**-8.8%**	-0.0863	0.1255		-4.9%
	Emotional Support	-0.3945	0.0892	***	**-9.5%**	-0.0861	0.1172		-4.9%
	Intact Family	0.2370	0.1811		5.7%	0.5829	0.1721	***	**33.0%**
	Family Income	-0.4645	0.1129	***	**-11.2%**	-0.0901	0.1061		-5.1%
	All Together Jointly	1.3216	0.6425	***	**32.0%**	1.0808	0.4401	***	**61.2%**

Source: Moon (2014)

Data: A balanced panel from Children of NLSY79.

Note: (a) "Mother's skill" denotes mother's AFQT score, Rosenberg Self-esteem scale, and Rotter Locus of Control scale obtained from NLSY79; (b) "Parental Investment" consists of three latent factors estimated by individual indicators in HOME-SF Inventory up to the corresponding age; (c) "Intact Family" is a continuous variable of fraction of childhood spent in a family headed by his/her biological parents in wedlock up to the age of test taking; (d) "Family Income" include all types of income in the household averaged over the whole childhood up to the age of test taking; (e) "Others" denote all other variables included in the regression such as dummy indicators for teenage mothers and mothers older than 30, dummy indicators for birth order, the number of siblings in the household, dummy indicators for birth cohorts, a dummy indicator for whether the town is in MSA or not, the county-level unemployment rate at child's birth, the county-level crime rate at child's birth, the teacher-student ratio at the county level, the per-pupil educational expenditure at the state-level, and dummy indicators for mother's educational attainment.

Table A.7: Contributions by Components to Racial Skill Gaps at age 8: Static Decomposition, Raw Scores

	Age 8	Math				Reading			
		Mean	s.e.		%Changes	Mean	s.e.		%Changes
Girls	**Actual Gap (=W-B)**	5.1382	0.6080	***		3.5628	0.6652	***	
	Contribution by								
	Mother's Skill	2.7338	0.5971	***	**53.2%**	3.2826	0.6781	***	**92.1%**
	Mother's Cog.	2.0687	0.4565	***	**40.3%**	2.4999	0.4463	***	**70.2%**
	Mother's Non-cog.	0.1091	0.2530		2.1%	0.5939	0.1534	***	**16.7%**
	Parental Investment	1.6231	0.4015	***	**31.6%**	0.5680	0.3167	***	**15.9%**
	Material Resource	0.7080	0.1620	***	**13.8%**	-0.3444	0.2347		-9.7%
	Cognitive Stimulation	0.1514	0.1946		2.9%	0.4042	0.2312		11.3%
	Emotional Support	-0.0113	0.2173		-0.2%	0.0922	0.1749		2.6%
	Intact Family	0.9514	0.2729	***	**18.5%**	0.2146	0.2404		6.0%
	Family Income	-0.0319	0.2054		-0.6%	0.4713	0.2168		13.2%
	All Together Jointly	8.4589	1.3849	***	**164.6%**	4.8014	1.2491	***	**134.8%**
Boys	**Actual Gap (=W-B)**	7.8927	0.6951	***		5.7689	0.7598	***	
	Contribution by								
	Mother's Skill	0.1581	0.4175		2.0%	1.3319	0.4175	***	**23.1%**
	Mother's Cog.	0.2596	0.4277		3.3%	1.4343	0.3437	***	**24.9%**
	Mother's Non-cog.	-0.0050	0.2447		-0.1%	0.0821	0.2251		1.4%
	Parental Investment	1.4969	0.4633	***	**19.0%**	1.3132	0.3847	***	**22.8%**
	Material Resource	0.6372	0.2557	***	**8.1%**	-0.2972	0.3007		-5.2%
	Cognitive Stimulation	0.2249	0.2361		2.9%	-0.4098	0.3123		-7.1%
	Emotional Support	-0.5604	0.2807		-7.1%	0.0465	0.2768		0.8%
	Intact Family	0.0615	0.4371		0.8%	0.0837	0.4296		1.5%
	Family Income	-0.0099	0.1697		-0.1%	0.7981	0.2578	*	**13.8%**
	All Together Jointly	1.0499	1.3322		13.3%	1.5758	1.6601	**	**27.3%**

Source: Moon (2014)

Data: A balanced panel from Children of NLSY79.

Note: (a) "Mother's skill" denotes mother's AFQT score, Rosenberg Self-esteem scale, and Rotter Locus of Control scale obtained from NLSY79; (b) "Parental Investment" consists of three latent factors estimated by individual indicators in HOME-SF Inventory up to the corresponding age; (c) "Intact Family" is a continuous variable of fraction of childhood spent in a family headed by his/her biological parents in wedlock up to the age of test taking; (d) "Family Income" include all types of income in the household averaged over the whole childhood up to the age of test taking; (e) "Others" denote all other variables included in the regression such as dummy indicators for teenage mothers and mothers older than 30, dummy indicators for birth order, the number of siblings in the household, dummy indicators for birth cohorts, a dummy indicator for whether the town is in MSA or not, the county-level unemployment rate at child's birth, the county-level crime rate at child's birth, the teacher-student ratio at the county level, the per-pupil educational expenditure at the state-level, and dummy indicators for mother's educational attainment.

Table A.8: Contributions by Components to Racial Skill Gaps at age 10: Static Decomposition, Raw Scores

	Age 10	Math				Reading			
		Mean	s.e.		%Changes	Mean	s.e.		%Changes
Girls	**Actual Gap (=W-B)**	4.9991	0.5573	***		5.4490	0.7313	***	
	Contribution by								
	Mother's Skill	2.4316	0.4193	***	**48.6%**	3.1203	0.4861	***	**57.3%**
	Mother's Cog.	1.5777	0.3434	***	**31.6%**	1.9647	0.4150	***	**36.1%**
	Mother's Non-cog.	0.5930	0.2144	**	**11.9%**	0.4168	0.3203	*	**7.6%**
	Parental Investment	1.2101	0.3112	***	**24.2%**	1.4945	0.2420	***	**27.4%**
	Material Resource	0.8562	0.3691	*	**17.1%**	0.9075	0.2961	*	**16.7%**
	Cognitive Stimulation	1.0006	0.3638	*	**20.0%**	0.5114	0.3193		9.4%
	Emotional Support	0.5475	0.2833		11.0%	0.2179	0.2407		4.0%
	Intact Family	0.9134	0.3906	**	**18.3%**	0.3798	0.5135		7.0%
	Family Income	0.0650	0.2297		1.3%	-0.3846	0.2187		-7.1%
	All Together Jointly	4.0526	0.9874	***	**81.1%**	3.9843	2.5116	***	**73.1%**
Boys	**Actual Gap (=W-B)**	8.0250	0.6575	***		8.6815	0.8423	***	
	Contribution by								
	Mother's Skill	1.3211	0.5350	**	**16.5%**	0.4754	0.4171		5.5%
	Mother's Cog.	1.2266	0.4371	***	**15.3%**	0.2970	0.6139		3.4%
	Mother's Non-cog.	0.1876	0.2032		2.3%	0.1242	0.2530		1.4%
	Parental Investment	1.6647	0.3630	***	**20.7%**	0.7054	0.3133	***	**8.1%**
	Material Resource	-0.1786	0.4423		-2.2%	0.8257	0.3458	**	**9.5%**
	Cognitive Stimulation	-0.4240	0.3327		-5.3%	0.5606	0.2828	**	**6.5%**
	Emotional Support	-0.2457	0.2440		-3.1%	0.3140	0.2844		3.6%
	Intact Family	-0.1441	0.3622		-1.8%	0.5578	0.4444		6.4%
	Family Income	0.1845	0.2943		2.3%	0.0647	0.2981		0.7%
	All Together Jointly	0.3526	1.0594		4.4%	1.7944	1.1283	***	**20.7%**

Source: Moon (2014)

Data: A balanced panel from Children of NLSY79.

Note: (a) "Mother's skill" denotes mother's AFQT score, Rosenberg Self-esteem scale, and Rotter Locus of Control scale obtained from NLSY79; (b) "Parental Investment" consists of three latent factors estimated by individual indicators in HOME-SF Inventory up to the corresponding age; (c) "Intact Family" is a continuous variable of fraction of childhood spent in a family headed by his/her biological parents in wedlock up to the age of test taking; (d) "Family Income" include all types of income in the household averaged over the whole childhood up to the age of test taking; (e) "Others" denote all other variables included in the regression such as dummy indicators for teenage mothers and mothers older than 30, dummy indicators for birth order, the number of siblings in the household, dummy indicators for birth cohorts, a dummy indicator for whether the town is in MSA or not, the county-level unemployment rate at child's birth, the county-level crime rate at child's birth, the teacher-student ratio at the county level, the per-pupil educational expenditure at the state-level, and dummy indicators for mother's educational attainment.

Table A.9: Contributions by Components to Racial Skill Gaps at age 12: Static Decomposition, Raw Scores

	Age 12	Math				Reading			
		Mean	s.e.		%Changes	Mean	s.e.		%Changes
Girls	**Actual Gap (=W-B)**	6.3731	0.2928	***		5.3663	0.3710	***	
	Contribution by								
	Mother's Skill	3.2826	0.6781	***	**51.5%**	4.1805	0.6452	***	**77.9%**
	Mother's Cog.	2.4999	0.4463	***	**39.2%**	3.2859	0.5356	***	**61.2%**
	Mother's Non-cog.	0.5939	0.1534	***	**9.3%**	0.7779	0.2289	***	**14.5%**
	Parental Investment	0.5680	0.3167	***	**8.9%**	1.4638	0.3502	***	**27.3%**
	Material Resource	-0.3444	0.2347		-5.4%	0.4033	0.2866		7.5%
	Cognitive Stimulation	0.4042	0.2312		6.3%	0.2156	0.2212		4.0%
	Emotional Support	0.0922	0.1749		1.4%	0.8420	0.2343	***	**15.7%**
	Intact Family	0.2146	0.2404		3.4%	1.0145	0.3455	***	**18.9%**
	Family Income	0.4713	0.2168		7.4%	-0.4191	0.2198		-7.8%
	All Together Jointly	4.8014	1.2491	***	**75.3%**	6.3158	0.8482	***	**117.7%**
Boys	**Actual Gap (=W-B)**	9.6089	0.3319	***		10.4059	0.4403	***	
	Contribution by								
	Mother's Skill	1.3319	0.4175	***	**13.9%**	-0.0897	0.7736		-0.9%
	Mother's Cog.	1.4343	0.3437	***	**14.9%**	0.0437	0.5204		0.4%
	Mother's Non-cog.	0.0821	0.2251		0.9%	-0.0802	0.2583		-0.8%
	Parental Investment	1.3132	0.3847	***	**13.7%**	0.7706	0.6831		7.4%
	Material Resource	-0.2972	0.3007		-3.1%	0.5569	0.2899	**	**5.4%**
	Cognitive Stimulation	-0.4098	0.3123		-4.3%	0.6429	0.4213		6.2%
	Emotional Support	0.0465	0.2768		0.5%	0.2388	0.2815	*	**2.3%**
	Intact Family	0.0837	0.4296		0.9%	1.2836	0.5101	*	**12.3%**
	Family Income	0.7981	0.2578	*	**8.3%**	0.4629	0.3622	*	**4.4%**
	All Together Jointly	1.5758	1.6601	*	**16.4%**	2.0414	2.3343		19.6%

Source: Moon (2014)

Data: A balanced panel from Children of NLSY79.

Note: (a) "Mother's skill" denotes mother's AFQT score, Rosenberg Self-esteem scale, and Rotter Locus of Control scale obtained from NLSY79; (b) "Parental Investment" consists of three latent factors estimated by individual indicators in HOME-SF Inventory up to the corresponding age; (c) "Intact Family" is a continuous variable of fraction of childhood spent in a family headed by his/her biological parents in wedlock up to the age of test taking; (d) "Family Income" include all types of income in the household averaged over the whole childhood up to the age of test taking; (e) "Others" denote all other variables included in the regression such as dummy indicators for teenage mothers and mothers older than 30, dummy indicators for birth order, the number of siblings in the household, dummy indicators for birth cohorts, a dummy indicator for whether the town is in MSA or not, the county-level unemployment rate at child's birth, the county-level crime rate at child's birth, the teacher-student ratio at the county level, the per-pupil educational expenditure at the state-level, and dummy indicators for mother's educational attainment.

Table A.10: Oaxaca Decomposition of Black-White Skill Gap: PIAT Math and Reading at Age 12

Age 12	**Girls**		**Boys**	
	Math	Reading	Math	Reading
Overall Gap (Raw Scores)	**6.618**	**5.256**	**9.811**	**10.163**
Contributions (in %)				
by Endowments				
Mother's Skills	81.3%	122.8%	58.4%	62.2%
Parental Investment	13.2%	45.2%	56.7%	47.7%
Intact Family	4.3%	15.7%	-14.6%	-13.2%
Family Income	4.2%	-2.0%	21.4%	44.1%
Others	8.7%	-2.4%	2.7%	8.6%
Total	**111.6%**	**179.3%**	**124.7%**	**149.4%**
by Coefficients				
Mother's Skills	46.3%	126.2%	9.1%	-14.2%
Parental Investment	-19.9%	-4.8%	22.4%	3.2%
Intact Family	-5.3%	-11.1%	6.8%	9.7%
Family Income	-8.6%	0.2%	-18.5%	-30.0%
Others	53.8%	-7.8%	80.1%	182.9%
Constant	-65.2%	-152.8%	-69.9%	-159.1%
Total	**1.1%**	**-50.1%**	**30.1%**	**-7.6%**
by E-C Interactions				
Mother's Skills	-37.2%	-58.6%	-22.1%	-23.2%
Parental Investment	45.6%	30.6%	-21.0%	6.6%
Intact Family	-7.5%	-14.4%	9.8%	14.1%
Family Income	-3.9%	4.3%	-24.0%	-44.2%
Others	-9.6%	8.9%	2.6%	4.8%
Total	**-12.7%**	**-29.2%**	**-54.8%**	**-41.9%**

Source : Moon (2014)

Data: A balanced panel from Children of NLSY79.

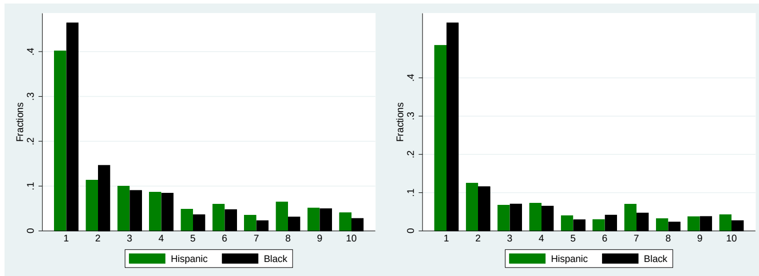
Note: (a) "Mother's skill" denotes mother's AFQT score, Rosenberg Self-esteem scale, and Rotter Locus of Control scale obtained from NLSY79; (b) "Parental Investment" consists of three latent factors estimated by individual indicators in HOME-SF Inventory up to the corresponding age; (c) "Intact Family" is a continuous variable of fraction of childhood spent in a family headed by his/her biological parents in wedlock up to the age of test taking; (d) "Family Income" include all types of income in the household averaged over the whole childhood up to the age of test taking; (e) "Others" denote all other variables included in the regression such as dummy indicators for teenage mothers and mothers older than 30, dummy indicators for birth order, the number of siblings in the household, dummy indicators for birth cohorts, a dummy indicator for whether the town is in MSA or not, the county-level unemployment rate at child's birth, the county-level crime rate at child's birth, the teacher-student ratio at the county level, the per-pupil educational expenditure at the state-level, and dummy indicators for mother's educational attainment.

B Evidence on Gaps in Family Environments and Investments in Child Care Across Socioeconomic Classes

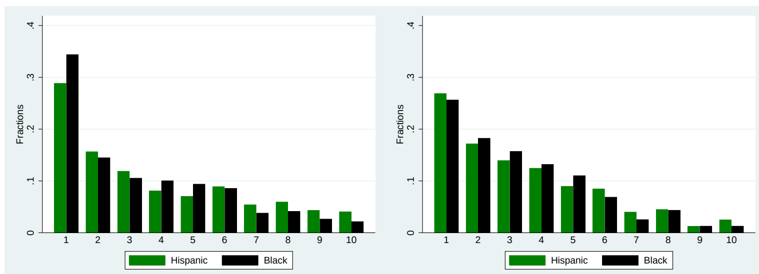
B.1 Comparison of Ability and Personality Measures by Race

Comparison of AFQT Distributions Figure A.42 places the Black and Hispanic scholastic ability distribution in the overall White distribution. The measures of ability is based on achievement tests for reading and math skills. The tests are taken in the teenage years. If abilities were distributed equally across groups, minorities would be distributed evenly across the deciles of the White ability distribution. (A decile is a measure of location in a distribution. The first decile is a measure of the average scores for persons in the bottom 10% of the White test score distribution. The tenth decile measures the average score for people at the top of the White distribution.) By construction, 10% of Whites are in each decile. Blacks and Hispanics are over-represented in the lower end of the White ability distribution with Blacks faring slightly worse than Hispanics.

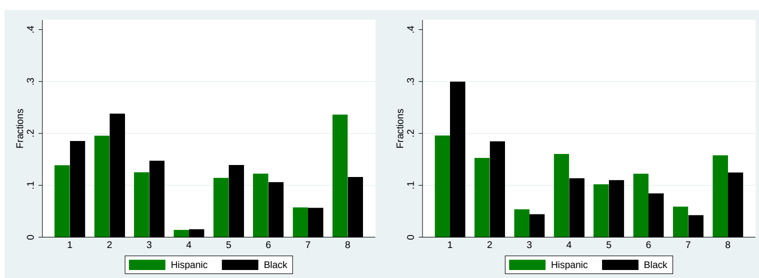
Figure B.1: Hispanic and Black Parental Investment in White Distribution: Unadjusted, Age 0-3



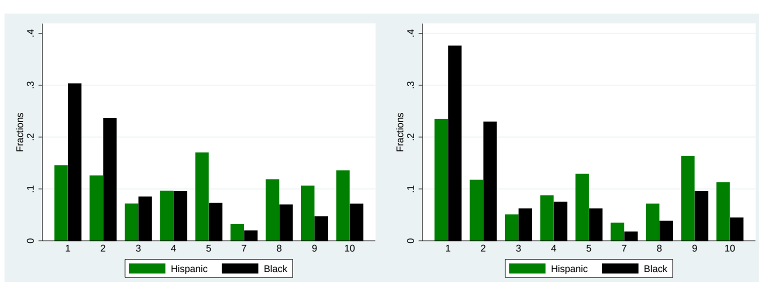
(a) Material Goods (Females) **(b)** Material Goods (Males)



(c) Cognitive Stimulation (Females) **(d)** Cognitive Stimulation (Males)



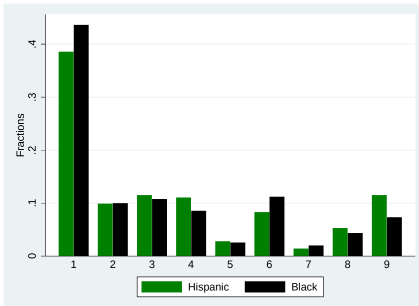
(e) Emotional Support (Females) **(f)** Emotional Support (Males)



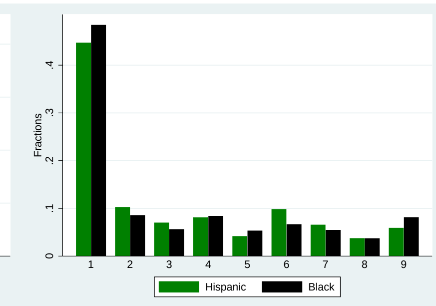
(g) Father Engagement (Females) **(h)** Father Engagement (Males)

Source: Moon (2014).

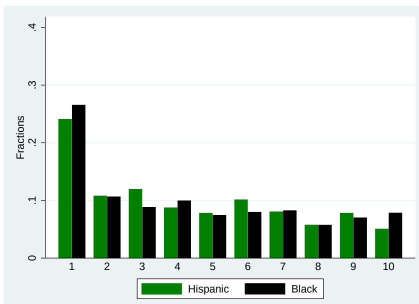
Figure B.2: Hispanic and Black Parental Investment in White Distribution: Unadjusted, Age 4-7



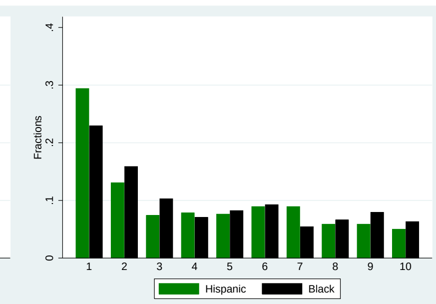
(a) Material Goods (Females)



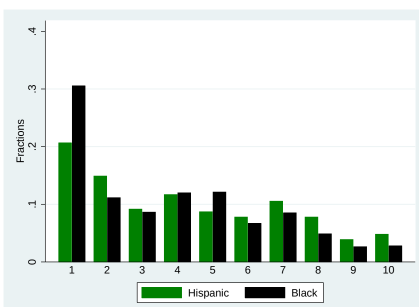
(b) Material Goods (Males)



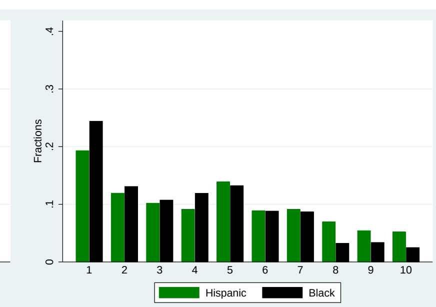
(c) Cognitive Stimulation (Females)



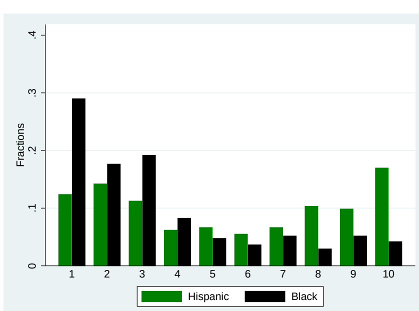
(d) Cognitive Stimulation (Males)



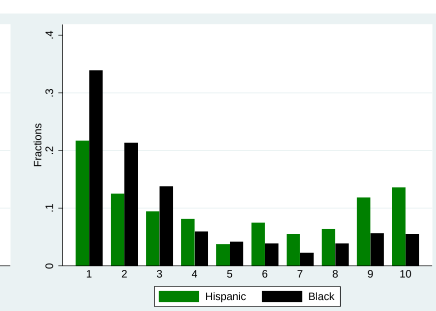
(e) Emotional Support (Females)



(f) Emotional Support (Males)



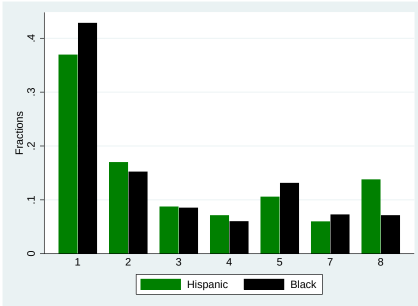
(g) Father Engagement (Females)



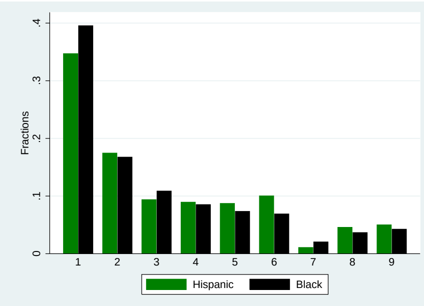
(h) Father Engagement (Males)

Source: Moon (2014).

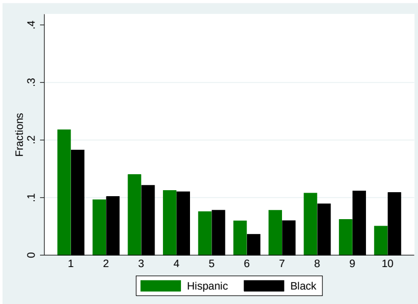
Figure B.3: Hispanic and Black Parental Investment in White Distribution: Unadjusted, Age 8-11



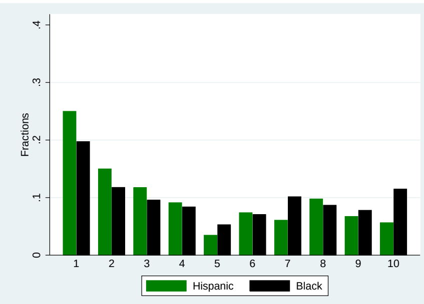
(a) Material Goods (Females)



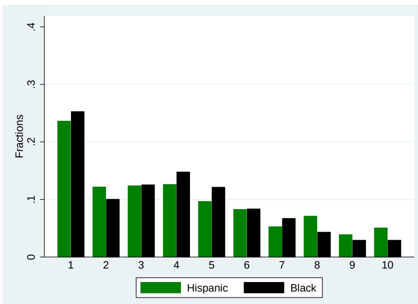
(b) Material Goods (Males)



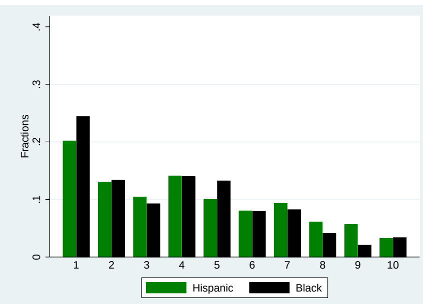
(c) Cognitive Stimulation (Females)



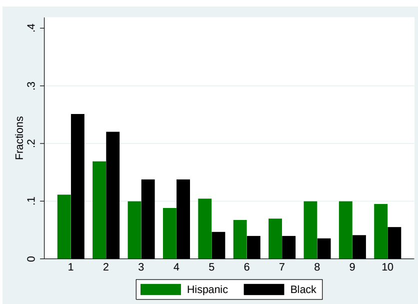
(d) Cognitive Stimulation (Males)



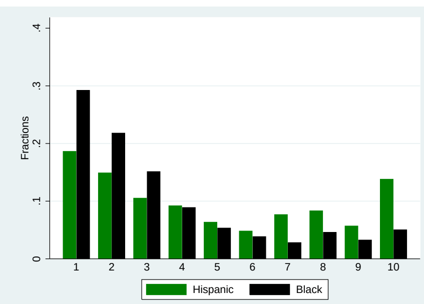
(e) Emotional Support (Females)



(f) Emotional Support (Males)



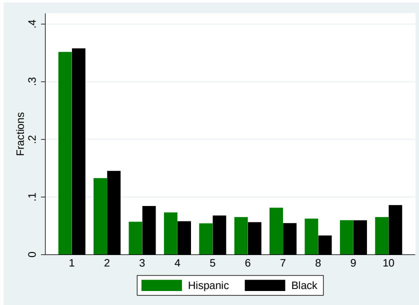
(g) Father Engagement (Females)



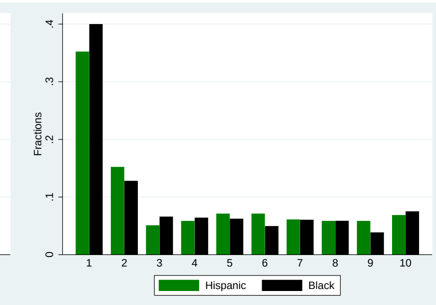
(h) Father Engagement (Males)

Source: Moon (2014).

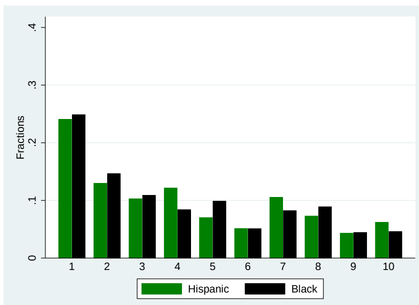
Figure B.4: Hispanic and Black Parental Investment in White Distribution: Adjusted for Mother's Education, Family Income, and Family Structure, Age 0-3



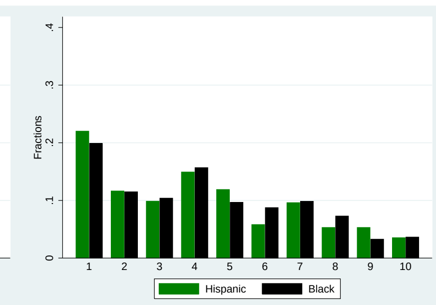
(a) Material Goods (Females)



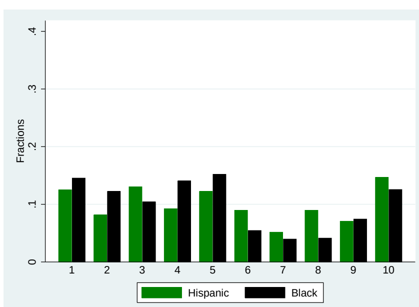
(b) Material Goods (Males)



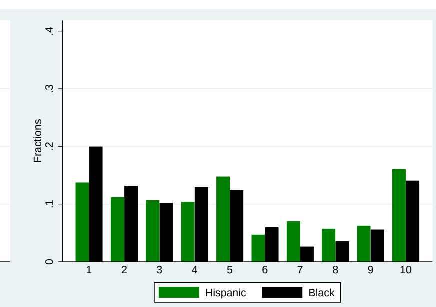
(c) Cognitive Stimulation (Females)



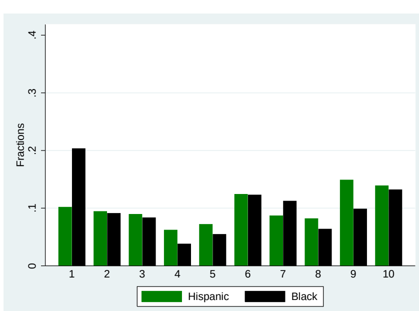
(d) Cognitive Stimulation (Males)



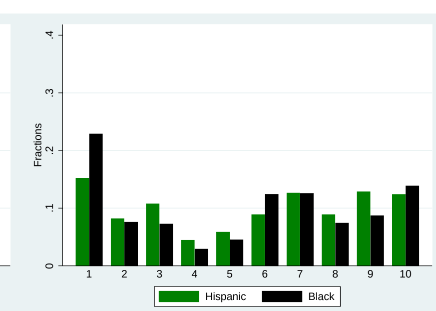
(e) Emotional Support (Females)



(f) Emotional Support (Males)



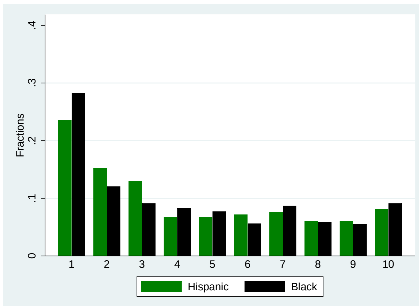
(g) Father Engagement (Females)



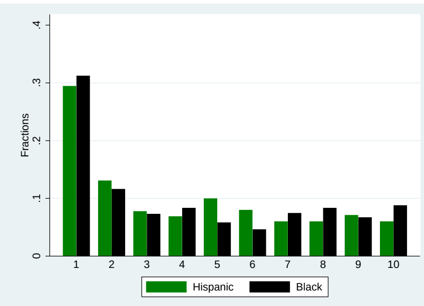
(h) Father Engagement (Males)

Source: Moon (2014).

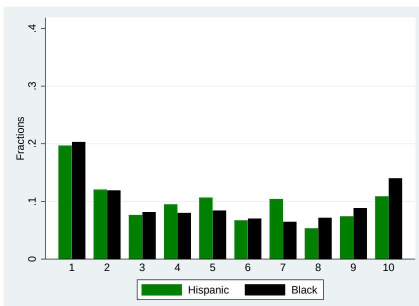
Figure B.5: Hispanic and Black Parental Investment in White Distribution: Adjusted for Mother's Education, Family Income, and Family Structure, age 4-7



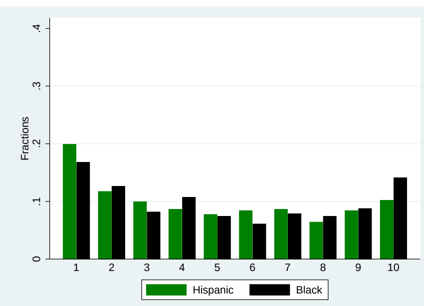
(a) Material Goods (Females)



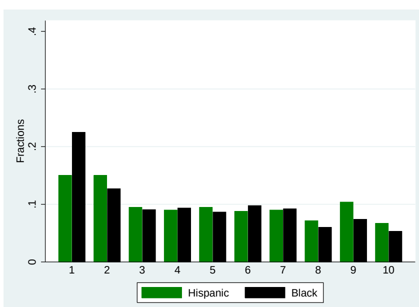
(b) Material Goods (Males)



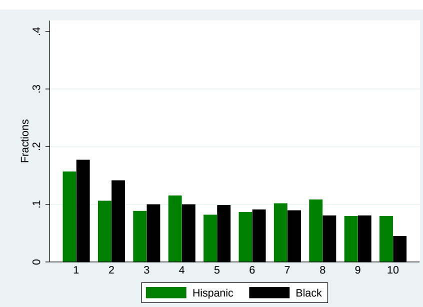
(c) Cognitive Stimulation (Females)



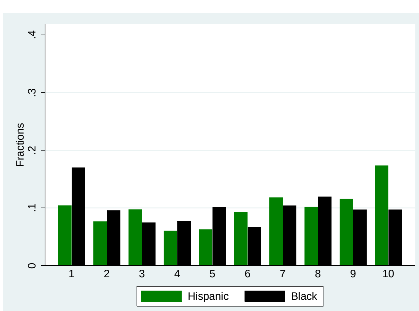
(d) Cognitive Stimulation (Males)



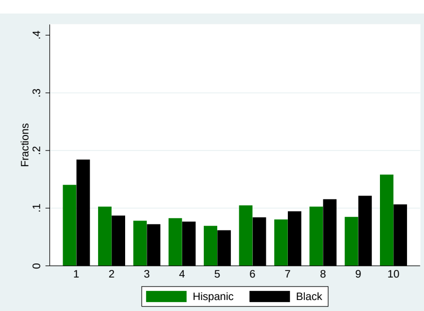
(e) Emotional Support (Females)



(f) Emotional Support (Males)



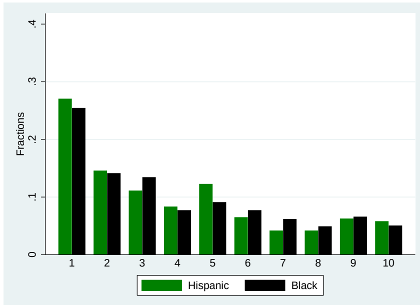
(g) Father Engagement (Females)



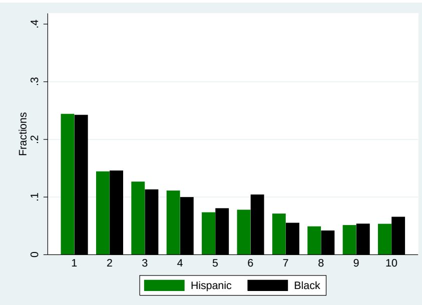
(h) Father Engagement (Males)

Source: Moon (2014).

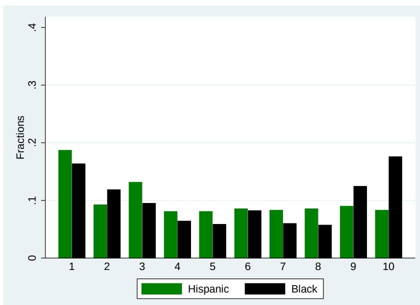
Figure B.6: Hispanic and Black Parental Investment in White Distribution: Adjusted for Mother's Education, Family Income, and Family Structure, age 8-11



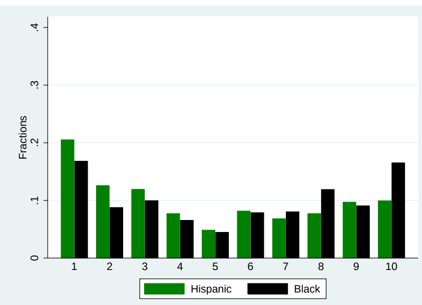
(a) Material Goods (Females)



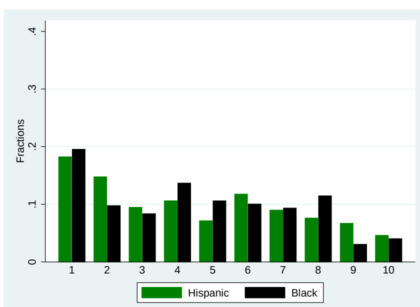
(b) Material Goods (Males)



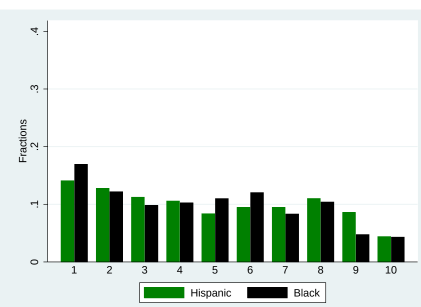
(c) Cognitive Stimulation (Females)



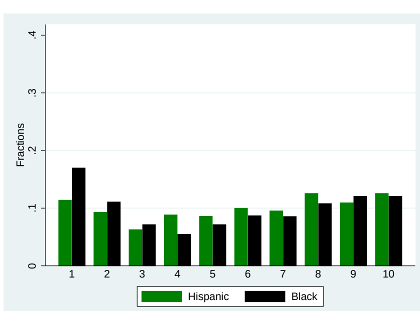
(d) Cognitive Stimulation (Males)



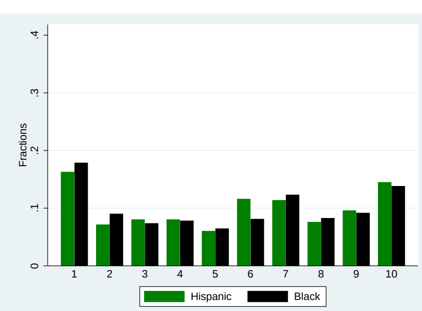
(e) Emotional Support (Females)



(f) Emotional Support (Males)



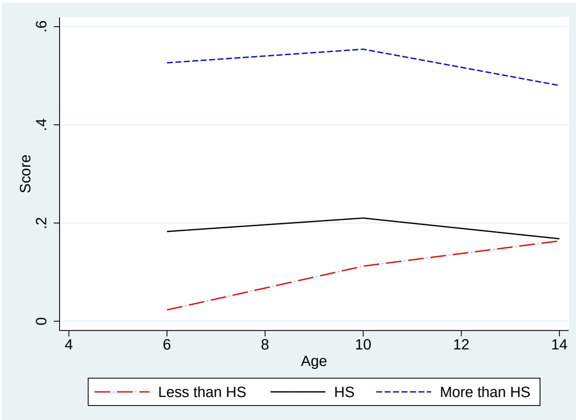
(g) Father Engagement (Females)



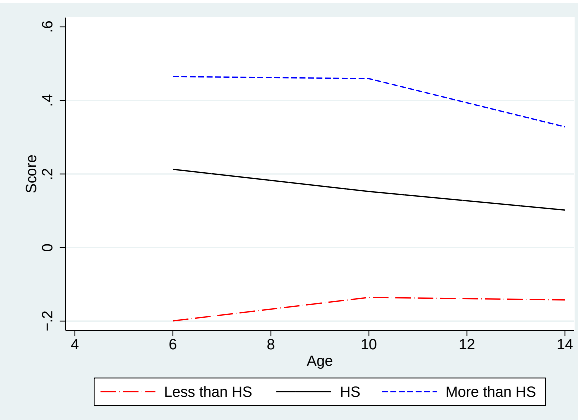
(h) Father Engagement (Males)

Source: Moon (2014).

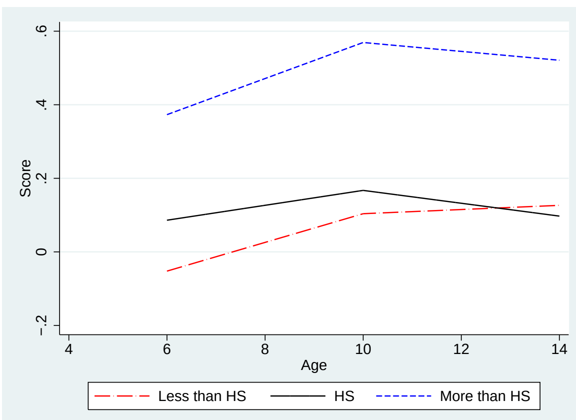
Figure B.7: Parental Investment over Childhood among Whites by Mother's Education



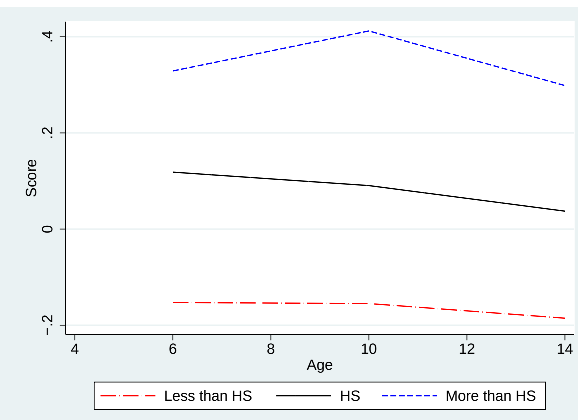
(a) Girls: Material Resource



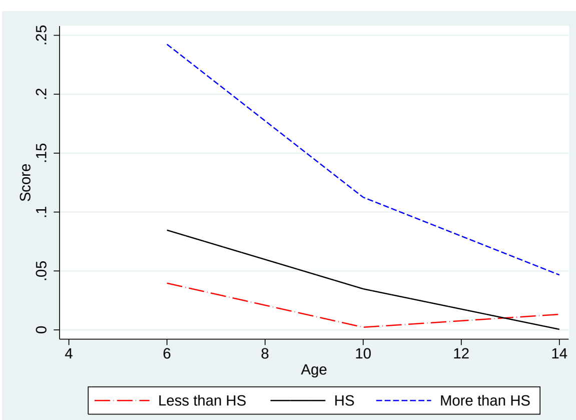
(b) Boys: Material Resource



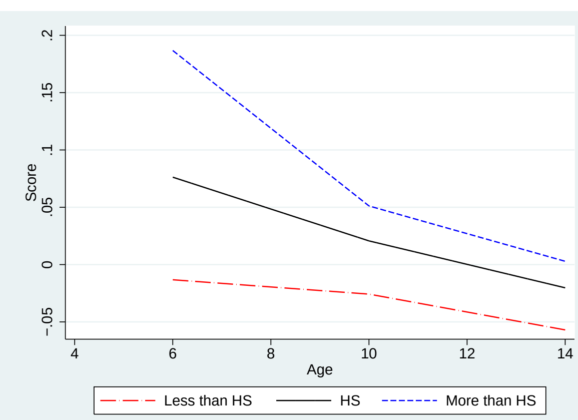
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



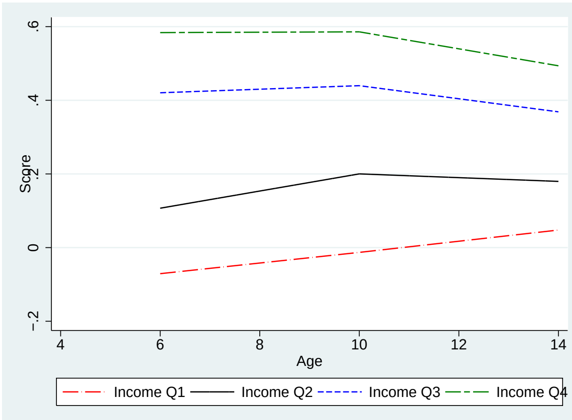
(e) Girls: Emotional Support



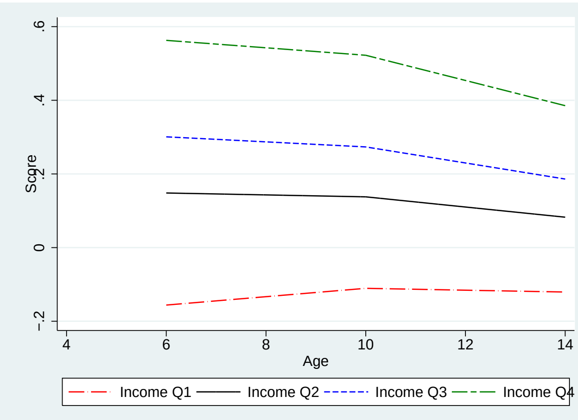
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

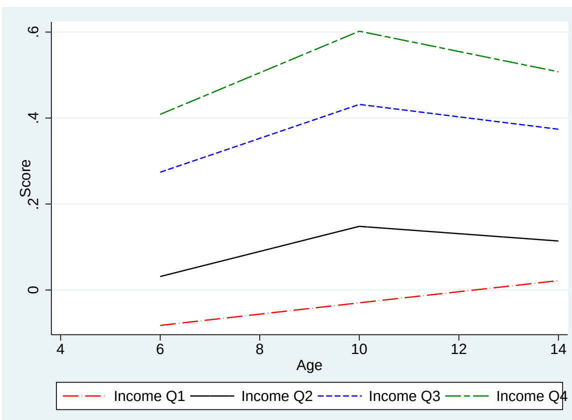
Figure B.8: Parental Investment over Childhood among Whites by Family Income Quartile



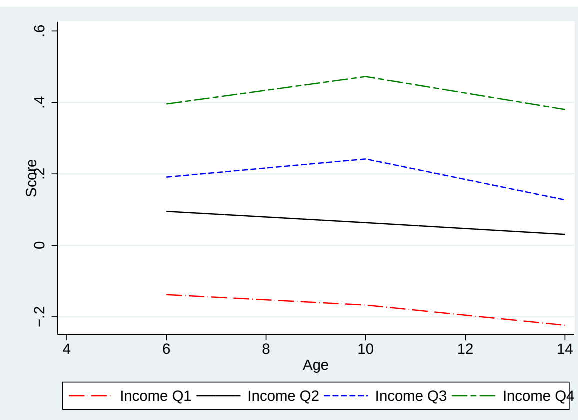
(a) Girls: Material Resource



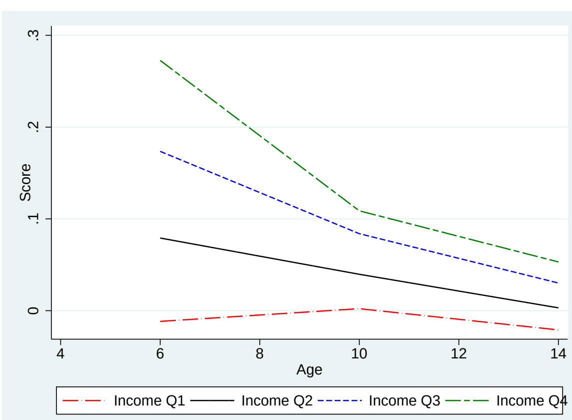
(b) Boys: Material Resource



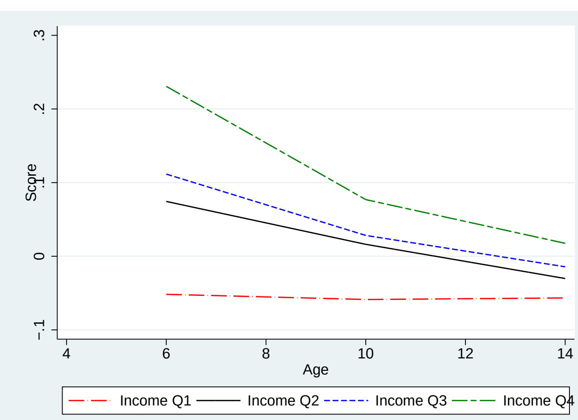
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



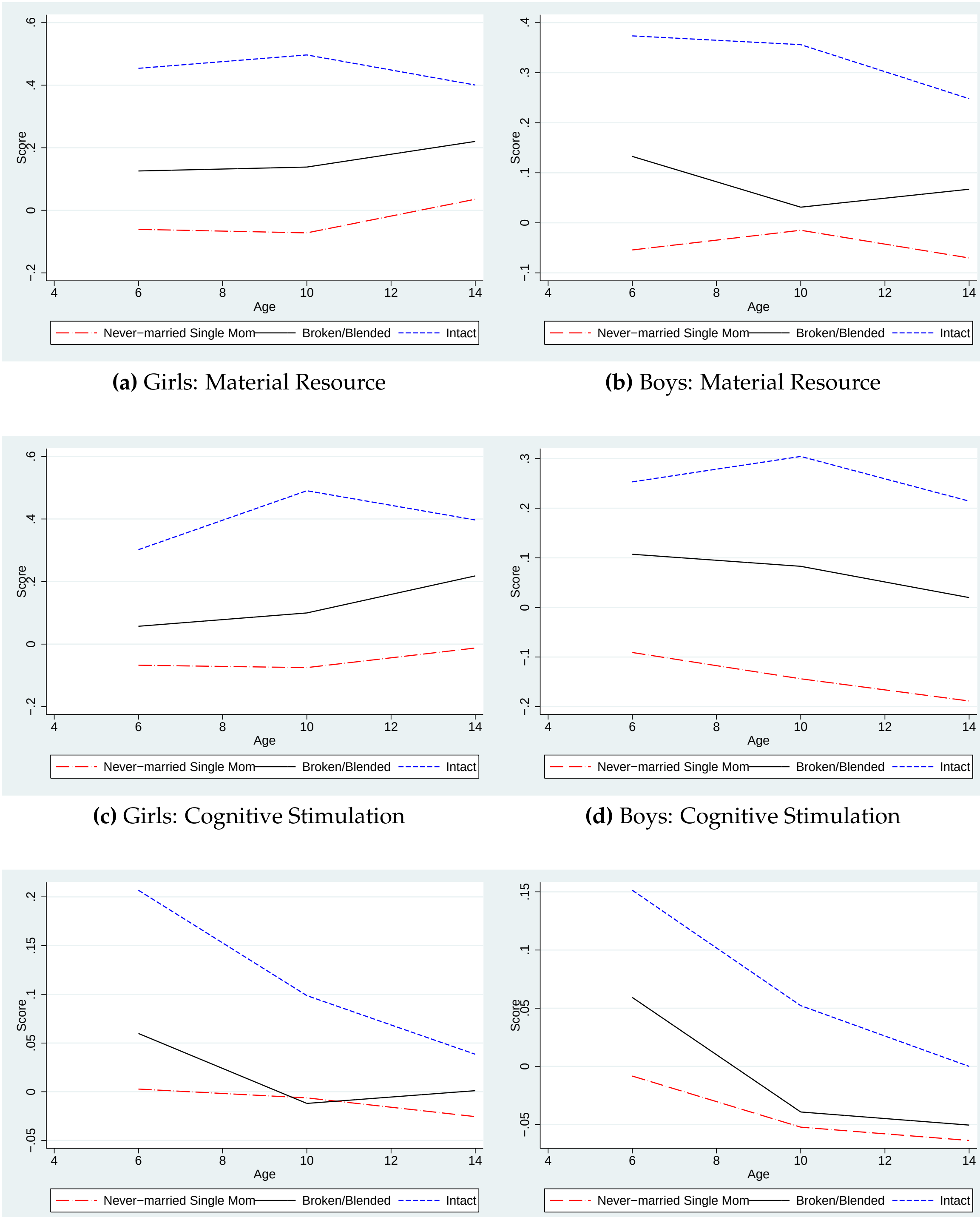
(e) Girls: Emotional Support



(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

Figure B.9: Parental Investment over Childhood among Whites by Family Type



(a) Girls: Material Resource

(b) Boys: Material Resource

(c) Girls: Cognitive Stimulation

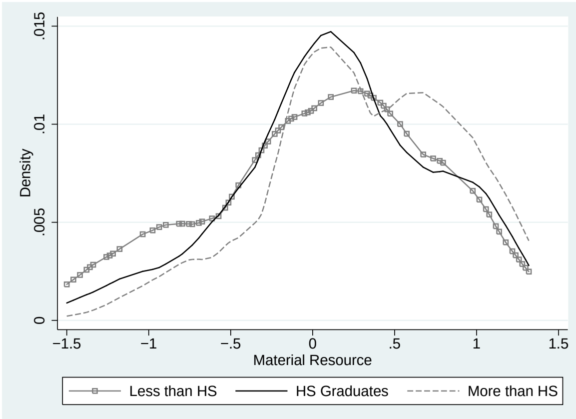
(d) Boys: Cognitive Stimulation

(e) Girls: Emotional Support

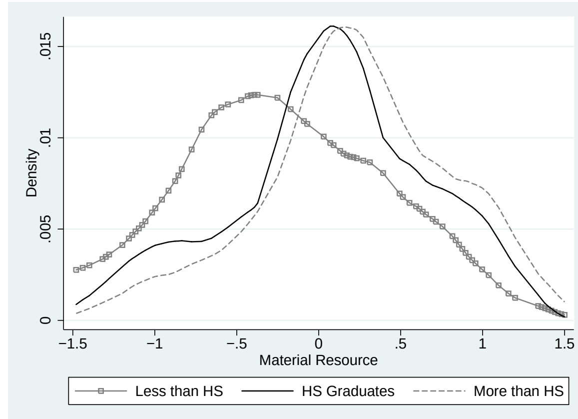
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

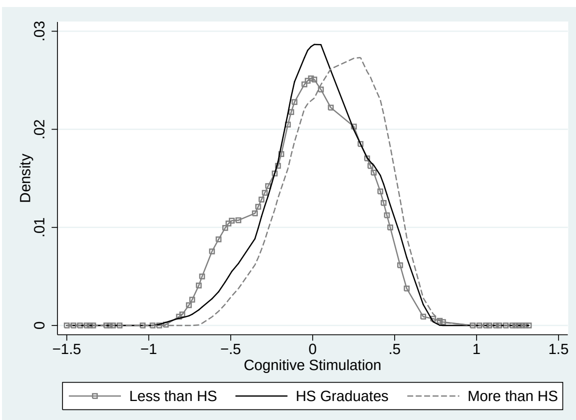
Figure B.10: Parental Investment among Whites by Mother's Education: Age 0-3



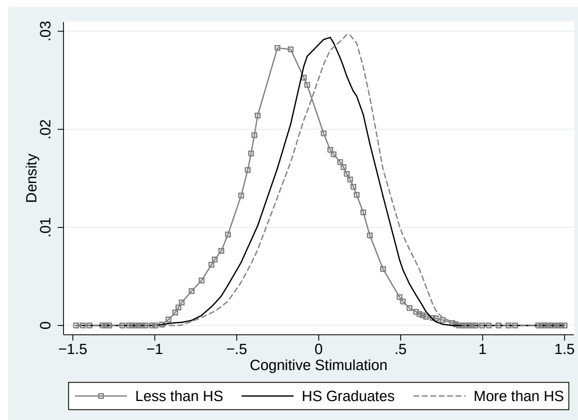
(a) Girls: Material Resource



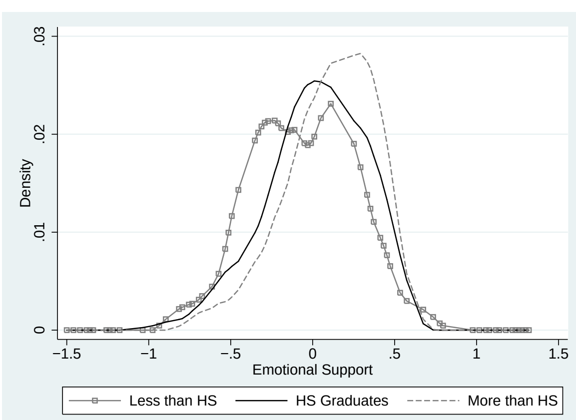
(b) Boys: Material Resource



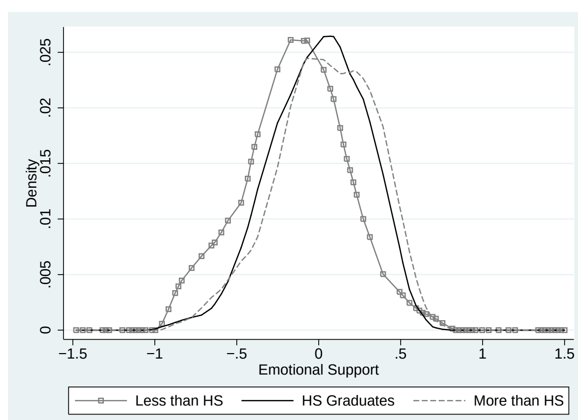
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



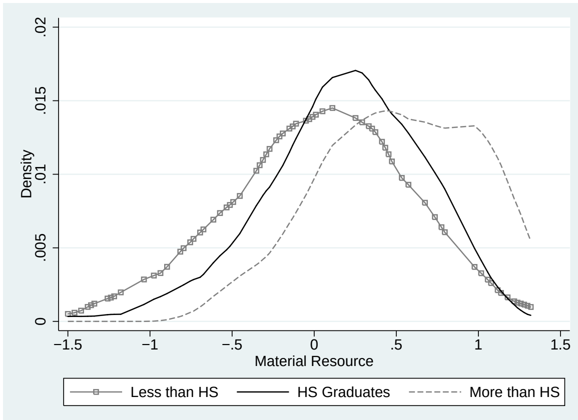
(e) Girls: Emotional Support



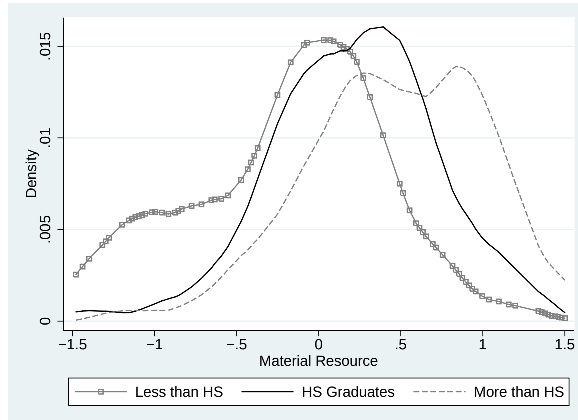
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

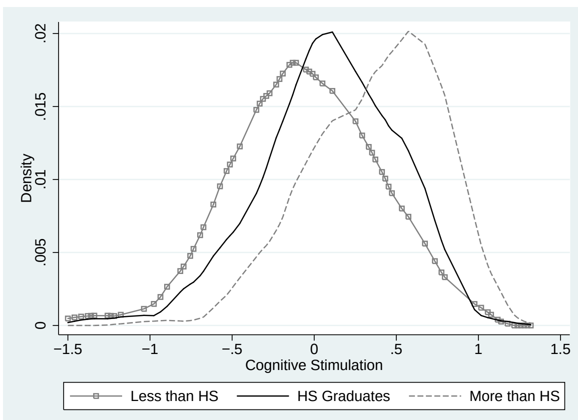
Figure B.11: Parental Investment among Whites by Mother's Education: Age 4-7



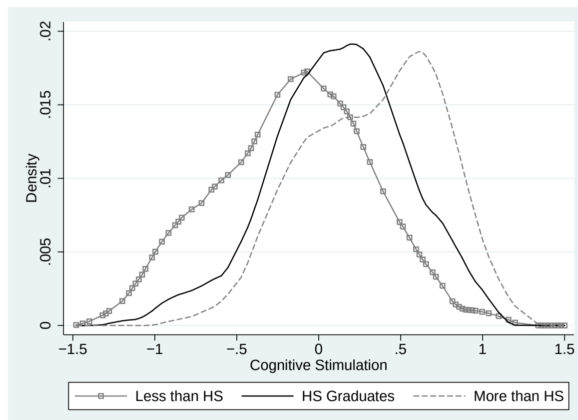
(a) Girls: Material Resource



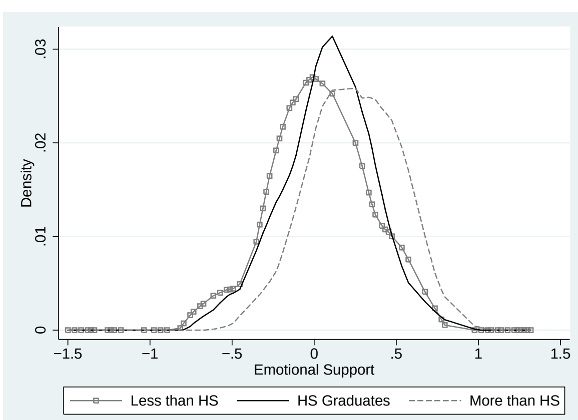
(b) Boys: Material Resource



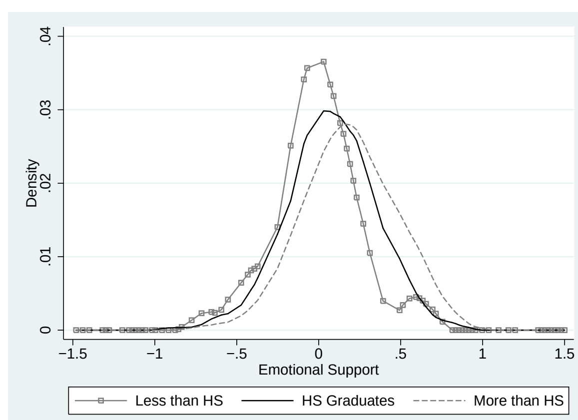
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



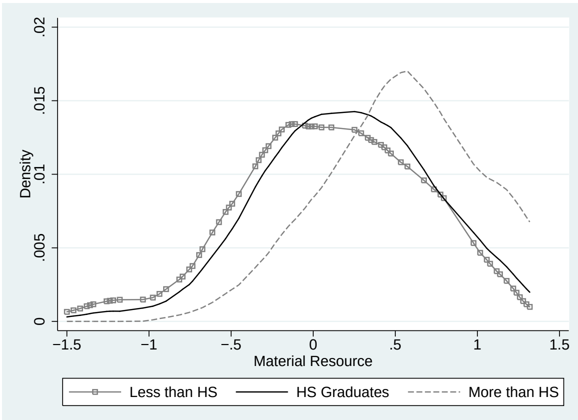
(e) Girls: Emotional Support



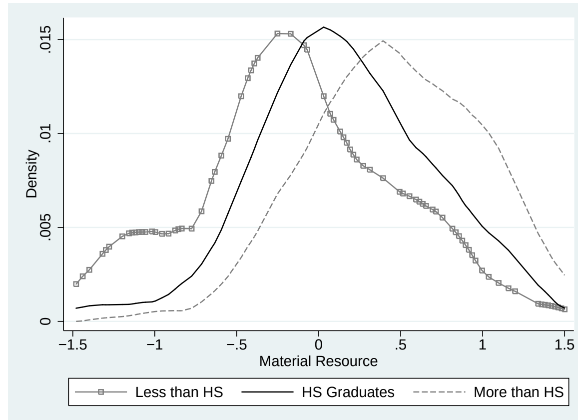
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

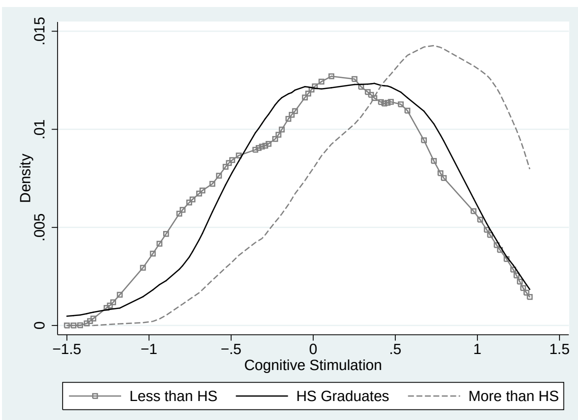
Figure B.12: Parental Investment among Whites by Mother's Education: Age 8-11



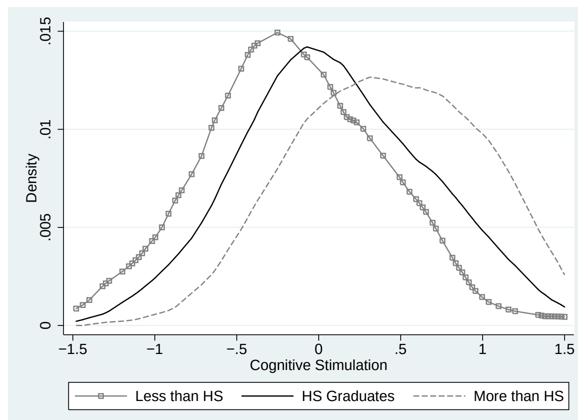
(a) Girls: Material Resource



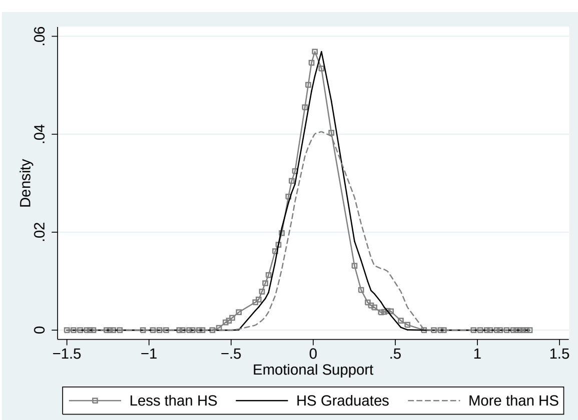
(b) Boys: Material Resource



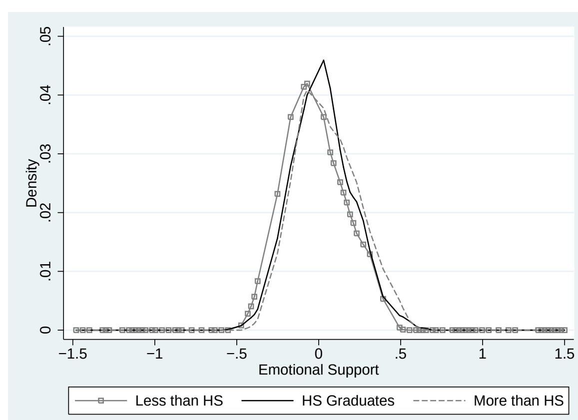
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



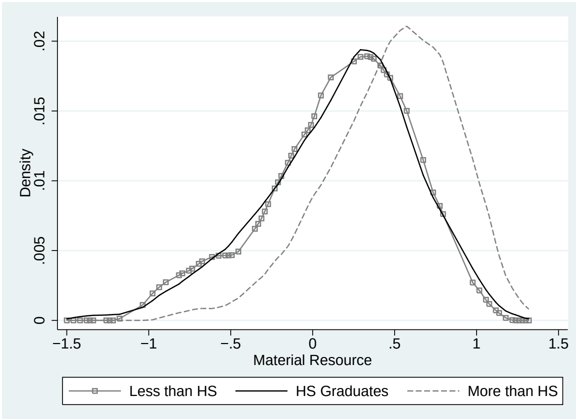
(e) Girls: Emotional Support



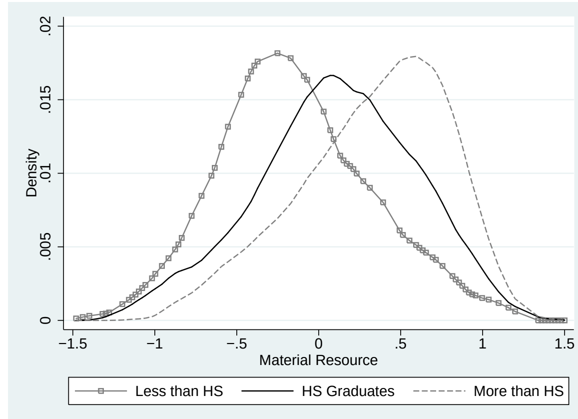
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

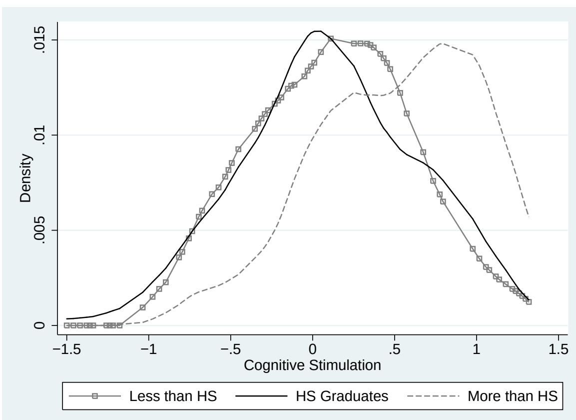
Figure B.13: Parental Investment among Whites by Mother's Education: Age 12-15



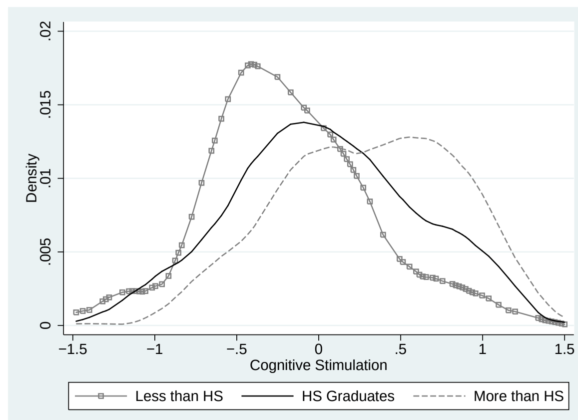
(a) Girls: Material Resource



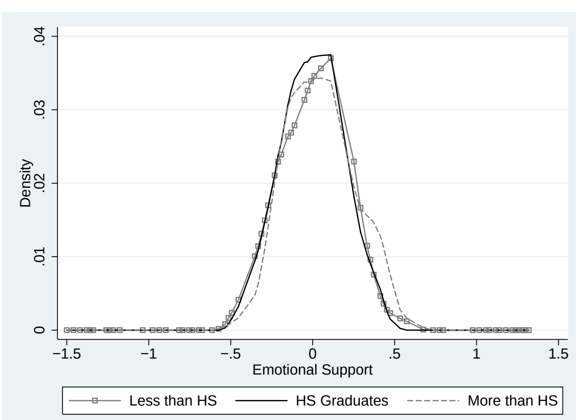
(b) Boys: Material Resource



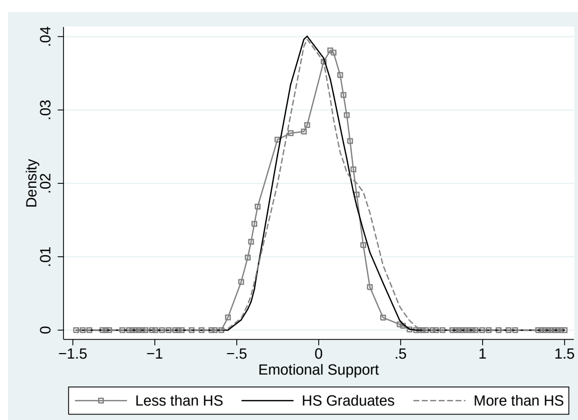
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



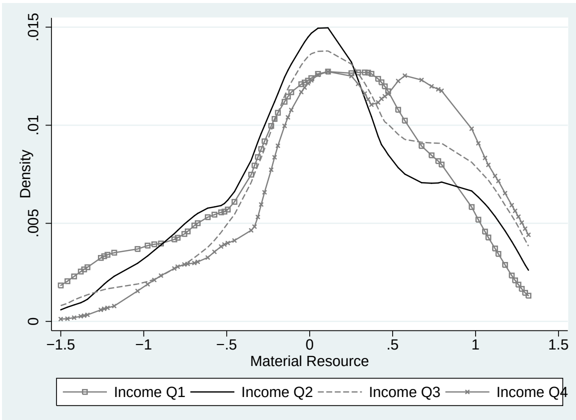
(e) Girls: Emotional Support



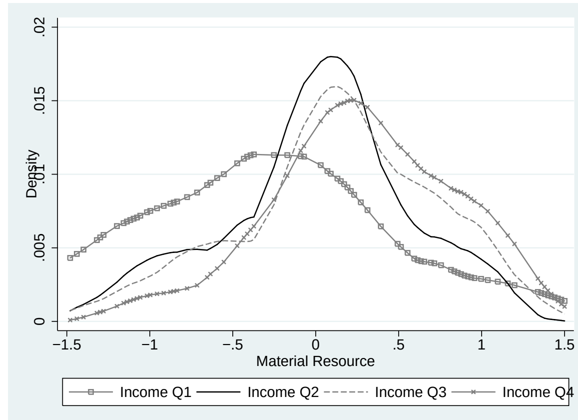
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

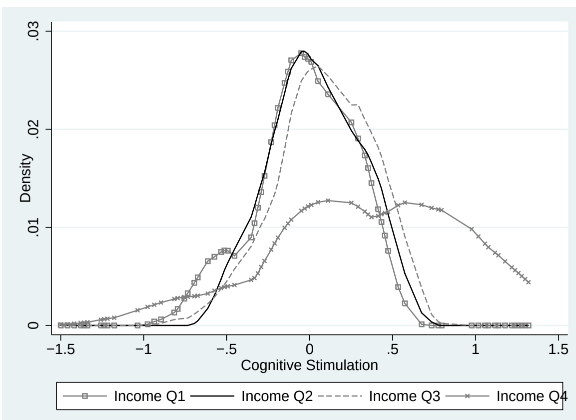
Figure B.14: Parental Investment among Whites by Family Income Quartile: Age 0-3



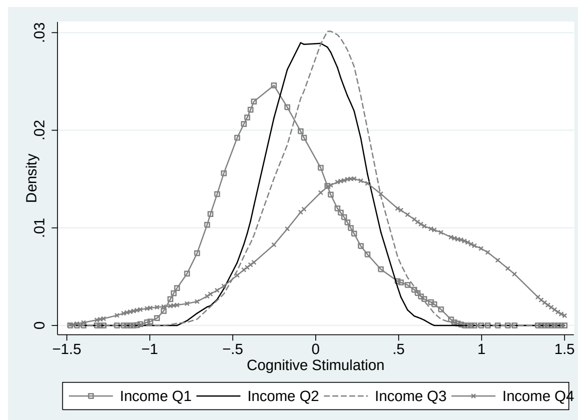
(a) Girls: Material Resource



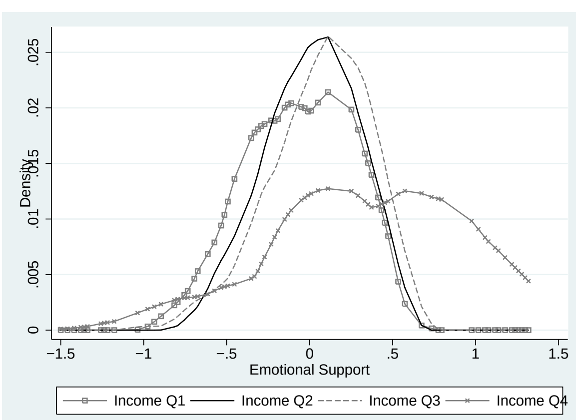
(b) Boys: Material Resource



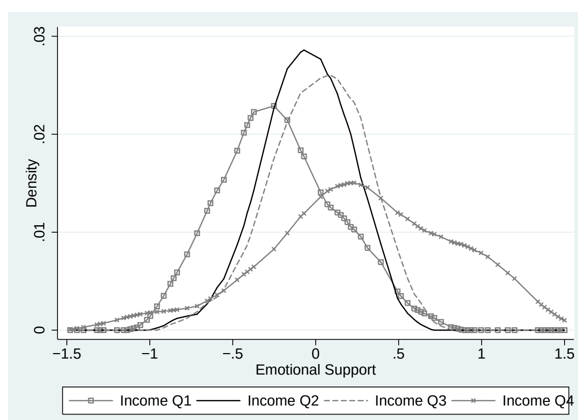
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



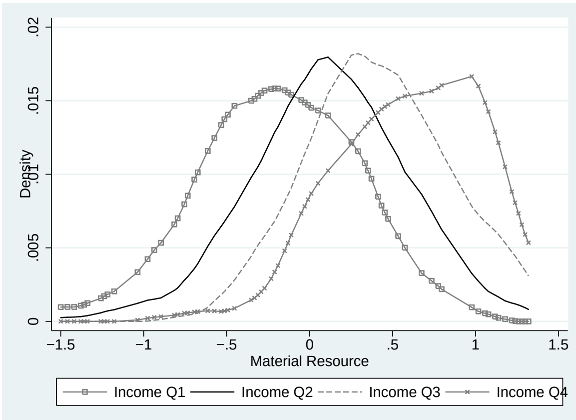
(e) Girls: Emotional Support



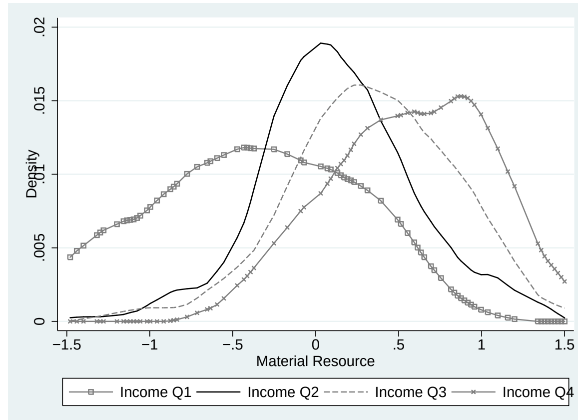
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

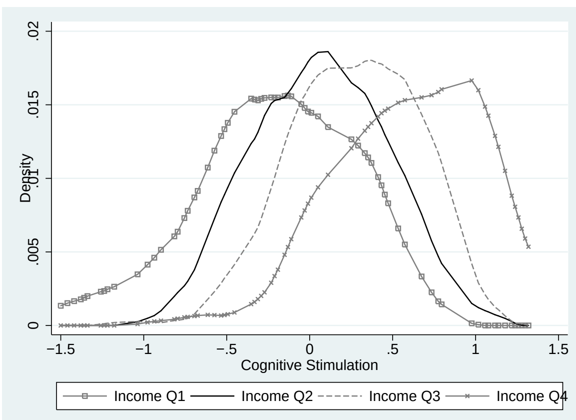
Figure B.15: Parental Investment among Whites by Family Income Quartile: Age 4-7



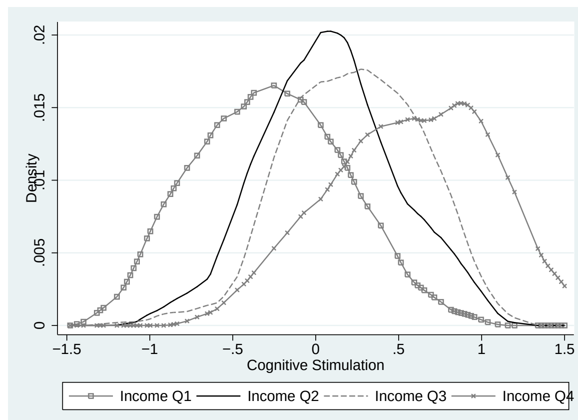
(a) Girls: Material Resource



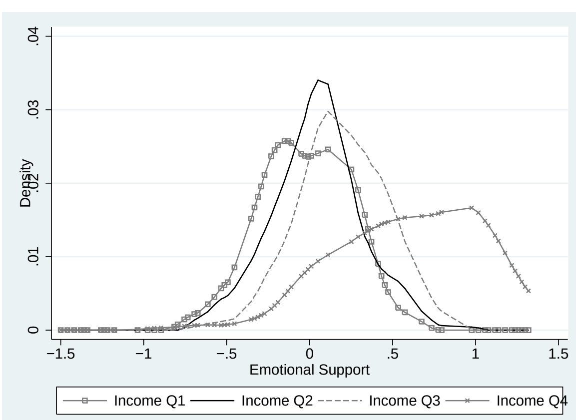
(b) Boys: Material Resource



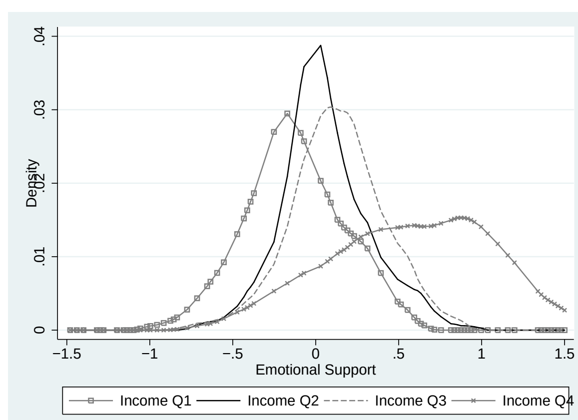
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



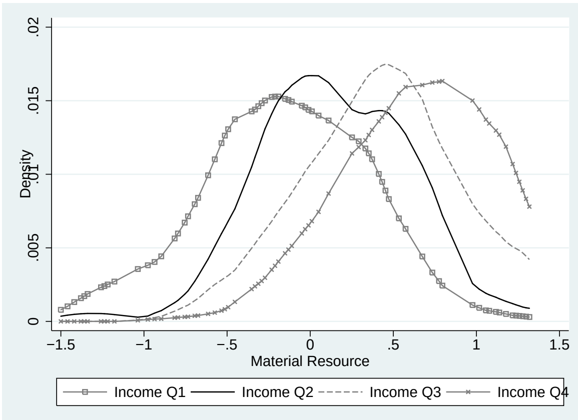
(e) Girls: Emotional Support



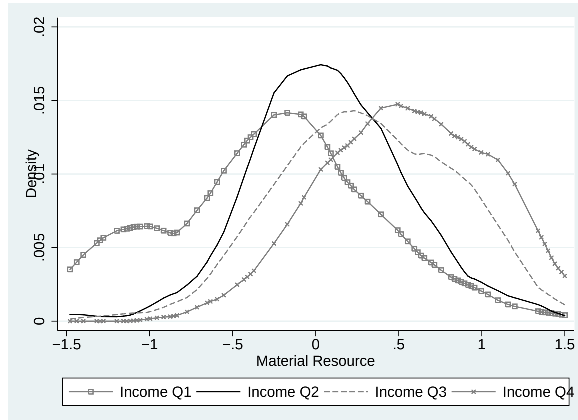
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

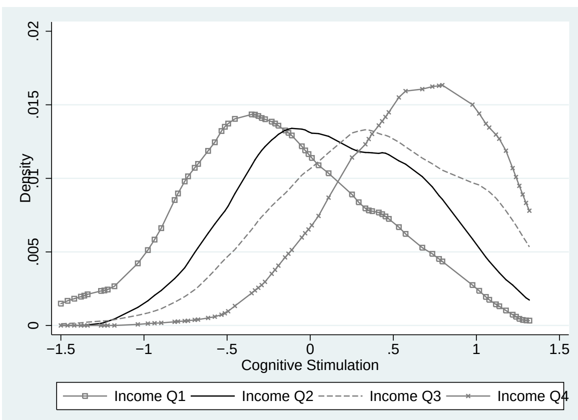
Figure B.16: Parental Investment among Whites by Family Income Quartile: Age 8-11



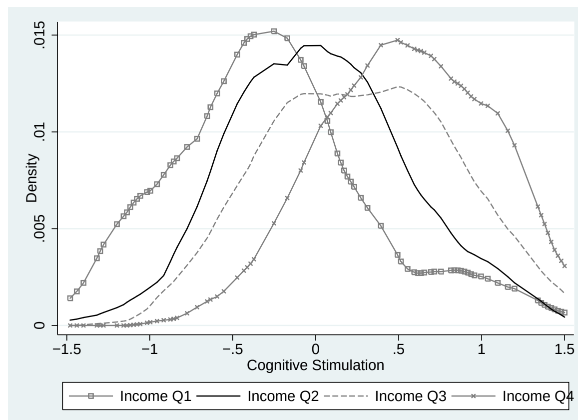
(a) Girls: Material Resource



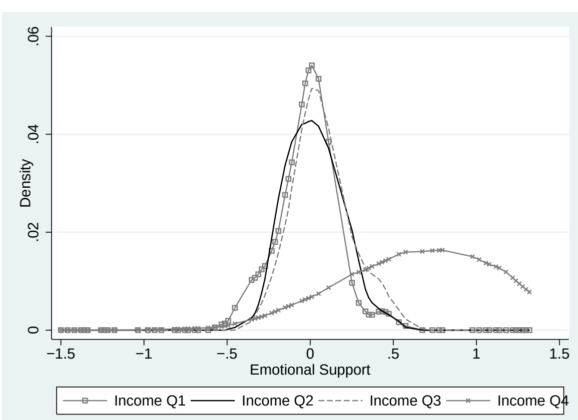
(b) Boys: Material Resource



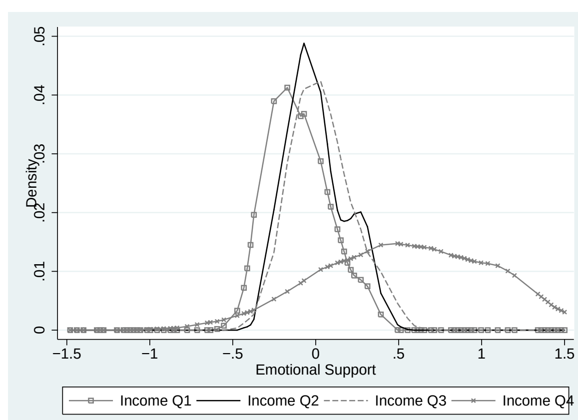
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



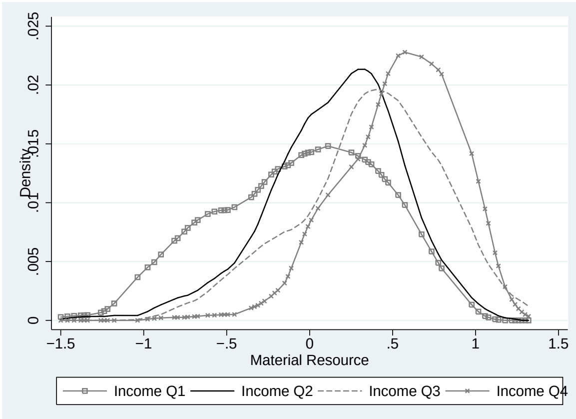
(e) Girls: Emotional Support



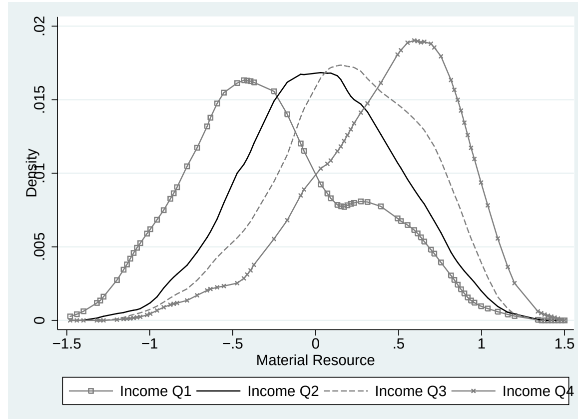
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

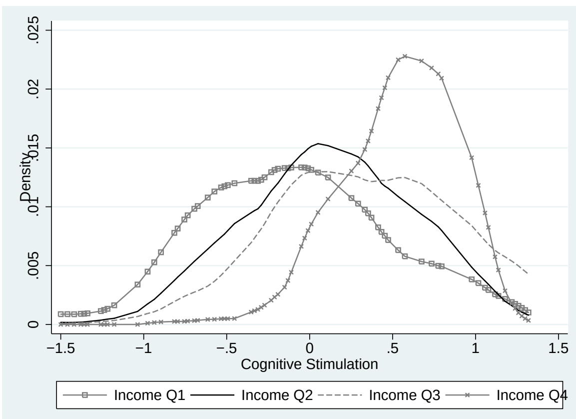
Figure B.17: Parental Investment among Whites by Family Income Quartile: Age 12-15



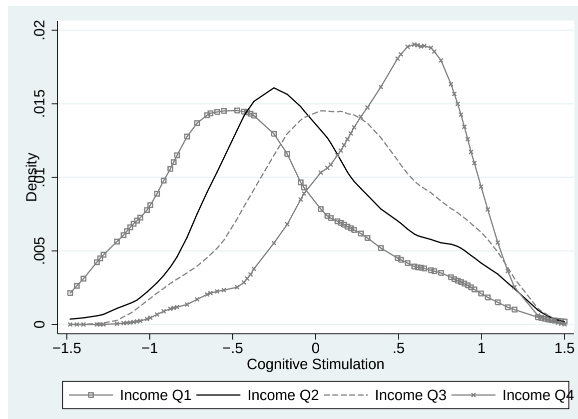
(a) Girls: Material Resource



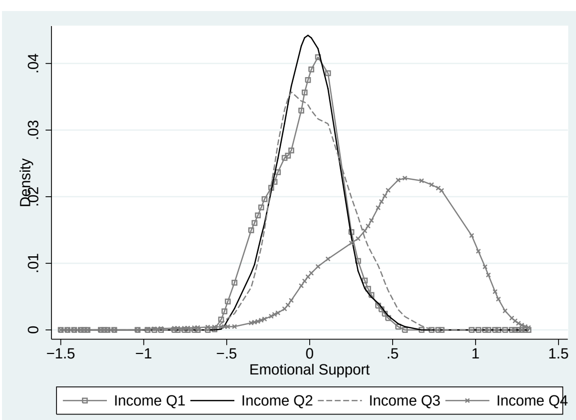
(b) Boys: Material Resource



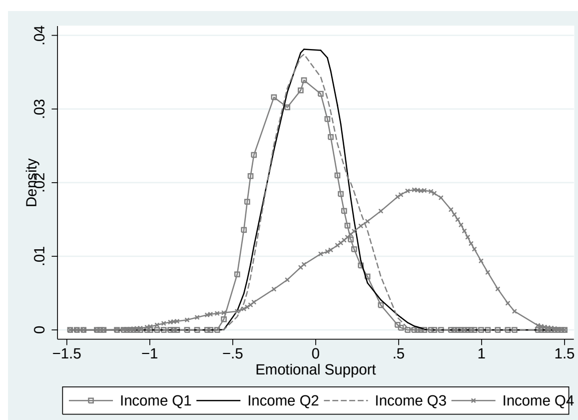
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



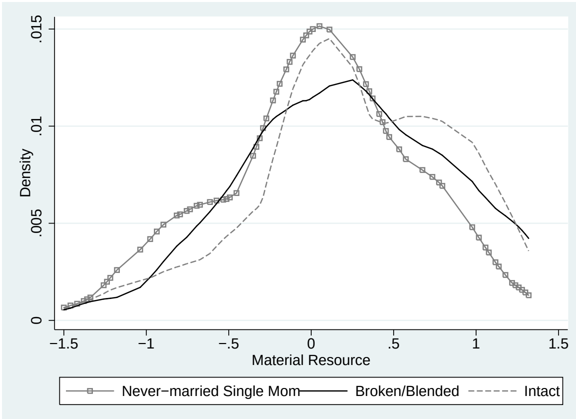
(e) Girls: Emotional Support



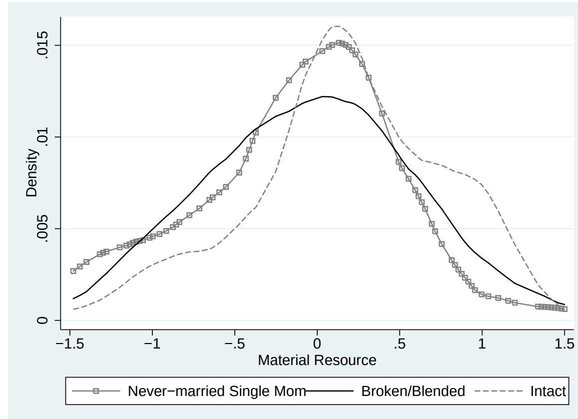
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

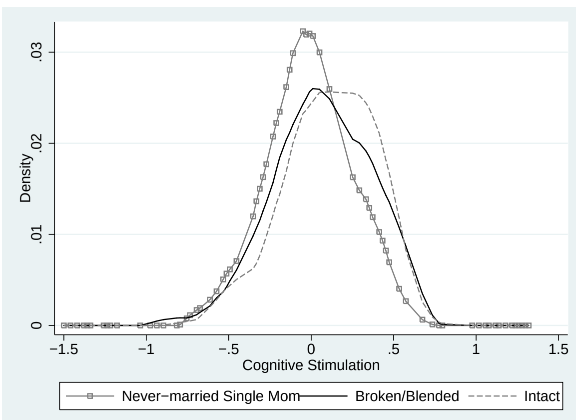
Figure B.18: Parental Investment among Whites by Family Structure: Age 0-3



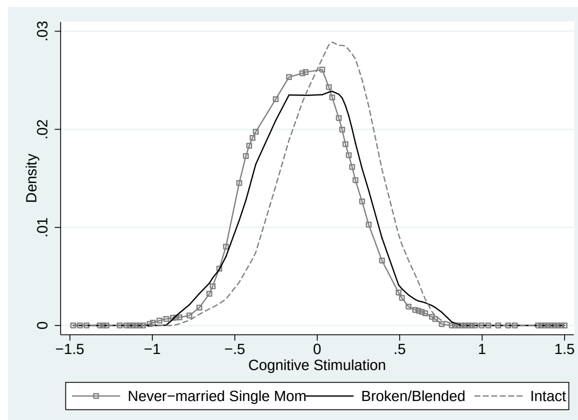
(a) Girls: Material Resource



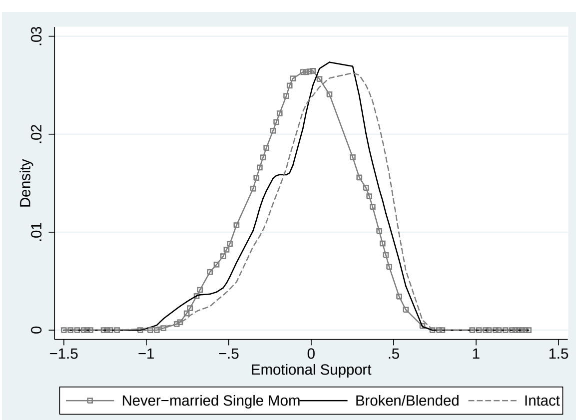
(b) Boys: Material Resource



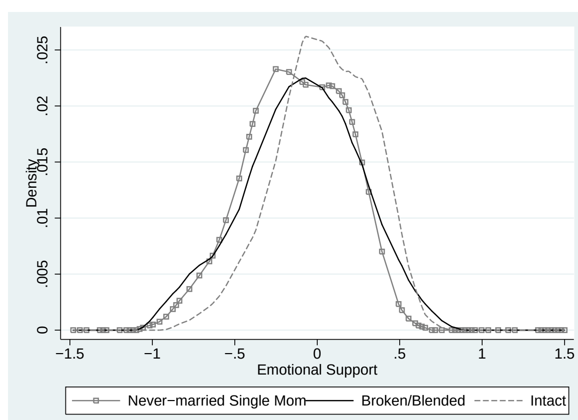
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



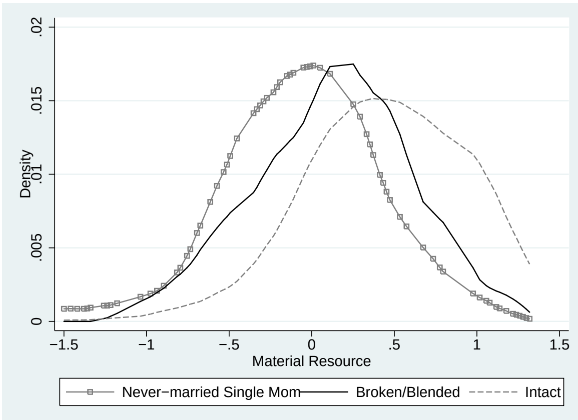
(e) Girls: Emotional Support



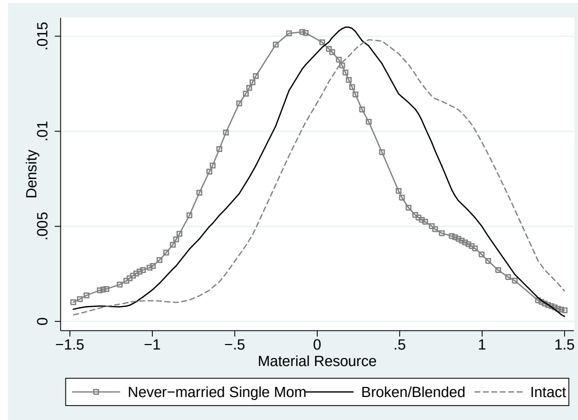
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

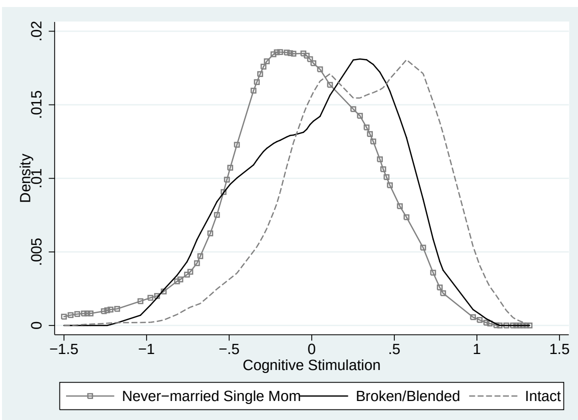
Figure B.19: Parental Investment among Whites by Family Structure: Age 4-7



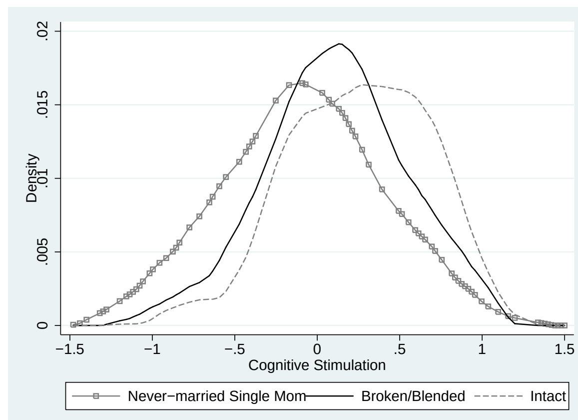
(a) Girls: Material Resource



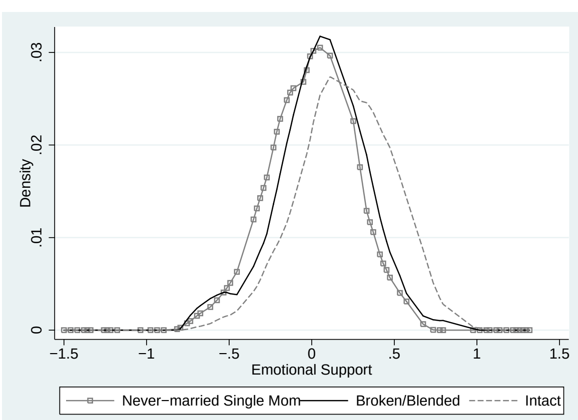
(b) Boys: Material Resource



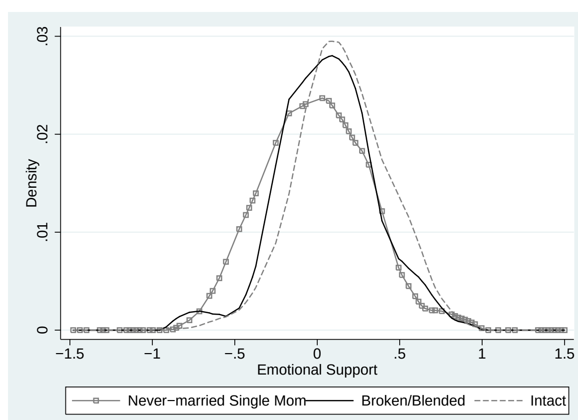
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



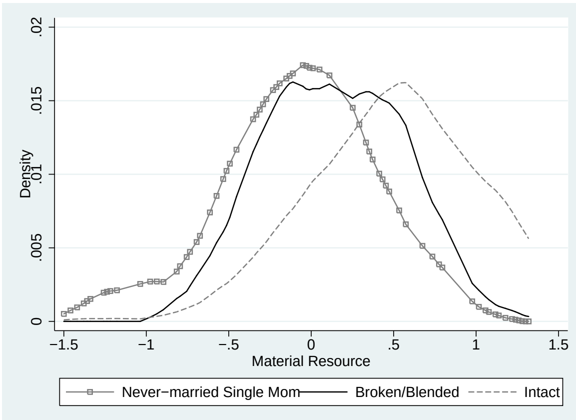
(e) Girls: Emotional Support



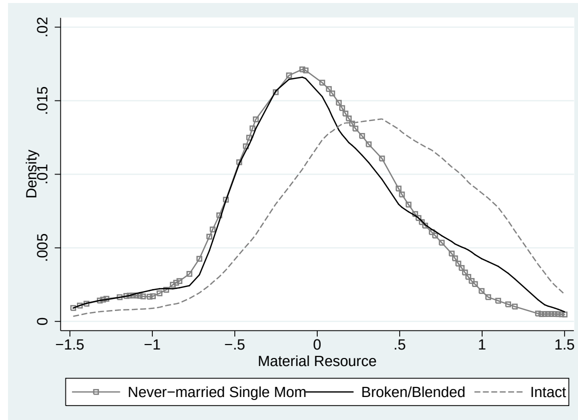
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

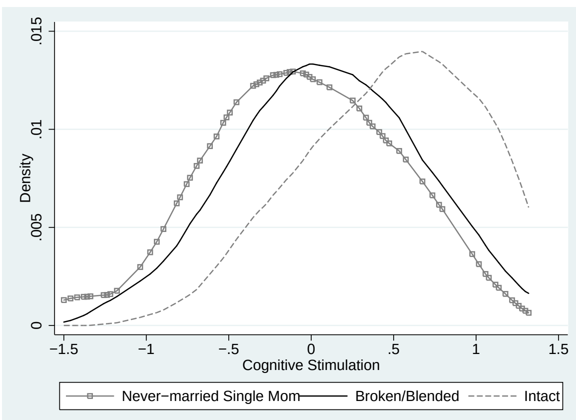
Figure B.20: Parental Investment among Whites by Family Structure: Age 8-11



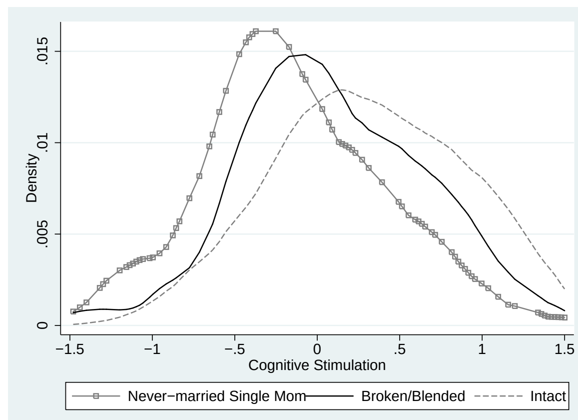
(a) Girls: Material Resource



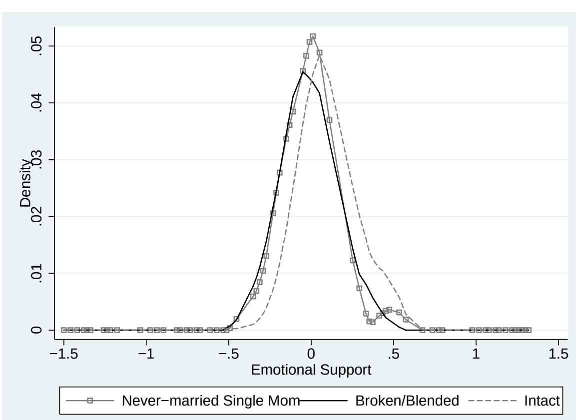
(b) Boys: Material Resource



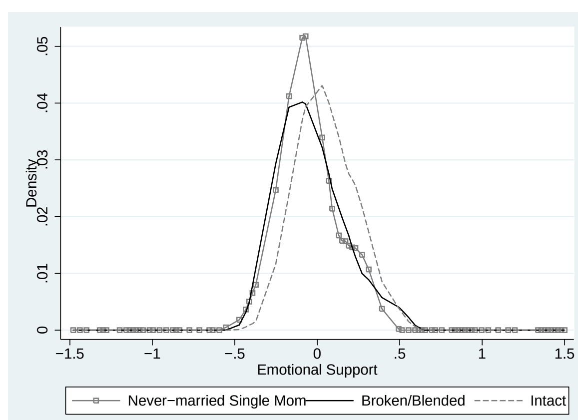
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



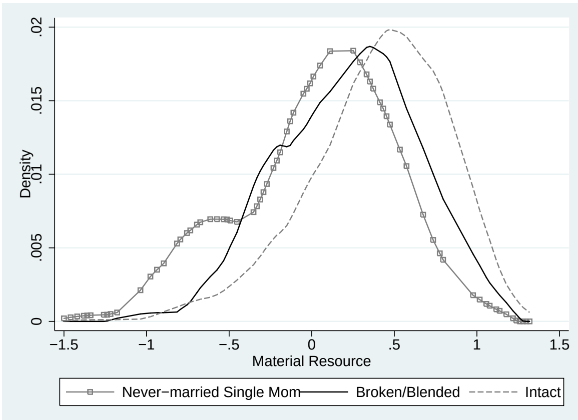
(e) Girls: Emotional Support



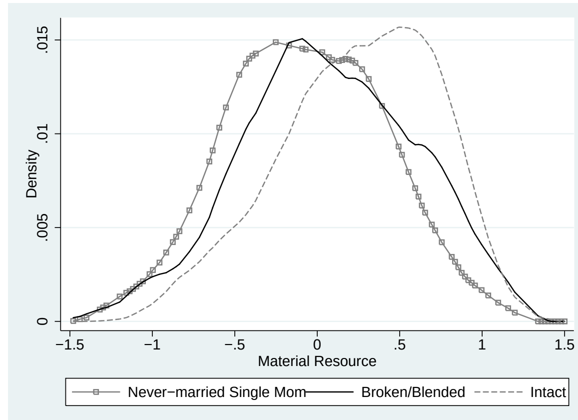
(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

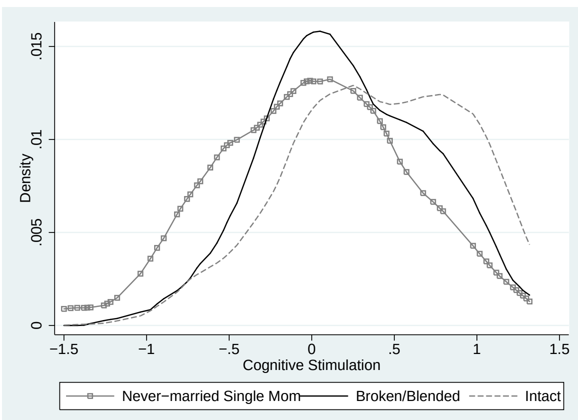
Figure B.21: Parental Investment among Whites by Family Structure: Age 12-15



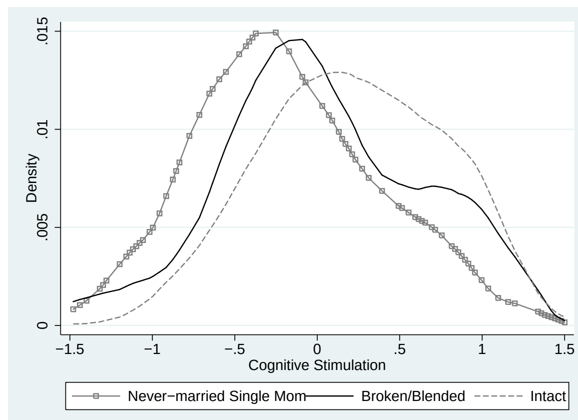
(a) Girls: Material Resource



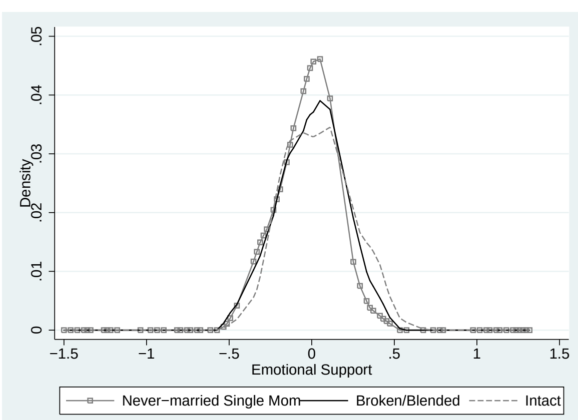
(b) Boys: Material Resource



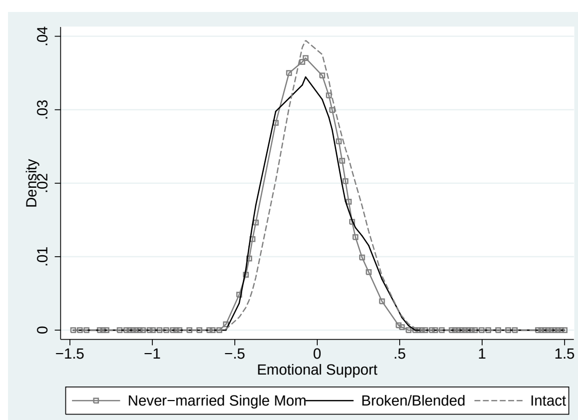
(c) Girls: Cognitive Stimulation



(d) Boys: Cognitive Stimulation



(e) Girls: Emotional Support



(f) Boys: Emotional Support

Data: A balanced panel from Children of National Longitudinal Survey of Youth 1979.
Source: Moon (2014).

B.2 Analyses of Lareau

Table B.1–B.4 (Lareau and Weininger (2008)) present evidence on the heterogeneity in parental behavior with their children according to family characteristics and maternal education.

Table B.1: Average Number of Organized Leisure Activities Child Participates in by Social Class: Lareau Data on 88 Children*

	Poor	Working Class	Middle Class
All Children			
Organized Activities	1.5	2.5	4.9
Items with Missing Data**	2.0	3.0	2.5
Count	26	26	36
Gender			
Organized Activities: Boys	1.5	2.6	5.1
Items with Missing Data: Boys**	2.1	3.8	3.4
Count	11	14	18
Organized Activities: Girls	1.5	2.5	4.7
Items with Missing Data: Girls**	1.9	2.1	1.5
Count	15	12	18
Race			
Organized Activities: Whites	1.4	2.3	4.6
Items with Missing Data: Whites**	0.9	2.3	2.9
Count	12	14	18
Organized Activities: Blacks	1.6	2.8	5.2
Items with Missing Data: Whites**	2.9	3.8	2.0
Count	14	12	18

Source: Lareau and Weininger (2008, Table 10.2).

*Organized activities include: Brownies or Cub Scouts, music lessons, team sports (soccer, Little League, etc.), non-team sports (gymnastics, karate, etc.), Tot Tumbling, dance lessons (ballet, tap, etc.), religious classes, choir, art classes, and any activity offered through a recreational center that requires formal enrollment.

**Not every respondent was asked about all of the activities that were eventually coded (though each was asked if his/her child participated in any activities not explicitly mentioned).

Table B.2: Children's Participation in Organized Leisure (yes/no) by Mother's Education, Gender, and Race: National Data

	Mother's Education				
	LT HS	HS Degree	Some College	Bachelor's or Higher	Total
All Children					
% who Participate	57.1	69.1	82.1	93.6	77.6
Count	253	630	460	290	1,633
Gender					
% Boys who Participate	62.5	69.1	75.8	93.6	75.8
Count	132	313	224	139	808
% Girls who Participate	50.4	69.0	88.3	93.6	79.4
Count	121	317	236	151	825
Race*					
% Whites who Participate	59.9	75.1	87.9	94.0	83.4
Count	66	294	240	243	843
% Blacks who Participate	54.2	51.8	59.0	88.3	57.0
Count	187	336	220	47	790

Taken from Lareau and Weininger (2008).

Source: 1997 Child Development Supplement to the Panel Study of Income Dynamics. Data are weighted. Includes children between the ages of six and twelve years old.

*Data reported on blacks and whites only due to low cell frequencies for other categories.

Table B.3: Children's Average Weekly Hours in Organized Leisure by Mother's Education, Gender, and Race: National Data

	Mother's Education				
	LT HS	HS Degree	Some College	Bachelor's or Higher	Total
All Children					
Mean Weekly Hours	2.02	2.91	3.38	4.82	3.45
Count	179	509	387	250	1,325
Gender					
Mean Weekly Hours: Boys	1.59	2.84	3.72	5.53	3.59
Count	91	258	187	121	657
Mean Weekly Hours: Girls	2.56	2.99	3.04	4.21	3.31
Count	88	251	200	129	668
Race*					
Mean Weekly Hours: Whites	0.90	3.25	3.52	5.03	3.73
Count	44	249	212	212	717
Mean Weekly Hours: Blacks	3.02	1.84	2.81	2.02	2.40
Count	135	260	175	38	608

Taken from Lareau and Weininger (2008).

Source: 1997 Child Development Supplement to the Panel Study of Income Dynamics. Data are weighted. Includes children between the ages of six and twelve years old.

*Data reported on blacks and whites only due to low cell frequencies for other categories.

Table B.4: Children's Average Weekly Hours in Organized Leisure by Mother's Education and Employment Status: National Data

	Mother's Education				
	LT HS	HS Degree	Some College	Bachelor's or Higher	Total
Mother's Employment Status					
Not Employed	2.80	4.35	3.79	5.96	4.28
Count	79	104	55	42	280
Employed less than 35 hrs/wk	2.58	3.74	3.30	5.31	4.03
Count	35	120	108	89	352
Employed 35 hrs/wk or more	0.95	2.01	3.29	3.92	2.76
Count	65	285	224	119	693

Taken from Lareau and Weininger (2008).

Source: 1997 Child Development Supplement to the Panel Study of Income Dynamics. Data are weighted. Includes children between the ages of six and twelve years old.

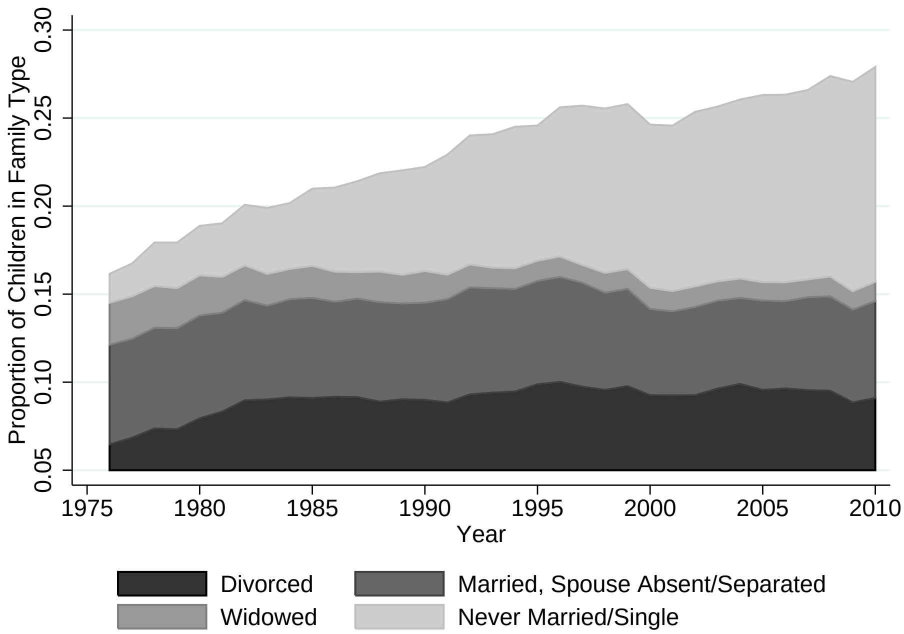
*Data reported on blacks and whites only due to low cell frequencies for other categories.

B.3 College Enrollment by Income and Ability

C Time Trends on Children in Single Parent Households

Trends by Marital Status

Figure C.1: Children in Single Parent Households by Marital Status—All Education Levels, All Races

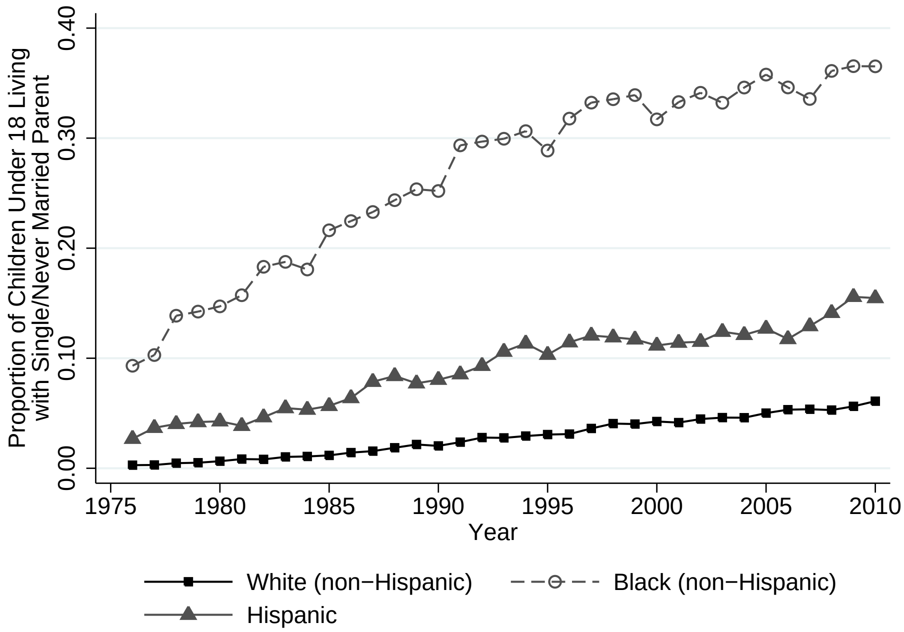


Source: Heckman (2011, Web Appendix).

Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of household. Children who have been married or are not living with their parents are excluded from the calculation. Separated parents are included in "Married, Spouse Absent" Category.

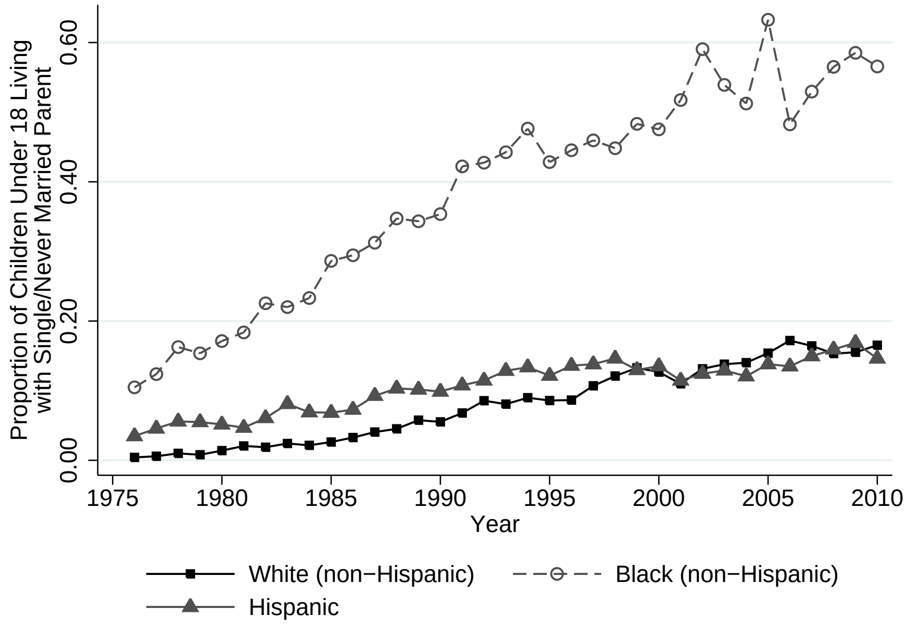
Trends for Children in Single/Never Married Households by Race

Figure C.2: Children in Households with Single, Never Married Parents by Race



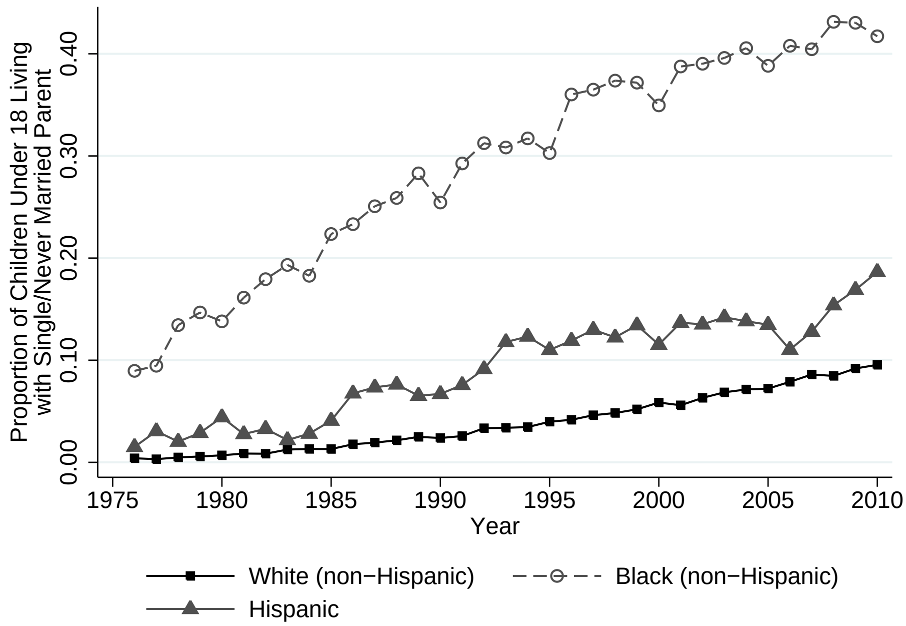
Source: Heckman (2011, Web Appendix).
Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation.

Figure C.3: Children in Households with Single, Never Married Parents by Race - Dropouts



Source: Heckman (2011, Web Appendix).
Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation. The "Dropout" category includes individuals who have finished 11 years of school or less.

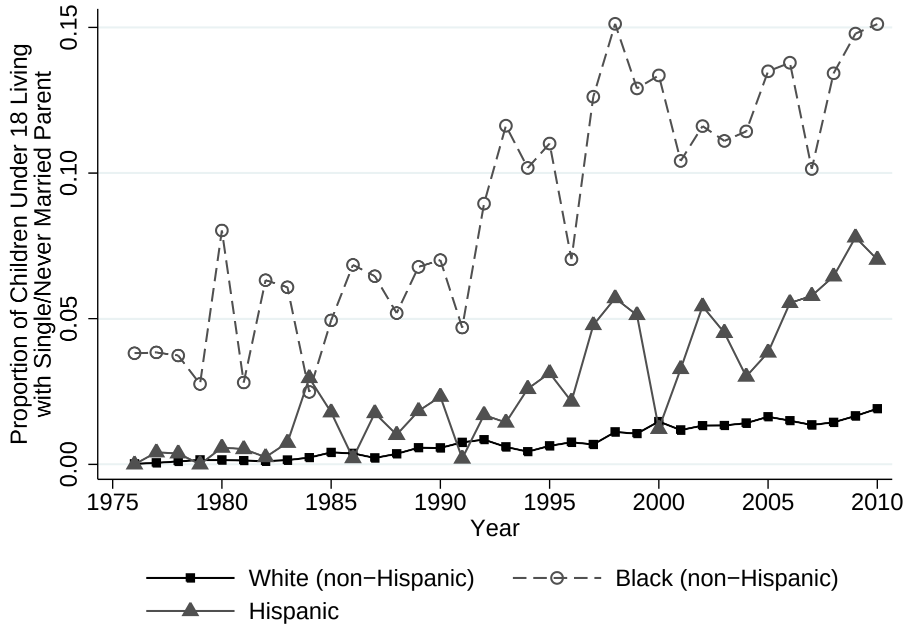
Figure C.4: Children in Households with Single, Never Married Parents by Race - High School Graduates



Source: Heckman (2011, Web Appendix).

Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation. For consistency across CPS waves, the "HS graduate" category is defined as any individual who completes 12 years of schooling, as specific degree status–whether having a high school diploma or an equivalency–is uncertain before 1992.

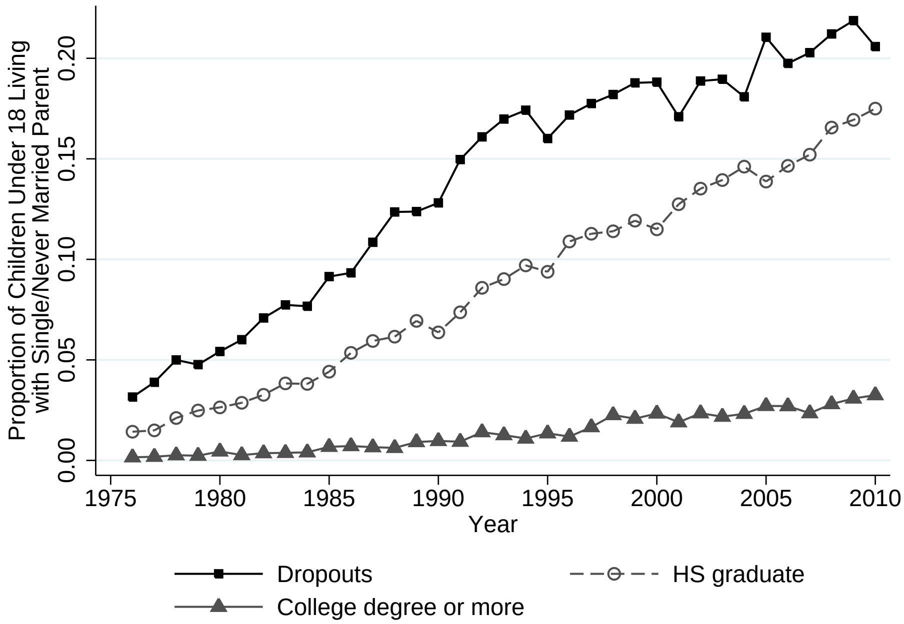
Figure C.5: Children in Households with Single, Never Married Parents by Race - College Graduates or More



Source: Heckman (2011, Web Appendix).
Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation. The "College degree or more" category is defined as individuals who have completed a Bachelor's or higher degree.

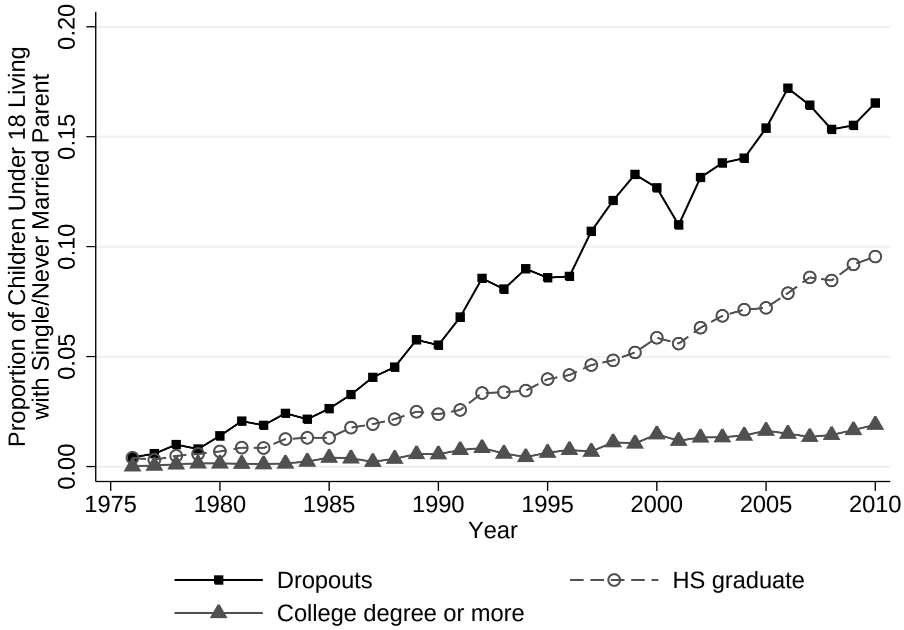
Trends in Children in Single/Never Married Households by Education

Figure C.6: Children in Households with Single, Never Married Parents by Education - All Races



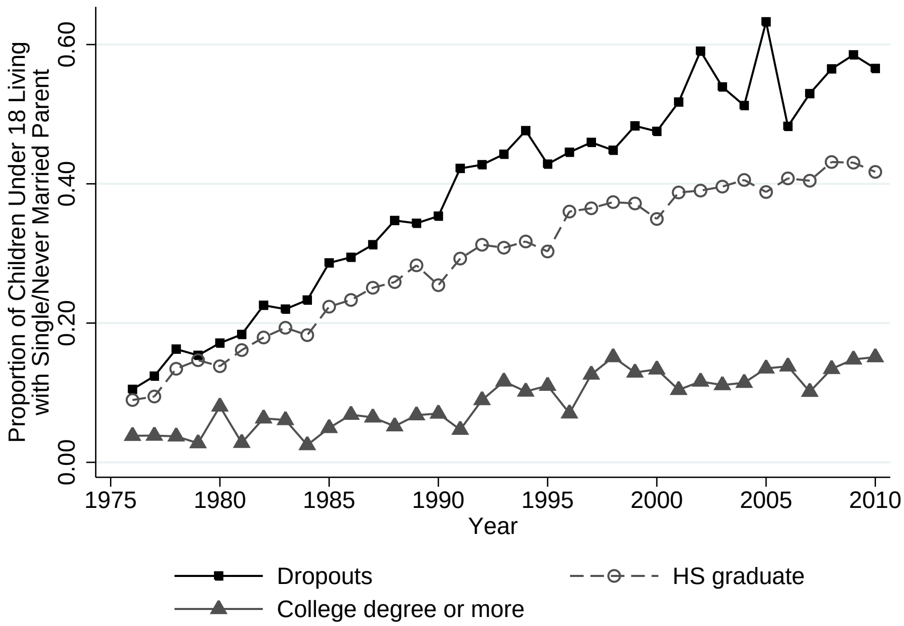
Source: Heckman (2011, Web Appendix).
Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation. The "Dropout" category includes individuals who have finished 11 years of school or less. For consistency across CPS waves, the "HS graduate" category is defined as any individual who completes 12 years of schooling, as specific degree status–whether having a high school diploma or an equivalency–is uncertain before 1992. The "College degree or more" category is defined as individuals who have completed a Bachelor's or higher degree.

Figure C.7: Children in Households with Single, Never Married Parents by Education - Non-Hispanic Whites



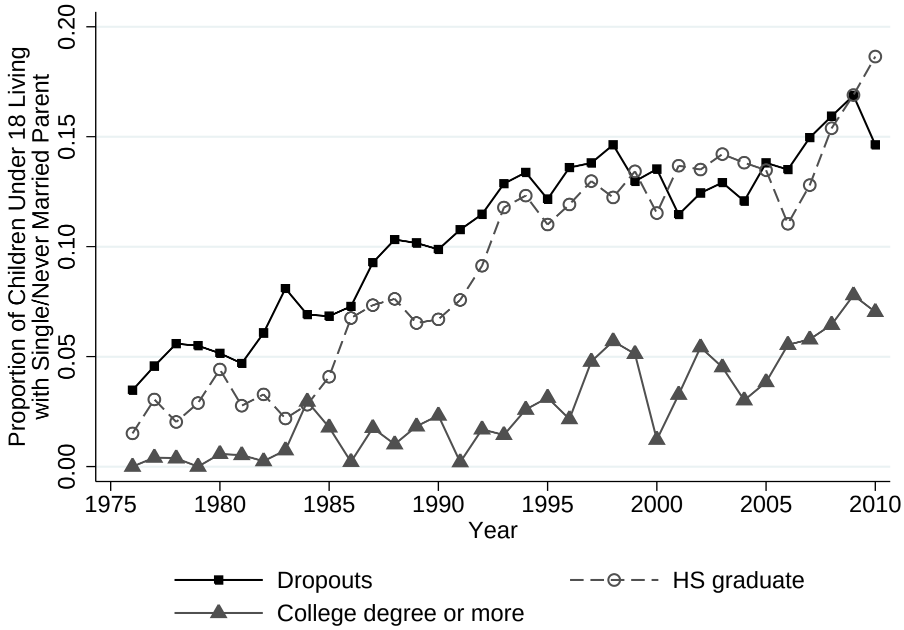
Source: Heckman (2011, Web Appendix).
Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation. The "Dropout" category includes individuals who have finished 11 years of school or less. For consistency across CPS waves, the "HS graduate" category is defined as any individual who completes 12 years of schooling, as specific degree status–whether having a high school diploma or an equivalency–is uncertain before 1992. The "College degree or more" category is defined as individuals who have completed a Bachelor's or higher degree.

Figure C.8: Children in Households with Single, Never Married Parents by Education - Non-Hispanic Blacks



Source: Heckman (2011, Web Appendix).
Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation. The "Dropout" category includes individuals who have finished 11 years of school or less. For consistency across CPS waves, the "HS graduate" category is defined as any individual who completes 12 years of schooling, as specific degree status–whether having a high school diploma or an equivalency–is uncertain before 1992. The "College degree or more" category is defined as individuals who have completed a Bachelor's or higher degree.

Figure C.9: Children in Households with Single, Never Married Parents by Education - Hispanics



Source: Heckman (2011, Web Appendix).

Note: Parents are defined as the head of the household. Children are defined as individuals under 18, living in the household, and the child of the head of the household. Children who have been married or are not living with their parents are excluded from the calculation. The "Dropout" category includes individuals who have finished 11 years of school or less. For consistency across CPS waves, the "HS graduate" category is defined as any individual who completes 12 years of schooling, as specific degree status–whether having a high school diploma or an equivalency–is uncertain before 1992. The "College degree or more" category is defined as individuals who have completed a Bachelor's or higher degree.

D Formal Models of Child Development

A basic model of skill formation that can rationalize many of the facts about the life cycle presented in Section 2 of the text was introduced into the literature in joint work with Flavio Cunha (Cunha and Heckman, 2007, 2009) and Cunha's highly original and important Ph.D. thesis (Cunha, 2007). It is a starting point for more general models. Multiple stages of childhood allow for a more nuanced policy analysis. Ability is scalar. It is partially inherited and partially created. The model explains the emergence of capability gaps over the life cycle. It relates gaps in capabilities to gaps in family investments. It models critical and sensitive periods in the life cycle and the importance of the early years. It explains why, everything else the same, remediation is less effective than prevention—why later life interventions for children born into disadvantage may not be as effective as early life interventions, a fact broadly consistent with the empirical literature. It justifies high returns for early investments in disadvantaged children and explains why early investment should be followed by later investment. This is a consequence of the emergence of dynamic complementarity. The model explains why investments in low-ability adolescents are not, in general, productive and why early credit constraints *might* have bigger effects on child outcomes than later constraints. It models parental influence on the intergenerational income elasticity and on educational choices even when credit markets are perfect.

D.1 A Framework for the Study of Capability Formation Over the Life Cycle

Capacities at t; $\boldsymbol{I_t}$: investment at t;
$\boldsymbol{\theta_{t+1}} = \boldsymbol{f^t}(\boldsymbol{\theta_t}, \boldsymbol{I_t}, \boldsymbol{\theta_{P,t}})$: Technology of Capability Formation

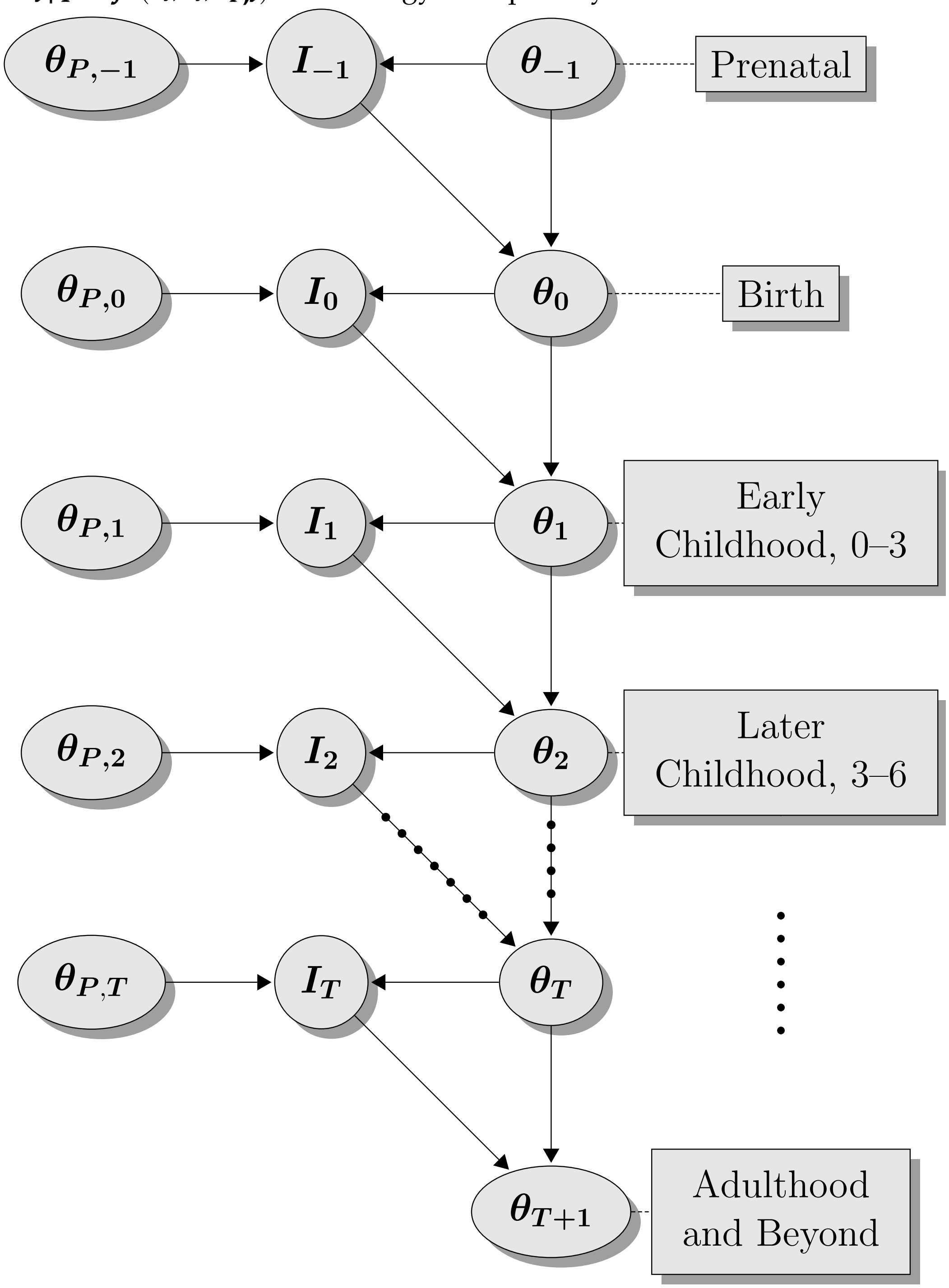


Figure D.1: The Empirical Challenge: A Life Cycle Framework for Organizing Studies and Integrating Evidence

D.2 Capability Formation in an Economy with Idiosyncratic Uncertainty and Liquidity Constraints

The model of Cunha (2007, revised in 2013) and Cunha and Heckman (2007, 2009) builds on Laitner's (1992) model. Altruistic households face two market imperfections:

(i) Parents can only transfer non-negative amounts of financial wealth in a risk-free asset to their children.

(ii) Parents cannot protect themselves against shocks to their own labor productivity. The heterogeneity in productivity featured in Laitner's model is exogenously determined.

Cunha (2007) builds on Laitner (1992) by allowing parents to invest in the **cognitive** skill of their children (noncognitive skills are ignored). One generation can transfer resources to the next through risk-free assets or investments in cognitive skill. The model is an overlapping generations economy with an infinite number of periods. Each generation consists of a continuum of agents with mass equal to unity. There is no population growth.

Each agent lives for $2T$ periods ($T = 1$ in Becker–Tomes–Solon (Becker and Tomes, 1986; Solon, 2004), henceforth BTS). During the first T years of life, the agent is a child and by assumption makes no economic decisions. Upon reaching age $T+1$, the agent becomes an adult and gives birth to a child. (Exogenous fertility.) The agent dies at the end of the calendar year in which she completes $2T$ years of age and is replaced in the beginning of the next calendar year by the generation of her grandchild.

D.3 The Technology of Skill Formation as in the Main Text

I_t **is investment;** θ_t **stock of child skills;** h **stock of parental skills.** In the notation of the text, $h = \theta_p$, which is assumed to be constant throughout adulthood. There are T distinct stages of development. The technology for capability formation for scalar θ_t, I_t, h is:

$$\theta_{t+1} = f^t\left(\theta_t, I_t, h\right). \tag{D.1}$$

f^t is increasing in each of its arguments, strictly concave, and twice-continuously differentiable.

The previous literature by Becker and Tomes (1979, 1986) and Solon (2004) (henceforth BTS) does not consider developmental stages of childhood in their model. They do not distinguish different types of investment over childhood. They focus on "human capital" or "ability," usually proxied by education or a test score. Investments have different impacts at different developmental stages. The technology allows for the degree of complementarity between investments, I_t, current stocks of skill, θ_t, and parental skill, h, to vary with the developmental stages of the child—features missing in the previous literature.

To develop some intuition about the skill formation process implied by the production function (D.1), consider the following parameterization:

$$\theta_{t+1} = \delta_t \left\{ \gamma_{1,t}\theta_t^{\phi_t} + \gamma_{2,t} I_t^{\phi_t} + \gamma_{3,t} h^{\phi_t} \right\}^{\frac{\rho_t}{\phi_t}}$$

with $0 < \gamma_{1,t}, \gamma_{2,t}, \gamma_{3,t}, \rho_t < 1$, $\phi_t \leq 1$, and $\sum_k \gamma_{k,n_t} = 1$. These conditions guarantee concavity and give well-defined properties for the production functions.

Consider the case of two periods of childhood ($T = 2$) and the special case $\rho_1 = \rho_2 = 1$, $\delta_1 = 1$, and $\phi_1 = \phi_2 = \phi \leq 1$. Substitute recursively. Skills at adulthood, $h' = \theta_3 = \theta_{T+1}$, can be expressed as

$$h' = \delta_2 \left[\gamma_{1,2}\gamma_{1,1}\theta_1^{\phi} + \underbrace{\gamma_{1,2}\gamma_{2,1}}_{\textbf{“Multiplier”}} I_1^{\phi} + \gamma_{2,2} I_2^{\phi} + \left(\gamma_{3,2} + \gamma_{1,2}\gamma_{3,1}\right) h^{\phi} \right]^{\frac{1}{\phi}} .$$

The multiplier arises because of self-productivity and productivity of investment in each period. It produces dynamic complementarity. The parameter $\gamma_{1,2}$ captures the notion of self-productivity of skills: it characterizes how much of the investment in period $t = 1$ propagates into skills at adulthood, θ_3. The parameter ϕ captures the intertemporal complementarity of investments. If $\phi = 1$, then investments in different periods are (almost) perfect substitutes. They are perfect substitutes if $\gamma_{1,2}\gamma_{2,1} = \gamma_{2,2}$, in which case the timing of investment in skills does not matter for the developmental process. This is the only circumstance in which collapsing childhood into one period as in BTS comes without loss of generality. The polar opposite case to perfect substitutability is the extreme case arising in the Leontief case where $\phi \to -\infty$, in which case we would write:

$$\theta_3 = \delta_2 \tau(\theta_1, h, \min(I_1, I_2)). \tag{D.2}$$

The extreme $\phi = -\infty$ is actually closer to the empirical truth than the case $\phi = 1$. Complementarity has a dual face. Early investment is essential but ineffective unless later investments are also made. The production function (D.2) is an extreme case that allows for no remediation. If parents are poor and unable to borrow against the future earnings of the children, and, as a result, I_1 is low, there is no amount of investments at later age, I_2, that can compensate for the early neglect.

D.4 The Problem of the Parent

The parent is assumed to be the decision-maker in the household. The child passively accepts investment. The consumption of the child is not modeled.[1] The problem solved by the parent depends on the age of the child. When the child is between ages 1 and $T-1$, he only receives investments and cannot work. When the child reaches age T, the parent may invest a minimum level or something beyond that minimum. If the parent invests the minimum amount, the child does not attend college but becomes a high school graduate and works full time. If the parent invests any amount beyond the minimum, the child attends school (college) full-time. At the end of the period, he becomes a college graduate.

[1]This is relaxed in work by Akabayashi (2006) and Cosconati (2013).

D.4.1 The Problem When the Child Is between 1 and $T-1$ Years Old

Parental labor supply is assumed to be perfectly inelastic. At each age t of the child, the parent is subject to productivity innovations ε_t, corresponding to labor market uncertainty. The shocks ε_t are independently and identically distributed across parents. The shocks follow a first-order Markov process:

$$\ln \varepsilon_{t+1} = \rho_\varepsilon \ln \varepsilon_t + \sigma_\eta \eta_t^\varepsilon. \tag{D.3}$$

Parents are assumed to have positive earnings. Productivity innovations are restricted so that there exists $\varepsilon_{\min}$ with the property that $\varepsilon_t \geq \varepsilon_{\min} > 0$ for any $t = T+1, \ldots, 2T$. The labor income of the parent is $wh\varepsilon_t$, where w is the efficiency wage and r is the risk-free discount rate. Innovations in wages and labor market uncertainty are missing in BTS.

The level of capability of the parent, h, is the outcome of investment decisions made by the grandparent. In similar fashion, the level of skill of the child when an adult, h', will also be the consequence of investments made by the parent, and satisfies $h' = \theta_{T+1}$. Defining s_t as the stock of savings of the parent at age t, the individual state variables for the parents of children who are between 1 and $T-1$ years old is $(h, \theta_t, s_t, \varepsilon_t, t)$.

Given the state variables, the parent chooses household consumption c_t, savings s_{t+1}, and investments I_t in the cognitive skill of the child. The savings of the parents are in a risk-free asset which pays a rate of interest r. p denotes the price of the investment goods in cognitive skill. Following Laitner (1992), the parents cannot leave debts to their children and have negative net worth, so savings are subject to the lower bound equal to $\frac{-wh\varepsilon_{\min}}{(1+r)}$ (the "natural" borrowing limit).

$V(t, h, \theta_t, s_t, \varepsilon_t)$ is the value function of the parent of a child at age t, $1 \leq t \leq T-1$. The problem of the parent is:

$$\begin{aligned} & V(t, h, \theta_t, s_t, \varepsilon_t) \\ & = \max_{c_t, I_t, s_{t+1}} \left\{ u(c_t) + \beta \mathbb{E}\left[V(t+1, h, \theta_{t+1}, s_{t+1}, \varepsilon_{t+1}) \middle| \varepsilon_t \right] \right\} \end{aligned}$$

subject to:

$$c_t + pI_t + s_{t+1} = wh\varepsilon_t + (1+r)\, s_t \tag{D.4}$$

$$s_{t+1} \geq -\left(wh\varepsilon_{\min}\right),\ I_t, c_t \geq 0 \tag{D.5}$$

and the technology for capability formation (D.1).

Associating multiplier μ_t to the borrowing constraint in stage t, the optimal conditions for consumption and investments are given by:

$$u_c(c_t) = \beta(1+r)\mathbb{E}\left[V_s\left(t+1, h, \theta_{t+1}, s_{t+1}, \varepsilon_{t+1}\right)|\varepsilon_t\right] + \mu_t \tag{D.6}$$

$$\beta\mathbb{E}\left[\frac{\partial\theta_{t+1}}{\partial I_t} V_\theta\left(t+1, h, \theta_{t+1}, s_{t+1}, \varepsilon_{t+1}\right)|\varepsilon_t\right]$$
$$= \beta(1+r)p\mathbb{E}\left[V_s\left(t+1, h, \theta_{t+1}, s_{t+1}, \varepsilon_{t+1}\right)|\varepsilon_t\right] + \mu_t \tag{D.7}$$

which imply that the marginal utility of investments is equated to the marginal utility of consumption and to the marginal utility of future wealth. Whenever the constraint binds ($\mu_t > 0$), consumption and investment will be reduced as the agent would like to borrow more than $(wh\varepsilon_{min})$, but she is constrained. Suppose now that the agent is not constrained in period t. Using the envelope condition for assets we can rewrite the optimal condition for investment and consumption making clear the dependence on expected future constraints:

$$\beta\mathbb{E}_t\left[\frac{\partial\theta_{t+1}}{\partial I_t} V_\theta\left(t+1, h, \theta_{t+1}, s_{t+1}, \varepsilon_{t+1}\right)|\varepsilon_t\right] = pu_c(c_t) \tag{D.8}$$
$$= \beta(1+r)p\mathbb{E}_t\left[V_s\left(t+1, h, \theta_{t+1}, s_{t+1}, \varepsilon_{t+1}\right)|\varepsilon_t\right]$$
$$= [\beta(1+r)]^2 p\mathbb{E}_t\left[\mathbb{E}_{t+1}\left[V_s\left(t+2, h, \theta_{t+2}, s_{t+2}, \varepsilon_{t+2}\right)|\varepsilon_{t+1}\right] + \mu_{t+1}|\varepsilon_t\right] \tag{D.9}$$
$$= [\beta(1+r)]^2 p\left[\mathbb{E}_t\left[V_s\left(t+2, h, \theta_{t+2}, s_{t+2}, \varepsilon_{t+2}\right)|\varepsilon_{t+1}\right]\right. \tag{D.10}$$
$$\left. + \mathbb{E}_t[\mu_{t+1} \,|\, s_{t+1} = -wh\varepsilon_{min}]\right] P(s^*_{t+1} < -wh\varepsilon_{min})$$

where s^*_{t+1} represents the optimal unconstrained amount of savings from stage $t+1$ to stage

$t+2$ and

$$P(s^*_{t+1} < -wh\varepsilon_{min}) = P\left(\varepsilon_{t+1}wh - c^*_{t+1}(\varepsilon_{t+1}wh) - pI^*_{t+1}(\varepsilon_{t+1}wh) \right.$$
$$\left. < -wh\varepsilon_{min} - (1+r)s_t\right) \qquad \text{(D.11)}$$

with c^*_{t+1} and I^*_{t+1} represent the optimal unconstrained levels of consumption and investments in period $t+1$ which depend on the realization of income. Even when the parent is not constrained in period t, the expectation of future constraints reduces current consumption and investments levels. The fear of hitting the constraint in the future induces a precautionary motive for savings which reduces current investments and consumption.

D.4.2 The Problem When the Child Is T Years Old: Go to College or Not?

Consider the decision to go to college (made by the parent). When the child reaches age T, the parent decides to invest the minimum amount, $\underline{I}$, or something beyond that amount. The parent uses the relevant information to make that decision, which is contained in the vector of state variables $(h, \theta_t, s_t, \varepsilon_t, n_t)$. Let κ be tuition cost. The parent's problem can be stated as:

$$V\left(T, h, \theta_T, s_T, \varepsilon_T\right)$$
$$= \max_{c_T, I_T, s'_1} \left\{ u\left(c_T\right) + \beta \mathbb{E}\left[V\left(1, h', \theta'_1, s'_1, \varepsilon'_1\right)\right]\right\}$$

subject to:

$$c_T + s'_1 + p\underline{I} = wh\varepsilon_T + w\theta_T + (1+r)\, s_T \text{ if } I_T = \underline{I} \qquad \text{(D.12)}$$

$$c_T + s'_1 + (pI_T + \kappa) = wh\varepsilon_T + (1+r)\, s_T \text{ if } I_T > \underline{I} \qquad \text{(D.13)}$$

$$s_T \geq 0 \qquad \text{(D.14)}$$

and the technology for the production of skills (D.1).

The budget constraint (D.12) states that a child who receives the minimum amount of investments $\underline{I}$ works full time. Refer to this child as a high school graduate. Note that the high-school-graduate child's earnings are pooled with the rest of the parental resources. Abstract from productivity shocks for the child before he reaches adulthood. If the parent decides to invest any amount above the minimum, so that $I_T > \underline{I}$, then the parent must pay the variable cost of the investment, which is p by unit, plus a fixed cost, φ—college tuition. A child who receives more than the minimum amount of investment does not work. This is described by the budget constraint (D.13). Note that equation (D.14) embodies the notion that the parent faces lifetime liquidity constraints. The parent dies and cannot leave debts to the child.

Following Cunha (2007), one can establish a steady state general equilibrium. Firms producing final output under constant returns to scale. Also a child investment good is produced. Cunha (2007) establishes a stochastic general equilibrium for the steady state, extending Laitner to include human capital.

D.5 Firms

Both education goods and final outputs are produced. The final output sector uses physical capital and labor, measured in efficiency units, to produce the consumption good. The education goods sector uses only labor, also measured in efficiency units, to produce the investment good for cognitive skills.

D.5.1 The Consumption Good Sector

The production function in the consumption good sector is assumed to exhibit constant returns to scale. Only stationary equilibrium is established. It is not necessary to use time subscripts.

Let $\boldsymbol{K}, \boldsymbol{L}$ denote the aggregate quantities of physical capital and labor, respectively. Let $\boldsymbol{Y}$ denote aggregate output. The production technology is represented by the production function $F : \boldsymbol{Y} = F(\boldsymbol{K}, \boldsymbol{L})$. Satisfies the Inada Conditions. It is twice-continuously differentiable.

The problem of the firm in the goods production sector is:

$$\pi_Y = \max \{F(\boldsymbol{K}, \boldsymbol{L}) - w\boldsymbol{L} - (r + \delta)\boldsymbol{K}\}$$

with first-order conditions:

$$w = \frac{\partial F(\boldsymbol{K}, \boldsymbol{L})}{\partial \boldsymbol{L}}$$

$$(r + \delta) = \frac{\partial F(\boldsymbol{K}, \boldsymbol{L})}{\partial \boldsymbol{K}}$$

D.5.2 The Education Good Sector

Let $\boldsymbol{E}$ denote the total supply of educational goods. This sector does not use physical capital as input, only labor $\boldsymbol{U}$. The production technology is

$$\boldsymbol{E} = \boldsymbol{U}.$$

The problem of the firm in this sector is to maximize π_E:

$$\pi_E = \max \{ p\boldsymbol{E} - w\boldsymbol{U} \}.$$

Problem has a solution with limited, positive production if, and only if

$$p = w.$$

D.5.3 Market-Clearing Conditions

Let $\zeta_t = (h, \theta_t, s_t, n_t, \varepsilon_t)$. This is the vector of state variables facing the parents. Define $\zeta = (\zeta_1, \ldots, \zeta_T)$. Let $g(\zeta)$ denote the joint probability density function of the state variables. Let $c_t(\zeta_t), s_t(\zeta_t)$ denote the consumption and savings functions when the child is t years old. Let $\boldsymbol{C}_t$, $\boldsymbol{S}_t$ denote the aggregate consumption and savings of households that have a child who is t years old, where $t = 1, 2, \ldots, T$.

By definition,

$$\boldsymbol{C}_t = \int c_t(\zeta_t)\, g(\zeta)\, d\zeta,$$
$$\boldsymbol{S}_t = \int s_t(\zeta_t)\, g(\zeta)\, d\zeta.$$

Denote the economy-wide investment in physical capital (conducted by the firm in the consumption good sector) by $\boldsymbol{Q}$. The market clearing in the consumption good sector is given by the condition

$$\sum_{t=1}^{T} \boldsymbol{C}_t + \boldsymbol{Q} = \boldsymbol{Y}.$$

Analogously, equilibrium in the physical capital sector the equilibrium condition is given by

$$\sum_{t=1}^{T} \boldsymbol{S}_t = \boldsymbol{K}.$$

Let $I_t(\zeta_t)$ denote the investments in cognitive skill when the child is t years old. Use $\boldsymbol{I}_t$ to denote the aggregate investment by households with a t-year-old child, $t = 1, 2, \ldots, T$. When the child is t years old, $t = 1, 2, \ldots, T-1$, aggregate investment is $\boldsymbol{I}_t = \int I_t(\zeta_t)\, g(\zeta)\, d\zeta$.

When the child is T years old, one must keep track of the fact that some children receive investments beyond the minimum amount and the others do not. The share of the children who receive investments is the share of children who become college graduates. Consequently, aggregate investment by households with a T-years-old child is:

$$\begin{aligned}\boldsymbol{I}_T = & \int_{\{\zeta_T / I_T(\zeta_T) = \underline{I}\}} \underline{I}\, g(\zeta)\, d\zeta \\ & + \int_{\{\zeta_T / I_T(\zeta_T) > \underline{I}\}} I_T(\zeta_T)\, g(\zeta)\, d\zeta.\end{aligned}$$

The market clearing condition for this sector is

$$\sum_{t=1}^{T} \boldsymbol{I}_t = \boldsymbol{E}.$$

To compute the aggregate stock of efficiency units, let $g_h(h)$ denote the probability density function of adult efficiency units. In households where children are t years old, $t = 1, 2, \ldots, T - 1$, they supply an amount of efficiency units that given by

$$\boldsymbol{H}_t = \int h g_h(h)\, dh.$$

In households where children are T years old, we may have two different types of persons supplying efficiency units: the parent and the child who is only receiving the minimum amount of investments, $\underline{I}$. Let $g_\theta, I_T(\theta_T, I_T(\varepsilon_T))$ denote the joint probability density function of efficiency units (determined by cognitive skills) for the children who are T years old and $I_T(\varepsilon_T)$. Then

$$\boldsymbol{H}_T = \int h g_h(h)\, dh + \int_{\{\zeta_T / I_T(\zeta_T) = \underline{I}\}} \theta_T g_\theta, (\theta_T, I_T(\zeta_T))\, d\theta_T d(\varepsilon_T).$$

The total supply of efficiency units in every calendar year in this economy is given by $\boldsymbol{H}$ is defined as

$$\boldsymbol{H} = \sum_{t=1}^{T} \boldsymbol{H}_t.$$

$\boldsymbol{L}, \boldsymbol{U}$ denote the aggregate amount of efficiency units allocated to the consumption and education good sector, respectively. Feasibility of the efficiency units allocation implies

$$\boldsymbol{L} + \boldsymbol{U} = \boldsymbol{H}.$$

Cunha (2007) establishes the existence of stationary equilibrium for this model.

Definition of Stationary Equilibrium. *A Stationary Recursive Competitive Equilibrium is a set of functions* $\{V(\zeta_t)\}_{t=1}^T$, $\{c_t(\zeta_t), I_t(\zeta_t), s_t(\zeta_t)\}_{t=1}^T$, $\boldsymbol{K}$, $\boldsymbol{L}$, $\boldsymbol{Y}$, $\boldsymbol{U}$, *wage rate* w, *interest rate* r, *prices of investment goods* p, *distributions of parents across states,* $g(\zeta)$ *such that:*

(a) Given prices w *and* r, *the functions* $\{V(\zeta_t)\}_{t=1}^T$, $\{c_t(\zeta_t), I_t(\zeta_t), s_t(\zeta_t)\}_{t=1}^T$ *solve the parent's maximization problem.*

(b) Given prices w *and* r, $\boldsymbol{K}$ *and* $\boldsymbol{L}$ *maximizes consumption-good firm's profits and* $\boldsymbol{U}$ *maximizes the*

education-good sector firm's profit.

(c) *Markets for consumption, investments in education, physical capital and efficiency units clear.*

(d) *The distributions of households across states* $\{\mu_t(\theta_t, h, s_t, \varepsilon_t)\}_{t=1}^{T}$ *are calendar-year invariant and are determined as a fixed point of an operator that maps current-calendar-year distributions into next-calendar-year distributions taking into account parent's optimal decisions and the evolution of exogenous states.*

D.6 Comparative Statics for the Problem of the Parent Facing Wage Uncertainty

By inspecting equation (D.6), (D.7), and (D.9) and defining $y_t \equiv (1+r)s_{t-1} + \varepsilon_t wh$, we derive the following comparative static results for the parent of a child at age $1 \le t \le t-2$. We assume that consumption and child ability are normal goods for the parent[2].

Changes in y_t:

1. $\left(\frac{\partial s_t}{\partial y_t} \in [0,1)\right)$ and is equal to 0 if the parent is constrained;
2. $\left(\frac{\partial c_t}{\partial y_t} \in (0,1]\right)$, $\left(\frac{\partial I_t}{\partial y_t} \in (0,1]\right)$, and $\left(\frac{\partial [c_t+I_t]}{\partial y_2} \in (0,1]\right)$. The last is equal to 1 if the parent is constrained;
3. The marginal utility of I_{t+1} increases in y_t if there is static complementarity between θ_{t+1} and I_{t+1} as this implies dynamic complementarity in investments and, therefore, I_t increases.

Changes in $\mathbb{E}_t[y_{t+1}]$:

1. $\left(\frac{\partial s_t}{\partial \mathbb{E}_t[y_{t+1}]} \in (-1,0]\right)$ and is equal to 0 if the parent is constrained;

[2]This is equivalent to the following assumptions on the value function:

$$V_\theta V_{ss} - V_s V_{\theta s} < 0 \quad \text{and}$$
$$V_s V_{\theta\theta} - V_s V_{\theta s} < 0.$$

2. $\left(\frac{\partial c_t}{\partial \mathbb{E}_t[y_{t+1}]} \in (0,1]\right)$, $\left(\frac{\partial I_t}{\partial \mathbb{E}_t[y_{t+1}]} \in (0,1]\right)$, and $\left(\frac{\partial [c_t+I_t]}{\partial \mathbb{E}_t[y_{t+1}]} \in (0,1]\right)$. The last is equal to 1 if the parent is constrained;

3. the marginal utility of I_t increases in $\mathbb{E}_t[y_{t+1}]$ if there is dynamic complementarity as $\mathbb{E}_t[I_{t+1}]$ increases. θ_{t+1} increases.

Supposing that the parent can face stricter borrowing limits than the natural one: $s_{t+1} \geq \underline{s}_t$, where $\underline{s} > -wh\varepsilon_{min}$, then if the probability of being constrained in period $t+1$ decreases (less tight borrowing constraint in period $t+1$):

1. $\left(\frac{\partial s_t}{\partial \underline{s}_{t+1}} \leq 0\right)$ and is equal to 0 if the parent is constrained in period 1;

2. $\left(\frac{\partial I_t}{\partial \underline{s}_{t+1}} > 0\right)$ and $\left(\frac{\partial c_t}{\partial \underline{s}_{t+1}} > 0\right)$ if the parent is unconstrained in period 1. $\left(\frac{\partial [c_t+I_t]}{\partial \underline{s}_{t+1}} \geq 0\right)$ if the parent is unconstrained in period t and $\left(\frac{\partial [c_t+I_t]}{\partial \underline{s}_{t+1}} = 0\right)$ if she is constrained.

This set of comparative statics mimic those proposed in the later model of Caucutt and Lochner (2012) based on Cunha (2007). Note however that in our context we cannot derive any result for the case in which the agent is constrained in period $t+1$. Allowing for income uncertainty implies that the agent is never sure of being constrained in the future. As stressed above what matters is the probability (and the possible changes in it) of being constrained and how this is affected by the various aspects of the model and in particular by the distribution of η_t^{ε}. In this model, the interaction between self-productivity of skills, complementarity of early and late investments and credit constraints can reduce the amount of investments in children. Parents who are constrained in the early phases or who expect to be constrained in the future will reduce their level of investments. Because of dynamic complementarily this will reduce future investments as well causing a possible serious lack of investments in children of constrained families.

D.7 Targeting Relatively More Investment Toward Disadvantaged Children Can Be Socially Efficient

D.7.1 Introduction

We analyze the problem of investing in children with different initial endowments that is stated in the text (Section 5) assuming that $\sigma = 1$ and that children are weighted equally ($\omega_k = 1$ for all k). Families are assumed to only care about productivity. Consider the following two-stage model of childhood investment:

$$\theta_3 = f^{(2)}(\theta_2, I_2) \tag{D.15}$$

$$\theta_2 = f^{(1)}(\theta_1, I_1) \tag{D.16}$$

where θ_3 represents the level of skill at the beginning of adulthood. The functions are assumed to be strictly concave in I_2 and I_1, respectively, and twice differentiable. Concavity in θ_2 or θ_1 is not required for an optimum, although it plays a role in signing terms in the comparative statics exercise below. The assumptions made below imply that all inputs are normal.[3] Total resources are E. The price of input i is p_i. There are two children: A and B. Their initial endowments are θ_1^A and θ_1^B, respectively. We write $\theta_1^A = \gamma\theta_1^B$ and consider how, from a position of initial equality ($\theta_1^A = \theta_1^B$ or $\gamma = 1$), raising the initial endowment of A affects Benthamite allocations of investment goods between A and B. Denote investment in the first period for child A by I_1^A and in the second period by I_2^A. I_1^B and I_2^B are defined analogously for child B.

[3] See Bear (1965).

In a one period of childhood problem where parents (or social planners) seek to maximize the aggregate of adult skills (θ_2):

$$\theta_2^A + \theta_2^B$$

$$\text{subject to} \quad E = p_1(I_1^A + I_1^B),$$

the first order condition is

$$\text{F.O.C.: } f_2^{(1)}\left(\gamma\theta_1^B, I_1^A\right) = f_2^{(1)}\left(\theta_1^B, I_1^B\right).$$

Notice that

$$\text{sign}\left(\frac{\partial I_1^A}{\partial \gamma}\right) = \text{sign}\left(f_{12}^{(1)}(\cdot)\right)_{\gamma=1},$$

where $f_{12}^{(1)}(\cdot)$ is the value of $f^{(12)}$ in the neighborhood of $(\cdot)$. Parents (social planners) invest more in the disadvantaged if inputs are substitutes with initial endowments and they invest less if they are complements.

In the multiperiod setting of Equations (D.15) and (D.16), the result that it is optimal to invest more in the child with the lower initial endowment continues to hold if $f_{12}^{(1)}(\cdot) < 0$ even though $f_{12}^{(2)}(\cdot) > 0$.[4] This pattern is consistent with the evidence discussed in Section 4 on the evolution of complementarity at later stages in the life cycle: $f_{12}^{(1)}(\cdot) < f_{12}^{(2)}(\cdot)$. However, targeting relatively more investment to the initially more disadvantaged child can still be efficient if $0 \leq f_{12}^{(1)}(\cdot) \leq f_{12}^{(2)}(\cdot)$.

To establish this and more general results for the two period case, suppose that parents (or social planners) seek to maximize

$$\theta_3^A + \theta_3^B$$

subject to

$$E = p_1(I_1^A + I_1^B) + p_2(I_2^A + I_2^B).$$

[4]This is proved in Part D.7.3.

The first order conditions are

$$f_1^{(2)}\left(f^{(1)}\left(\theta_1^A, I_1^A\right), I_1^A\right) f_2^{(1)}\left(\theta_1^A, I_1^A\right) = \lambda p_1$$

$$f_2^{(2)}\left(f^{(1)}\left(\theta_1^A, I_1^A\right), I_2^A\right) = \lambda p_2$$

$$f_1^{(2)}\left(f^1\left(\theta_1^B, I_1^B\right), I_2^B\right) f_2^{(1)}\left(\theta_1^B, I_1^B\right) = \lambda p_1$$

$$f_2^{(2)}\left(f^1\left(\theta_1^B, I_1^B\right), I_2^B\right) = \lambda p_2$$

$$p_1\left(I_1^A + I_1^B\right) + p_2\left(I_2^A + I_2^B\right) = E.$$

Consider an enhancement of the endowment of A in the neighborhood of initial equality ($\theta_1^A = \theta_1^B$). As before, let $\theta_1^A = \gamma\theta_1^B$. We perturb γ. Take total differentials of the system of first order conditions:

$$\left.\begin{aligned}
&\left\{f_{11}^{(2)}(\cdot)\left[f_2^{(1)}(\cdot)\right]^2 + f_1^{(2)}(\cdot) f_{22}^{(1)}(\cdot)\right\} dI_1^A + \left[f_{12}^{(2)}(\cdot) f_2^1(\cdot)\right] dI_2^A + \theta_1^B \underbrace{\left[f_{11}^{(2)} f_1^{(1)}(\cdot) f_2^{(1)}(\cdot) + f_1^{(2)}(\cdot) f_{21}^{(1)}(\cdot)\right]}_{\text{"Term 1"}} d\gamma \\
&\qquad = (d\lambda)p_1 + \lambda dp_1 \\
&\left\{f_{21}^{(2)}(\cdot) f_2^{(1)}(\cdot)\right\} dI_1^A + \left\{f_{22}^{(2)}(\cdot)\right\} dI_2^A + \theta_1^B \underbrace{\left\{f_{21}^{(2)}(\cdot) f_1^{(1)}(\cdot)\right\}}_{\text{"Term 2"}} d\gamma = (d\lambda)p_2 + \lambda dp_2 \\
&\left\{f_{11}^{(2)}(\cdot)\left[f_2^{(1)}(\cdot)\right]^2 + f_1^{(2)}(\cdot) f_{21}^{(1)}(\cdot)\theta_1^B\right\} dI_1^B + \left[f_{12}^2(\cdot) f_2^1(\cdot)\right] dI_2^B = (d\lambda)p_1 + \lambda dp_1 \\
&\left\{f_{21}^{(2)}(\cdot) f_2^{(1)}(\cdot)\right\} dI_1^B + \left\{f_{22}^{(2)}(\cdot)\right\} dI_2^B = (d\lambda)p_2 + p_2 d\lambda \\
&-dE + p_1 dI_1^A + p_2 dI_2^A + p_1 dI_1^B + p_2 dI_2^B + I_1^A dp_2 + I_2^A dp_2 + I_1^B dp_1 + I_2^B dp_2 = 0.
\end{aligned}\right\} \quad \text{(D.17)}$$

D.7.2 A Three-Stage Analysis

It is fruitful to analyze the problem in three stages. In the first stage, we consider, for a single agent, how as $\gamma \uparrow$, the allocation of a fixed bundle of resources between investment in the first period and investment in the second period is affected. Then in the second stage we consider how, as $\gamma \uparrow$, the productivity of expenditure changes and how resources are allocated across A and B. Clearly, resources shift to where they become more productive. Finally, in the third stage, we consider how an increase in resources is allocated between the first and the second periods. We use fictitious child A specific prices (p_1^A and p_2^A) and child B specific prices p_1^B and p_2^B.

Let the expenditures on child A and child B be

$$\begin{aligned} E_A &= p_1 I_1^A + p_2 I_2^A \\ E_B &= p_1 I_1^B + p_2 I_2^B. \end{aligned}$$

Maximize each of θ_3^A and θ_3^B separately subject to E_A and E_B respectively, then allocate E_A and E_B to equalize marginal productivity of expenditure across A and B.

We do not require concavity of the production functions in terms of θ_1 or θ_2. This allows us to use standard results from consumer theory.

The "$d\gamma$" terms act like income-compensated price changes. They do not affect total resources E. Assuming interior solutions, $\gamma \uparrow$ is like a change in the (child-specific) input prices p_1 and p_2.

D.7.2.1 The effect of $\gamma \uparrow$ on the allocation of investments across periods holding E_A fixed.

Consider the effect of an increase in γ on the allocation of period one and period two investment of child A while E_A is fixed. (We will consider the allocation of E_A and E_B across A and B later). The displacement system derived from the first order conditions for this problem may be written as

$$\underbrace{\begin{bmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_1 & -p_2 & 0 \end{bmatrix}}_{\substack{|M| \\ (+)}} \begin{bmatrix} dI_1^A \\ dI_2^A \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda dp_1 - \theta_1^B(\text{Term } 1)d\gamma \\ \lambda dp_2 - \theta_1^B(\text{Term } 2)d\gamma \\ 0 \end{bmatrix}. \tag{D.18}$$

Observe that the income compensated own price changes are negative. Cross effects can be shown to be positive under the conditions specified below. $|M| > 0$ from the assumption of a regular optimum. To simplify the notation here and throughout the rest of the appendix, we suppress the "$(\cdot)$" notation. We can sign

$$c = \left[f_{11}^{(2)}[f_2^{(1)}]^2 + f_1^{(2)} f_{22}^{(1)}\right] \leq 0$$

if period 2 production is concave in θ_2 and period 1 production is concave in I_1. (We assume that all marginal products are strictly positive unless otherwise noted.) But c might still be negative if period 2 production is convex in θ_2 ($f_{11}^{(2)} > 0$) provided $f_1^{(2)} f_{22}^{(1)}$ is sufficiently negative.

$$d = f_{12}^{(2)} f_2^{(1)} > 0 \quad \text{if there is second period complementarity}$$

and

$$e = f_{22}^{(2)} < 0 \quad \text{from concavity in } I_2.$$

Observe that in displacement system (D.17)

$$\text{Term 1} \equiv \left[f_{11}^{(2)} f_1^{(1)} f_2^{(1)} + f_1^{(2)} f_{21}^{(1)}\right]$$

may be of either sign. The second grouping of terms in Term 1 is positive under first period complementarity. It is negative under substitutability. The first grouping is negative under concavity of $f^{(2)}$ in θ_2. Under second period complementarity ($f_{21}^{(2)} > 0$) and have

$$\text{Term 2} = \left[f_{21}^{(2)} f_1^{(1)}\right] \geq 0.$$

The change associated with Term 1 alone is opposite in sign to the change in the income-constant price of I_1^A which is negative. Similarly, a change associated with Term 2 alone is opposite in sign from a change in the price of I_2^A.

Using standard results in price theory,

$$\frac{\partial I_1^A}{\partial \gamma} = \frac{\begin{vmatrix} -\theta_1^B(\text{Term } 1) & d & -p_1 \\ -\theta_1^B(\text{Term } 2) & e & -p_2 \\ 0 & -p_2 & 0 \end{vmatrix}}{|M|}$$

$$= \left(\frac{(\text{Term } 1)p_2^2 - (\text{Term } 2)p_1 p_2}{|M|} \right) \theta_1^B.$$

Focus on the numerator of the preceding expression (the denominator is positive). Substitute out for p_1 and p_2 using the first order conditions (D.17). The numerator can be written as

$$\theta_1^B \left[\frac{1}{\lambda^2} \right] \left\{ \left[f_{11}^{(2)} f_1^{(1)} f_2^{(1)} + f_1^{(2)} f_{21}^{(1)} \right] \left[f_2^{(2)} \right]^2 - \left[f_{21}^{(2)} f_1^{(1)} f_1^{(2)} f_2^{(1)} f_2^{(2)} \right] \right\}.$$

Focusing further on the term in braces (which is multiplied by a positive term), we obtain

$$\left\{ \left[f_{11}^{(2)} f_1^{(1)} f_2^{(1)} \left[f_2^{(2)} \right]^2 + f_1^{(2)} f_{21}^{(1)} \left(f_2^{(2)} \right)^2 - f_{21}^{(2)} \left(f_1^{(2)} \right)^2 f_2^{(2)} f_1^{(1)} f_2^{(1)} \right] \right\}$$

$$= f_2^{(1)} \left(f_2^{(2)} \right)^2 f_1^{(2)} \left[\frac{f_{11}^{(2)}}{f_1^{(2)}} \left(f_1^{(1)} \right) + \frac{f_{21}^{(1)}}{f_2^{(1)}} - \frac{f_{21}^{(2)}}{f_2^{(2)}} f_1^{(2)} \right]$$

$$= f_2^{(1)} \left(f_2^{(2)} \right)^2 f_1^{(2)} \left[f_1^{(1)} \underbrace{\underset{(-)}{\left[\frac{\partial \ln f_1^{(2)}}{\partial \theta_2^A} \right]}}_{\substack{\text{Diminishing marginal} \\ \text{productivity of } \theta_2}} + \underbrace{\left(\frac{\partial \ln f_2^{(1)}}{\partial \theta_1^A} \right)}_{\substack{\text{Effect of } \theta_1^A \\ \text{on marginal} \\ \text{productivity of } I_1^A}} - \underbrace{\left(\frac{\partial \ln f_2^{(2)}}{\partial \theta_2^A} \right)}_{\substack{\text{Effect of } \theta_2^A \\ \text{on marginal} \\ \text{productivity of } I_2^A}} f_1^{(1)} \right]$$

Note that $f_1^{(1)} = \frac{\partial \theta_2^A}{\partial \theta_1^A}$. This is the marginal self productivity of θ_1.

Thus the term in brackets may be written as

$$\left[\underbrace{\frac{\partial lnf_1^{(2)}}{\partial \theta_1^A}}_{\substack{\text{The effect of } \theta_1^A \\ \text{on the marginal} \\ \text{productivity of } \theta_2^A}} + \underbrace{\frac{\partial lnf_2^{(1)}}{\partial \theta_1^A}}_{\substack{\text{The effect of } \theta_1^A \\ \text{on the marginal} \\ \text{productivity of } I_1^A}} - \underbrace{\frac{\partial lnf_2^{(2)}}{\partial \theta_1^A}}_{\substack{\text{The effect of } \theta_1^A \\ \text{on the marginal} \\ \text{productivity of } I_2^A}} \right]$$
$$= \frac{\partial}{\partial \theta_1} \left[lnf_1^{(2)} + lnf_2^{(1)} - lnf_2^{(2)} \right] \tag{D.19}$$

Consider the three effects inside the bracket going from left to right. The first term is the effect of θ_1^A on the marginal product of θ_2^A in period 2 production. From concavity (in terms of θ_2^A), this term is negative. Diminishing returns is a force toward investing less in the first period. This term reflects how first period stocks of skills augment second period stocks of skills. If, example, $f_1^{(1)} = 0$ (so $\frac{\partial \theta_2^A}{\partial \theta_1^A} = 0$), this term is zero. This could occur if there is 100% depreciation of skills or if there is a threshold value of θ_1 beyond which increases in θ_1 do not affect θ_2 and the agent is at or beyond the threshold. If θ_2^A has a low or zero productivity in second period production, this term is small or zero.

The second term is the effect of increasing θ_1^A on augmenting the productivity of first period investment in producing θ_2^A. This is the term that drives the analysis in a one period model of childhood.

The third term is the effect of increasing θ_1^A on augmenting the productivity of second period investment. Again, if there is no self-productivity ($\frac{\partial \theta_2^A}{\partial \theta_1^A} = 0$), this term is zero. Greater complementarity with later stages in the life cycle is a force toward investing less in the first period.

In the absence of self-productivity ($f_1^1 = \frac{\partial \theta_2^A}{\partial \theta_1^A} = 0$), the effect is driven solely by the second term. Under complementarity, the sign of the effect is positive.

Thus, we conclude that

$$\frac{\partial I_1^A}{\partial \gamma} < 0$$

if (a) $f^{(2)}$ concave in θ_2^A, $f_{21}^{(1)} < 0$, $f_{21}^{(2)} > 0$, and/or (b) $f^{(2)}$ is concave in θ_2 and $\frac{\partial ln f_2^{(1)}}{\partial \theta_1} < f_1^{(1)} \frac{\partial ln f_2^{(2)}}{\partial \theta_2}$, or if there are other configurations so that the term in brackets in (D.19) is positive. Because of the budget constraint it follows that

$$\frac{\partial I_2^A}{\partial \gamma} > 0 \quad \text{if} \quad \frac{\partial I_1^A}{\partial \gamma} < 0$$

and the effects are offsetting. This is an analysis for allocation of investment *within* the life cycle of child A.

D.7.2.2 The Effects on Productivity: Allocation over A and B

Let λ_A be the productivity of expenditure on A. λ_B is defined analogously for B. If, as $\gamma \uparrow$, $\lambda_A \uparrow$, it is optimal to allocate to $A (E_A \uparrow)$. If $\lambda_A \downarrow$ it is optimal to allocate less to A $(E_A \downarrow)$. The sign of this relationship hinges on the sign of Term 1 as we now show.

$$\frac{\partial \lambda_A}{\partial \gamma} = \frac{\begin{vmatrix} c & d & (-\text{Term } 1) \\ d & e & (-\text{Term } 2) \\ -p_1 & -p_2 & 0 \end{vmatrix}}{|M|} \theta_1^B$$

Collecting terms and using the first order conditions (D.17), using

$$p_1 = \frac{1}{\lambda} f_1^{(2)} f_2^{(1)} \quad \text{and} \quad p_2 = \frac{1}{\lambda} f_2^{(2)}$$

$$\frac{\partial \lambda_A}{\partial \gamma} = \frac{\theta_1^B}{\lambda}\frac{1}{|M|}\left[\underbrace{(\text{Term 1})}_{?}[\underbrace{f_2^{(2)}d - f_1^{(2)}f_2^{(1)}e}_{\substack{Q_1 \\ (+)}}] - \underbrace{\underbrace{(\text{Term 2})}_{(+)}[\underbrace{f_2^{(2)}c - df_1^{(2)}f_2^{(1)}}_{\substack{Q_2 \\ (-)}}]}_{+} \right],$$

where

$$Q_1 = f_2^{(1)}\left[f_2^{(2)}f_{12}^{(2)} - f_1^{(2)}f_{22}^{(2)}\right] > 0$$

and

$$Q_2 = f_2^{(2)}f_{11}^{(2)}[f_2^{(1)}]^2 + f_2^{(2)}f_1^{(2)}f_{22}^{(1)} - f_{12}^{(2)}f_2^{(1)}f_1^{(2)}f_2^{(1)} < 0.$$

Thus

$$\frac{\partial \lambda}{\partial \gamma} = \frac{\theta_1^B}{\lambda}\frac{1}{|M|}\left[\underset{?}{(\text{Term 1})}\underset{(+)}{(Q_1)} + \underset{(+)}{(\text{Term 2})}\underset{(+)}{Q_2}\right].$$

So if Term 1 $(+)$, then $\frac{\partial \lambda_A}{\partial \gamma} > 0$. This is a sufficient condition. In this case, as $\gamma \uparrow$ it is efficient to allocate *more* to $A(E_A \uparrow)$.

If Term 1 is sufficiently negative, it is optimal to allocate *less* to $A(E_A) \downarrow$. Recall that a sufficient condition for Term 1 to be negative is that $f_{21}^{(1)} < 0$. But even if $f_{21}^{(1)} > 0$, if there is sufficiently strong diminishing returns in $\theta_1(f_{11}^2 < 0)$, the optimal response of an increase in γ is to reduce I_1^A (i.e. to favor the disadvantaged child).

D.7.2.3 Allocation of Changes in Endowments over Periods

From standard results in consumer theory,

$$\frac{\partial I_1^A}{E_A} = \frac{(-1)}{|M|}\begin{vmatrix} d & -p_1 \\ e & -p_2 \end{vmatrix} = \frac{dp_2 - p_1 e}{|M|} = \frac{(f_2^{(1)})}{\lambda|M|}\left[f_{12}^2 f_2^2 - f_1^2 f_{22}^2\right] \geq 0$$

Recall we always assume $f_{12}^{(2)} > 0$ and from concavity it follows that $f_{22}^{(2)} < 0$. Thus $\frac{\partial I_1^A}{\partial E_A} > 0$.

$$\frac{\partial I_2^A}{\partial E_A} = \frac{\begin{vmatrix} c & -p_1 \\ d & -p_2 \end{vmatrix}}{|M|} = \frac{1}{\lambda |M|} \left\{ (f_2^1)^2 f_1^2 f_{12}^2 - (f_2^2)[f_{11}^2 (f_2^1)^2 + f_1^2 f_{22}^1] \right\}$$

This expression is also positive. Thus inputs are normal under our assumptions. For the case $p_1 = p_2 = 1$ (which we can assume with no loss of generality)

$$\frac{\partial I_1^A}{\partial E_A} = \frac{f_{12}^{(2)} f_2^{(1)} - f_{22}^{(2)}}{|M|}$$

$$\frac{\partial I_2^A}{\partial E_A} = \frac{f_{11}^{(2)} (f_2^{(1)})^2 - f_1^{(2)} f_{22}^{(1)} + f_{12}^{(2)} f_2^{(1)}}{|M|}.$$

Observe that $\frac{\partial I_1^A}{\partial E_A}$ is larger

(a) the greater the second period complementarity $(f_{12}^{(2)})$ (so that I_1^A has greater productivity in producing final output θ_3^A),

(b) the larger $f_2^{(1)} (= \frac{\partial \theta_2^A}{\partial I_1^A})$ (so that I_1^A is more productive in producing the intermediate product θ_2^A);

(c) the more rapidly the decline in the productivity of I_2^A.

Intuitively, relatively more is allocated to first period investment the more productive is the first period investment.

D.7.2.4 Putting it All Together

The second step is the key one. It determines the allocation of expenditure across children in response to an increase in endowment ($\gamma \uparrow$). The greater the decline in self productivity with increases in θ_1 (the more negative $f_{11}^{(2)}$), the more likely it is that more resources are devoted to the less advantaged child. This negative effect is amplified by greater productivity of θ_1 in period 1 ($f_1^{(1)}$) and greater productivity of I_1 in period 1. These effects are reinforced if there is substitutability between θ_1 and $I_1(f_{21}^{(1)} < 0)$. If $f_{21}^{(1)}$ is positive, the redistributive effect is attenuated. This offsetting effect is weaker the smaller the productivity of θ_2 in period 2 production.

The first step explores substitution effects arising from the change in γ. The third step explores income effects arising from transfers across children. The other steps determine the allocation of investment across periods for each child. The analysis of the third step for each child informs us that resources are differentially allocated to the more productive period. The analysis of the first step makes a similar claim but investigates how changes in γ affect the relative productivity of investment in each period.

In Section D.7.3 we establish that if first period investment (I_1) and initial endowment (θ_1) are substitutes, $(f_{12}^{(1)} < 0)$, but θ_2 is complementary with second period investments $(f_{12}^{(2)} > 0)$, first period investments are greater for the more disadvantaged child.

But even if $(f_{12}^{(1)} > 0)$, greater first period investment in the initially disadvantaged child may be optimal. This is more likely (ceteris paribus)

(a) the more steeply diminishing is the productivity of second period skills $(f_{22}^{(2)})$;

(b) the greater the self productivity of the stock of skills in the first period $(f_1^{(1)} = \frac{\partial \theta^2}{\partial \theta_1})$;

(c) the smaller first period complementarity $(f_{21}^{(1)})$ relative to second period complementarity and absolutely

(d) the more rapidly diminishing the marginal productivity of $\theta_1(f_{11}^{(1)})$;

(e) the greater the second period complementarity $(f_{12}^{(2)})$;

(f) the greater the first period productivity of investment $(f_2^{(1)})$ and

(g) the more rapidly diminishing the productivity of second period investment $(f_{22}^{(2)})$.

Roughly speaking, the more concave are the technologies in terms of stocks of skills, the more favorable is the case for investing relatively more in the disadvantaged child. The greater the second period complementarity $(f_{12}^{(2)})$, the greater the case for investing more in the initially disadvantaged child to allow the child to benefit from greater second period complementarity of the stock of skills with second period investment. In general, even when investment is greater in the first period for the disadvantaged child, second period investment is greater for the initially advantaged child. It is generally not efficient to make the initially disadvantaged child whole as it enters the second period when the effect of greater second period complementarity kicks in.

D.7.3 Proof that $f_{12}^{(1)} < 0$ is sufficient for $\frac{\partial I_A^1}{\partial \gamma} < 0$.

Consider the bordered Hessian displacement system associated for the problem for both children treated together:

$$\begin{bmatrix} c & d & 0 & 0 & -p_1 \\ d & e & 0 & 0 & -p_2 \\ 0 & 0 & c & d & -p_1 \\ 0 & 0 & d & e & -p_2 \\ -p_1 & -p_2 & -p_1 & -p_2 & 0 \end{bmatrix} \begin{bmatrix} dI_1^A \\ dI_2^A \\ dI_1^B \\ dI_2^B \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda dp_1 - \theta_A^1(\text{Term } 1)d\gamma \\ \lambda dp_2 - \theta_1^B(\text{Term } 2)d\gamma \\ \lambda dp_1 \\ \lambda dp_2 \\ -dE + \sum\limits_{\substack{j\in\{A,B\} \\ l\in\{1,2\}}} I_l^j dp_l \end{bmatrix} \tag{D.20}$$

where as before

$$c = \left[f_{11}^{(2)} [f_2^{(1)}]^2 + f_1^{(2)} f_{22}^{(1)} \right] \leq 0$$

if period 2 production is concave in θ_2 and period 1 production is concave in I_1. But it might also arise if period 1 production is convex in θ_2.

$$d = f_{12}^{(2)} f_2^{(1)} > 0 \quad \text{if there is second period complementarity}$$

$$e = f_{22}^{(2)} < 0 \quad \text{from concavity in } I_2.$$

Recall that

$$T_1 \equiv \text{Term } 1 \equiv \left[f_{11}^{(2)} f_1^{(1)} f_2^{(1)} + f_1^{(2)} f_{21}^{(1)} \right]$$

may be of either sign. The second grouping of terms in Term 1 is positive under complementarity in the first period; negative under substitutability. The first grouping is negative under concavity of $f^{(2)}$ in θ_2 (but it might be positive if there are increasing returns). Under second period complementarity ($f_{21}^{(2)} > 0$)

$$T_2 \equiv \text{Term 2} = \left[f_{21}^{(2)} f_1^{(1)}\right] \geq 0.$$

Let H be the bordered Hessian associated with displacement system (D.20) and let $|H|$ be the determinant of the Hessian. $|H| > 0$ under the assumption of a regular optimum.

Then the income-compensated effect of a change in p_2^A on I_1^A is

$$\frac{\partial I_1^A}{\partial p_2^A} = \lambda \begin{vmatrix} d & 0 & 0 & -p_1 \\ 0 & c & d & -p_1 \\ 0 & d & e & -p_2 \\ 0 & -p_1 & -p_2 & 0 \end{vmatrix} / |H|$$

$$= \lambda d \underbrace{\begin{vmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix}}_{<0} / |H|. \tag{D.21}$$

The numerator of (D.21) is negative from the sufficiency conditions for an optimum for the two stage budgeting problem for A and from second period dynamic complementarity ($d > 0$).

Hence both inputs are Hicks-compensated cross substitutes:

$$\frac{\partial I_1^A}{\partial p_2^A} < 0.$$

and from symmetry

$$\frac{\partial I_1^A}{\partial p_2^A} = \frac{\partial I_2^A}{\partial p_1^A} \quad < 0.$$

Collecting results,

$$\text{let } S_{ij} = \frac{\partial I_i^A}{\partial p_j^A} \qquad i, j \in \{1, 2\}$$

$$\frac{\partial I_1^A}{\partial \gamma} = -\left\{ \underset{(-)}{[S_{11}]} \underset{(?)}{[\text{Term } 1]} + \underset{(-)}{[S_{12}]} \underset{(+)}{[\text{Term } 2]} \right\} d\gamma \tag{D.22}$$

$$\frac{\partial I_2^A}{\partial \gamma} = -\left\{ \underset{(-)}{[S_{12}]} \underset{(?)}{[\text{Term } 1]} + \underset{(-)}{[S_{22}]} \underset{(+)}{[\text{Term } 2]} \right\} d\gamma. \tag{D.23}$$

If Term 1 is sufficiently negative, which could happen even if $f_{21}^{(1)}(\cdot) > 0$, then

$$\frac{\partial I_1^A}{\partial \gamma} < 0.$$

(Term 1 would be negative if $f_{21}^{(1)} < 0$) and possibly even

$$\frac{\partial I_2^A}{\partial \gamma} < 0.$$

Term 1 positive $\Rightarrow \frac{\partial I_1^A}{\partial \gamma} > 0$ and $\frac{\partial I_1^B}{\partial \gamma} < 0$. Thus it may be efficient to allocate more to the less endowed, even in both periods.

We can say something stronger. If $f_{12}^{(1)} < 0$, but $f_{12}^{(2)} > 0$, then as $\gamma \uparrow, I_A \downarrow$ and the term in braces in (D.22) is positive. To prove this define $T_1 =$ Term 1 and $T_2 =$ Term 2 and notice that

$$\frac{\partial I_1^A}{\partial \gamma} = \frac{\begin{vmatrix} -T_1\theta_1^B & d & 0 & 0 & -p_1 \\ -T_2\theta_1^B & e & 0 & 0 & -p_2 \\ 0 & 0 & c & d & -p_1 \\ 0 & 0 & d & e & -p_2 \\ 0 & -p_2 & -p_1 & -p_2 & 0 \end{vmatrix}}{\underbrace{|H|}_{(+)}} = \frac{|N|}{|H|}\theta_1^B,$$

where

$$|N| = \left\{ -T_1 e \underbrace{\begin{vmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_2 & -p_2 & 0 \end{vmatrix}}_{|M|>0} - T_1 p_2 \begin{vmatrix} 0 & 0 & -p_2 \\ c & d & -p_1 \\ d & e & -p_2 \end{vmatrix} \right.$$

$$\left. + T_2 d \underbrace{\begin{vmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix}}_{|M|>0} + T_2 p_2 \begin{vmatrix} 0 & 0 & -p_1 \\ c & d & -p_1 \\ d & e & -p_2 \end{vmatrix} \right\} \theta_1^B$$

$$|N| = \left[(-T_1 e + T_2 d)|m| - \underbrace{\underset{(?)}{(T_1)}\, \underset{(+)}{p_2}\, \underset{(-)}{(-p_2)} \underset{(+)}{\begin{vmatrix} c & d \\ d & e \end{vmatrix}}}_{-\text{if } (T_1)<0} + \underbrace{\underset{(+)}{T_2}\, \underset{(+)}{p_2}\, \underset{(-)}{(-p_1)} \underset{(+)}{\begin{vmatrix} c & d \\ d & e \end{vmatrix}}}_{(-)} \right] \theta_1^B$$

Thus it follows as a sufficient condition that

$$|N| < 0 \text{ if } \left[(-T_1 e + T_2 d) < 0 \right].$$

Writing out $(-T_1 e + T_2 d)$,

$$(-T_1 e + T_2 d) = -f_{11}^{(2)} f_1^{(1)} f_2^{(1)} f_{22}^{(2)} - f_1^{(2)} f_{21}^{(1)} f_{22}^{(2)} + f_{21}^{(2)} f_1^{(1)} f_{12}^{(2)} f_2^{(1)},$$

and collecting the first and the last terms:

$$\underbrace{- \underbrace{f_1^{(1)} f_2^{(1)}}_{(+)} \underbrace{\left[f_{11}^{(2)} f_{22}^{(2)} - [f_{12}^{(2)}] \right]}_{(+)\text{ by concavity}}}_{(-)} - \underbrace{\underset{(+)}{f_1^{(2)}} \underset{(?)}{f_{21}^{(1)}} \underset{(-)}{f_{22}^{(2)}}}_{(-)\text{ if } f_{21}^{(1)}<0} \tag{D.24}$$

so

$$(-T_1 e + T_2 d) < 0 \quad \text{if} \quad f_{21}^{(1)} < 0,$$

and hence

$$|N| < 0 \quad \text{if} \quad f_{21}^{(1)} < 0,$$

so

$$\frac{\partial I_1^A}{\partial \gamma} < 0 \quad \text{if} \quad f_{21}^{(1)} < 0.$$

Notice, however, that even if $f_{21}^{(1)}(\cdot) > 0$, it is possible that

$$\frac{\partial I_1^A}{\partial \gamma} < 0.$$

(See the second term in equation (D.24).) Notice that the more negative $f_{22}^{(2)}$ (i.e., the more sharply are the diminishing returns to I_2^A in period 2), the more negative is $\frac{\partial I_1^A}{\partial \gamma}$. The intuition for this offsetting effect is that as second period investments become less effective, then it is more productive to invest relatively more in the first period. Concavity in terms of θ_2 is not strictly required.

Next consider

$$\frac{\partial I_2^A}{\partial \gamma} =$$

$$\frac{\begin{vmatrix} c & -T_1\theta_1^B & 0 & 0 & -p_1 \\ d & -T_2\theta_1^B & 0 & 0 & -p_2 \\ 0 & 0 & c & d & -p_1 \\ 0 & 0 & d & e & -p_2 \\ -p_1 & 0 & -p_1 & -p_2 & 0 \end{vmatrix}}{|H|}$$

$$= \frac{\tilde{N}}{|H|}\theta_1^B$$

$$\tilde{N} = T_1 \begin{vmatrix} d & 0 & 0 & -p_2 \\ 0 & c & d & -p_1 \\ 0 & d & e & -p_2 \\ -p_1 & -p_1 & -p_2 & 0 \end{vmatrix}$$

$$-T_2 \begin{vmatrix} c & 0 & 0 & -p_1 \\ 0 & c & d & -p_1 \\ 0 & d & e & -p_2 \\ -p_1 & -p_1 & -p_2 & 0 \end{vmatrix}$$

$$= T_1 \left[d \begin{vmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix} + p_1 \begin{vmatrix} 0 & 0 & -p_2 \\ c & d & -p_1 \\ d & e & -p_2 \end{vmatrix} \right]$$

$$-T_2 \left[c \begin{vmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix} + p_1 \begin{vmatrix} 0 & 0 & -p_1 \\ c & d & -p_1 \\ d & e & -p_2 \end{vmatrix} \right]$$

$$= (T_1 d - T_2 c) \underset{(+)}{\begin{vmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix}}$$

$$- \underbrace{T_1 p_1 p_2}_{(-)\text{ if } T_1<0} \underset{(+)}{\begin{vmatrix} c & d \\ d & e \end{vmatrix}} + \underset{(+)}{T_2 (p_1)^2} \underset{(+)}{\begin{vmatrix} c & d \\ d & e \end{vmatrix}}$$

Focus on the term $(T_1 d - T_2 c)$

$$= (\underset{(-)}{T_1} \underset{(+)}{d} - \underset{(+)}{T_2} \underset{(-)}{c}) \underset{(+)}{\begin{vmatrix} c & d & -p_1 \\ d & e & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix}} - (\underset{(-)}{T_1} \underset{(+)}{p_1 p_2} - \underset{+}{T_2} \underset{+}{p_1^2}) \underset{(+)}{\begin{vmatrix} c & d \\ d & e \end{vmatrix}}$$

Observe that

$$(T_1 d - T_2 c) = \left[f_{11}^{(2)} f_1^{(1)} f_2^{(1)} + f_{12}^{(1)} f_1^2 \right] f_{12}^{(2)} f_2^{(1)}$$
$$- f_{21}^{(2)} f_1^{(1)} \left[f_{11}^{(2)} (f_2^{(1)})^2 + f_1^{(2)} f_{22}^{(1)} \right]$$

$$= \cancel{f_{11}^{(2)} f_1^{(1)} f_2^{(1)} f_{12}^{(2)} f_2^{(1)}}$$
$$+ f_{12}^{(1)} f_1^{(2)} f_{12}^{(2)} f_2^{(1)}$$
$$\cancel{- f_{21}^{(2)} f_1^{(1)} f_{11}^{(2)} \left(f_2^{(1)} \right)^2}$$
$$- f_{21}^{(2)} f_1^{(1)} f_1^{(2)} f_{12}^{(1)}$$
$$= \underset{(+)}{f_{12}^{(2)} f_1^{(2)}} \underbrace{\left[f_{12}^{(1)} f_2^{(1)} - f_1^{(1)} f_{22}^{(1)} \right]}_{T_3}$$

and the last term is positive ($T_3 > 0$), if in the period 1 production function $f_{12}^{(1)} > 0$ (first period complementarity). This is a sufficient condition for

$$\frac{\partial I_2^A}{\partial \gamma} > 0.$$

Notice that when Term 1 (T_1) is negative, then T_3 can be negative.[5] Thus, it is possible that the efficient policy redistributes to the less endowed in period 1 but to the more endowed in period 2. It is also possible that as $\gamma \uparrow$, it is socially efficient to invest in the disadvantaged child in both periods, although this seems unlikely. In general, it is not efficient to make the initially disadvantaged child whole by the start of the second period, and second period complementarity reinforces starting of second period discrepancies.

D.8 Some Evidence from Simulations on Why Dynamic Complementarity is a Force Toward Targeting Disadvantaged Children in the Early Years

Dynamic complementarity is a force toward equalization of early stage investments even in the absence of family inequality aversion. To illustrate the mechanism underlying this claim, suppose that, for each child k, the outcome of interest for parents are children's earnings E_k and that they are a function of children's adult human capital determined by genes ($\theta_{1,k}$) and early ($I_{1,k}$) and late ($I_{2,k}$) parental investments.

$$E_k = wf^2(\theta_{2,k}, I_{2,k}) = \ f^2\left(\gamma_2\theta_{2,k}^{\phi_2} + (1-\gamma_2)I_{2,k}^{\phi_2}\right)^{\frac{\rho_2}{\phi_2}} \tag{D.25}$$

with

$$\theta_{2,k} = f^1(\theta_{1,k}, I_{1,k}) = \ f^1\left(\gamma_1\theta_{1,k}^{\phi_1} + (1-\gamma_1)I_{1,k}^{\phi_1}\right)^{\frac{\rho_1}{\phi_1}} \tag{D.26}$$

[5] Notice that T_3 is always positive whenever the marginal rate of substitution between initial ability (θ_1) and initial investments (I_1) is increasing in investments (I_1) i.e. if

$$\frac{\partial}{\partial I_1}\left[\frac{f_1^{(1)}}{f_2^{(1)}}\right] > 0.$$

where w is the payment to skill corresponding to one unit of human capital which is determined by equilibrium in the factor markets. Since w is common across families and siblings we assume that the measurement of human capital is chosen so that $w = 1$. The budget constraint faced by the parents with total resources R^e is:

$$p_1 \sum_{k=1}^{n} I_1 + p_2 \sum_{k=1}^{n} I_2 = R^e. \tag{D.27}$$

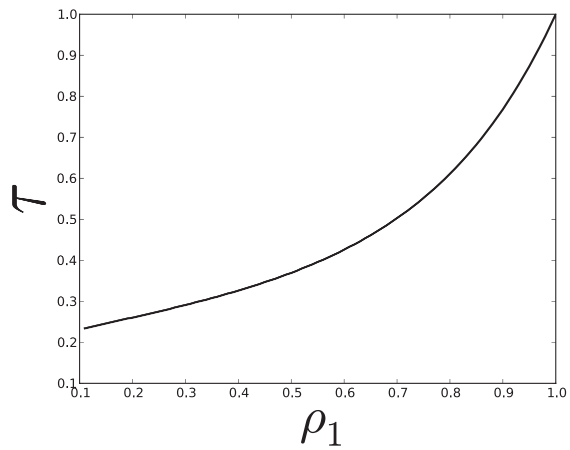
Consider the case of a parent with two children i and j. We show that even in the absence of inequality aversion, the shape of the technology, and in particular the presence of decreasing returns in at least one of the two periods, might induce parents to follow a compensating strategy devoting more resources to the less endowed child, say j ($\theta_{1,i} > \theta_{1,j}$).

As a measure of parental compensation with respect to initial inequality we define the parameter τ as:

$$\tau \equiv \left(\frac{E_i}{E_j}\right) \Big/ \left(\frac{\theta_i}{\theta_j}\right), \tag{D.28}$$

which captures how much earnings differences are inflated compared to initial endowment differences. If $\tau = 1$, the parents perfectly translate genetic differences into earnings. In results from a simulation exercise, Figure D.2 shows that earnings differences are dampened compared to differences in initial endowments whenever $\rho_1 < 1$.

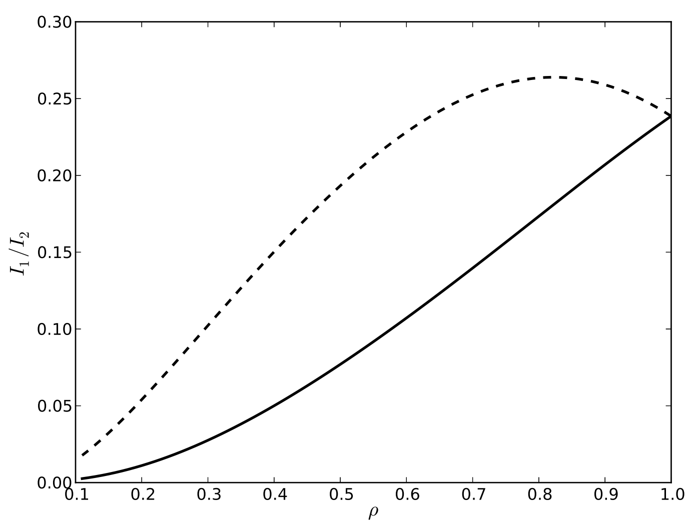
Figure D.2: Earnings Equalization



Notes: The parental preference parameters used in the simulation are $\sigma = 1$ and $\omega_i = \omega_j = 0.5$. Total resources are $R^e = 4$. The technology of skill formation parameters, capturing increasing complementarity between skills and investments over time, are: $\gamma_1 = \gamma_2 = 0.5$, $\phi_1 = 0.6$, $\phi_2 = -0.5$, $\rho_2 = 1$. The parameter ρ_1 defines the degree of homogeneity of the first period technology. We vary the value of ρ_1 over the range $[0.1, 1]$. Child i has a skill endowment of 5 while child j of 1.

We also consider the how changes in ρ_1 affect parental behavior in Figures D.3, D.4, and D.5. Figure D.3 shows the ratio of early (I_1) to late (I_2) investments.

Figure D.3: Ratio Early to Late Investments

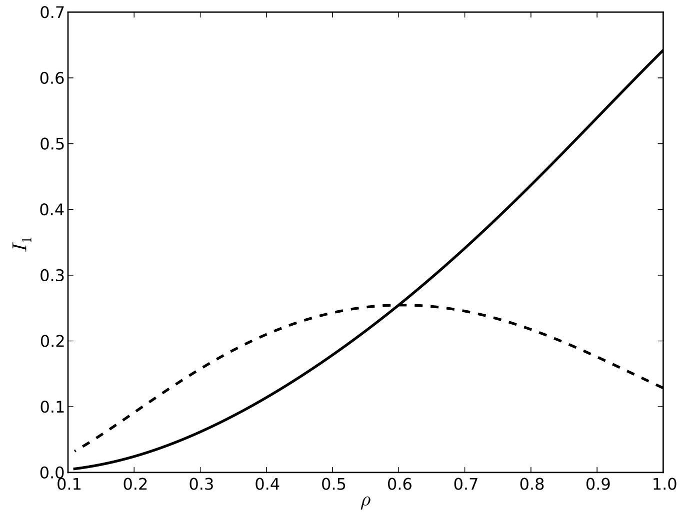


Notes: The solid line refers to the most endowed child, the dashed line to the least endowed chid. The parameters used are as in Figure D.2.

This ratio is always higher for the less endowed child j whenever ρ_1 is smaller than one. Figure D.4 shows that the *less* endowed child receives a *higher* amount of early investment whenever the period 1 technology exhibits substitutability[6] between skills (initial endowments) and investments (i.e. when $\rho_1 < \phi_1$).

[6]In the Edgeworth sense of a negative cross derivative.

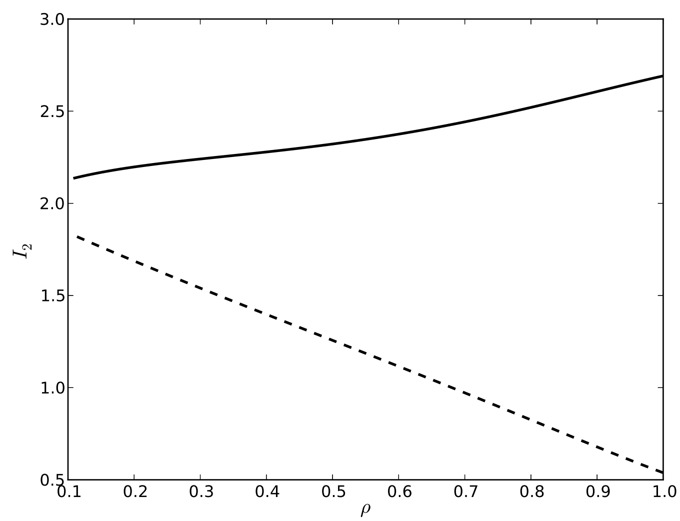
Figure D.4: Levels of Early Investments



Notes: The solid line refers to the most endowed child, the dashed line to the least endowed chid. The parameters used are as in Figure D.2.

Figure D.5 shows that the most endowed child always receives a higher level of late investment.

Figure D.5: Levels of Late Investments



Notes: The solid line refers to the most endowed child, the dashed line to the least endowed chid. The parameters used are as in Figure D.2.

Late investments are an increasing function of ρ_1 for the more endowed child while they are decreasing in ρ_1 for the less endowed child. As ρ_1 decreases the less endowed child receives a higher level of early investments and a level of late investments which is increasingly closer to the one of his more endowed brother. This explains why earnings tend to be equalized as ρ_1 decreases.

We conclude that if the technology of skill formation is defined over more than one period, parents might exhibit compensating behavior in investments in children's human capital even in absence of inequality aversion. In particular, less endowed children receive a higher level of early investment than their more endowed siblings if the technology of skill formation exhibits substitutability between initial (genetic) endowments and the level of early investments.

D.9 Review of Literature on Multichild Families

In a pioneering study, Behrman et al. (1982) estimate the coefficient of inequality aversion σ in Equation (**??**) and find that parents tend to compensate for initial differences across children. Del Boca et al. (2014) consider two-child households assuming $\sigma = 1$. They allow the weights on child cognitive ability, the outcome of interest for the parents, to differ across the two children and find that parents manifest a slight preference for the younger child (weight = 0.54). Gayle et al. (2013) also consider multi child families in an overlapping generation context where parents value the future utility of their children. They assume $\sigma = 1$ but the weights are decreasing in the number of children and are given by $\omega_k = N^{\rho}/N$ where $\rho \leq 1$. They use this setup to analyze quantity-quality trade-offs in parental choices. Considering hypothetical families with same sex children, they show that for boys the average quality declines with each child, while for girls the trade-off emerges only after the third child. The quantity-quality trade-off is more pronounced for blacks than for whites as fertility rates are higher among single black mothers and the cost of time for single mothers is higher than for married couples.[7]

[7] Yi (2013) develops a model in which parents invest in shaping a child's altruism toward siblings as an insurance device for protecting less endowed siblings.

E Evidence on The Predictive Power of Cognitive and Socioemotional Traits

The Big Five Traits are considered the "latitude and longitude of personality by personality" psychologists. They are defined in Table E.1.

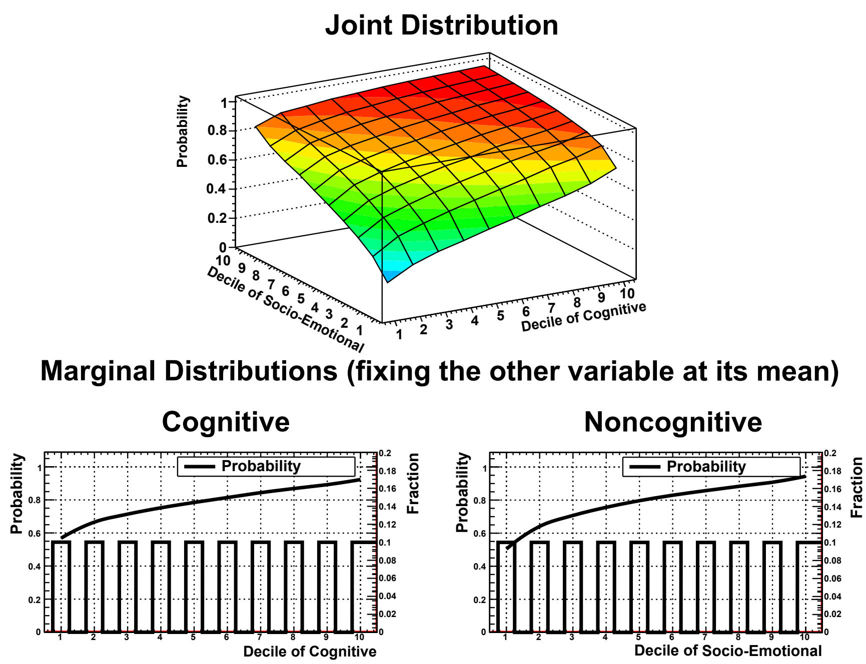
Table E.1: The Big Five Domains and Their Facets

Big Five Personality Factor	American Psychology Association Dictionary Description	Facets (and correlated skill adjective)	Related Skills	Analogous Childhood Temperament Skills
Conscientiousness	"The tendency to be organized, responsible, and hardworking"	Competence (efficient), Order (organized), Dutifulness (not careless), Achievement striving (ambitious), Self-discipline (not lazy), and Deliberation (not impulsive)	Grit, Perseverance, Delay of gratification, Impulse control, Achievement striving, Ambition, and Work ethic	Attention/(lack of) distractibility, Effortful control, Impulse control/delay of gratification, Persistence, Activity*
Openness to Experience	"The tendency to be open to new aesthetic, cultural, or intellectual experiences"	Fantasy (imaginative), Aesthetic (artistic), Feelings (excitable), Actions (wide interests), Ideas (curious), and Values (unconventional)		Sensory sensitivity, Pleasure in low-intensity activities, Curiosity
Extraversion	"An orientation of one's interests and energies toward the outer world of people and things rather than the inner world of subjective experience; characterized by positive affect and sociability"	Warmth (friendly), Gregariousness (sociable), Assertiveness (self-confident), Activity (energetic), Excitement seeking (adventurous), and Positive emotions (enthusiastic)		Surgency, Social dominance, Social vitality, Sensation seeking, Shyness*, Activity*, Positive emotionality, and Sociability/affiliation
Agreeableness	"The tendency to act in a cooperative, unselfish manner"	Trust (forgiving), Straight-forwardness (not demanding), Altruism (warm), Compliance (not stubborn), Modesty (not show-off), and Tender-mindedness (sympathetic)	Empathy, Perspective taking, Cooperation, and Competitiveness	Irritability*, Aggressiveness, and Willfulness
Neuroticism/ Emotional Stability	Emotional stability is "Predictability and consistency in emotional reactions, with absence of rapid mood changes." Neuroticism is "a chronic level of emotional instability and proneness to psychological distress"	Anxiety (worrying), Hostility (irritable), Depression (not contented), Self-consciousness (shy), Impulsiveness (moody), Vulnerability to stress (not self-confident)	Internal versus External, Locus of control, Core self-evaluation, Self-esteem, Self-efficacy, Optimism, and Axis I psychopathologies (mental disorders) including depression and anxiety disorders	Fearfulness/behavioral inhibition, Shyness*, Irritability*, Frustration, (Lack of) soothability, Sadness

Notes: *These temperament attributes may be related to two Big Five factors. Facets specified by the NEO-PI-R personality inventory (Costa and McCrae, 1992). Adjectives in parentheses from the Adjective Check List (Gough and Heilbrun, 1983).
Source: Table adapted from John and Srivastava (1999).

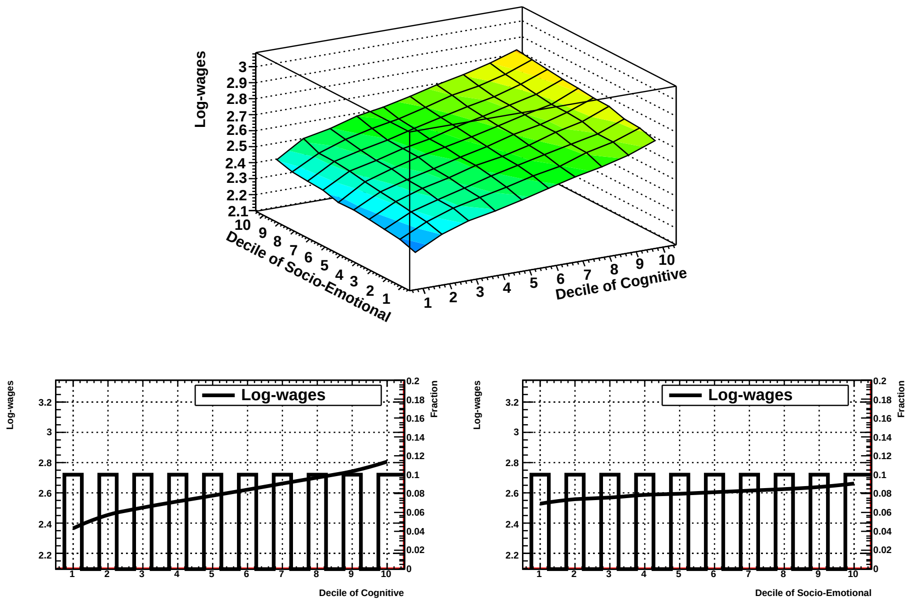
Borghans et al. (2008) and Almlund et al. (2011) present evidence on the predictive power of cognitive and social and emotional traits outcomes. The following figures taken from Heckman et al. (2011) shows the effect of child capacities on diverse outcomes correcting for the effect of schooling on capacities and the effect of capacities on schooling. There is a causal effect of schooling on these capacities. These empirical relationships account for reverse causality — measured capacities may be determined in part by schooling. The graphs show outcomes graphed against deciles of the cognitive and personality distributions. For a detailed description of the methodology see Heckman et al. (2011) and Almlund et al. (2011).

Figure E.1: The Probability of Educational Decisions, by Endowment Levels, Dropping from Secondary School vs. Graduating



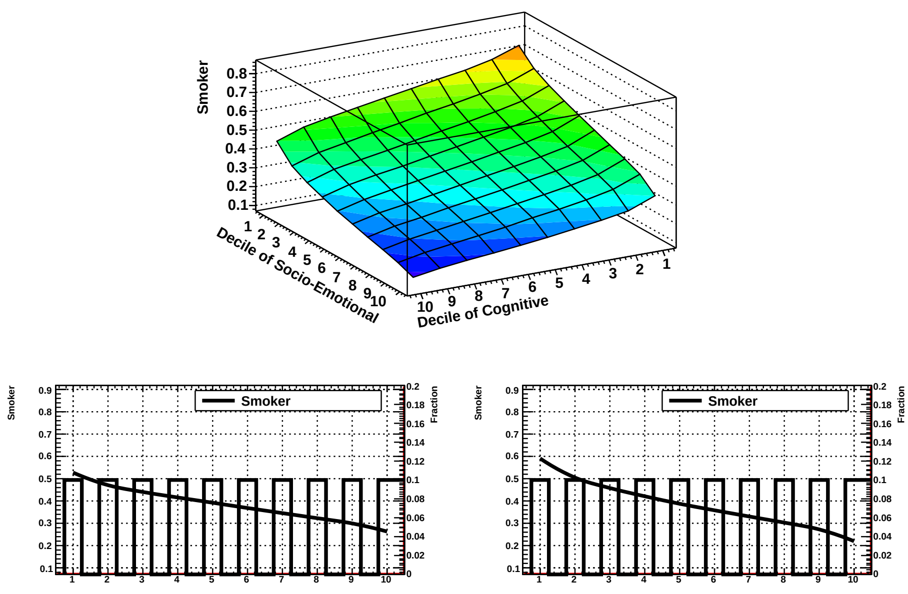
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure E.2: The Effect of Cognitive and Socio-emotional endowments, (log) Wages



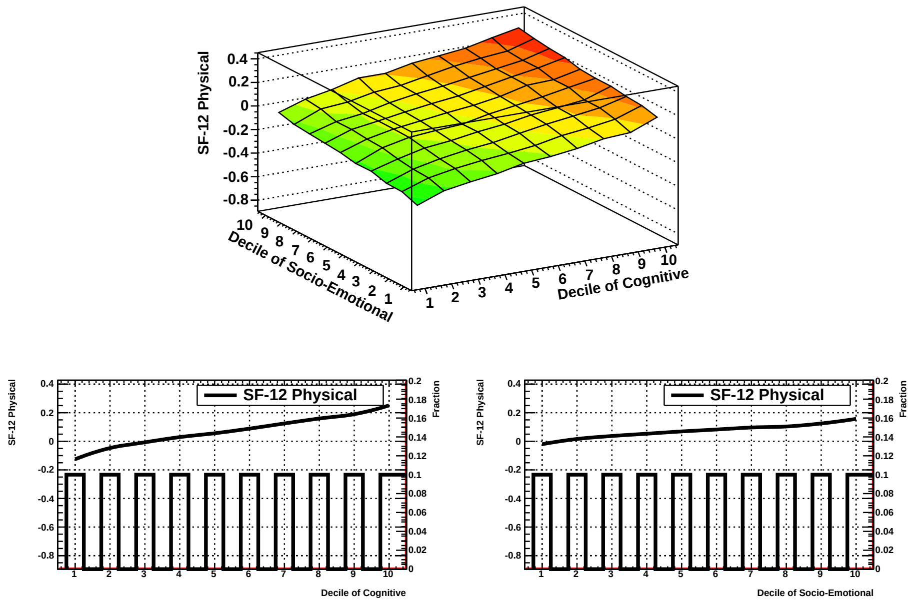
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure E.3: The Effect of Cognitive and Socio-emotional endowments, Daily Smoking



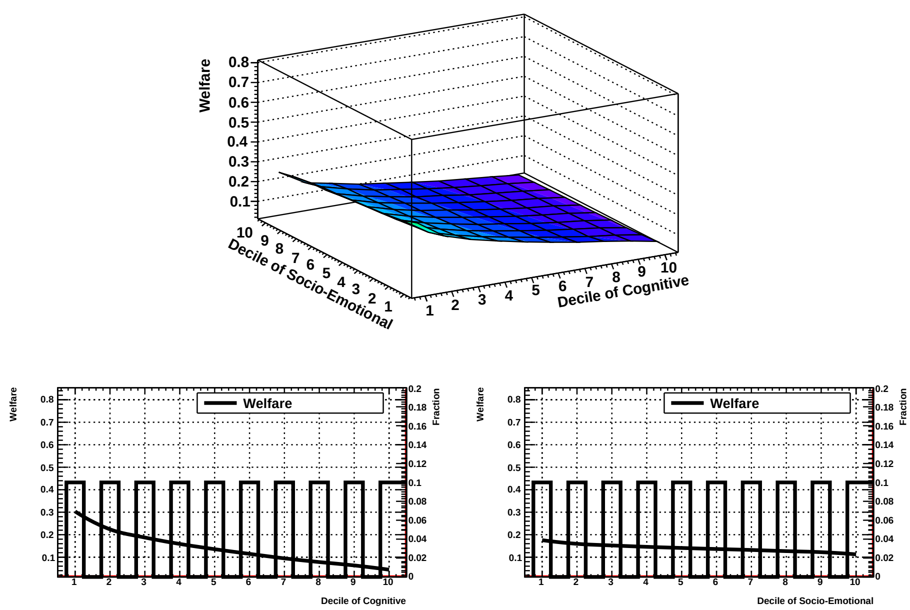
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure E.4: The Effect of Cognitive and Socio-emotional endowments on Physical Health at age 40 (PCS-12)



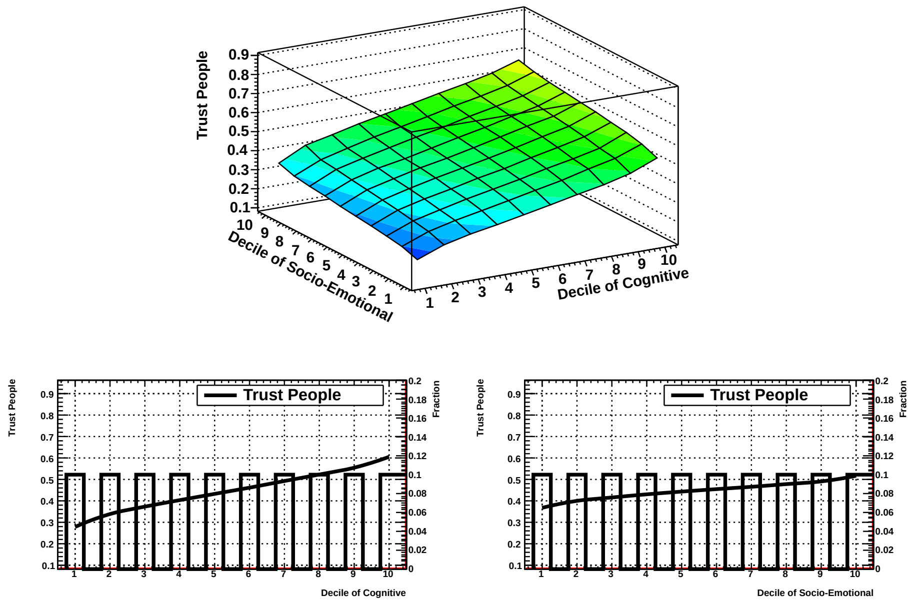
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure E.5: The Effect of Cognitive and Socio-emotional endowments on Ever Participated in Welfare (1996-2006)



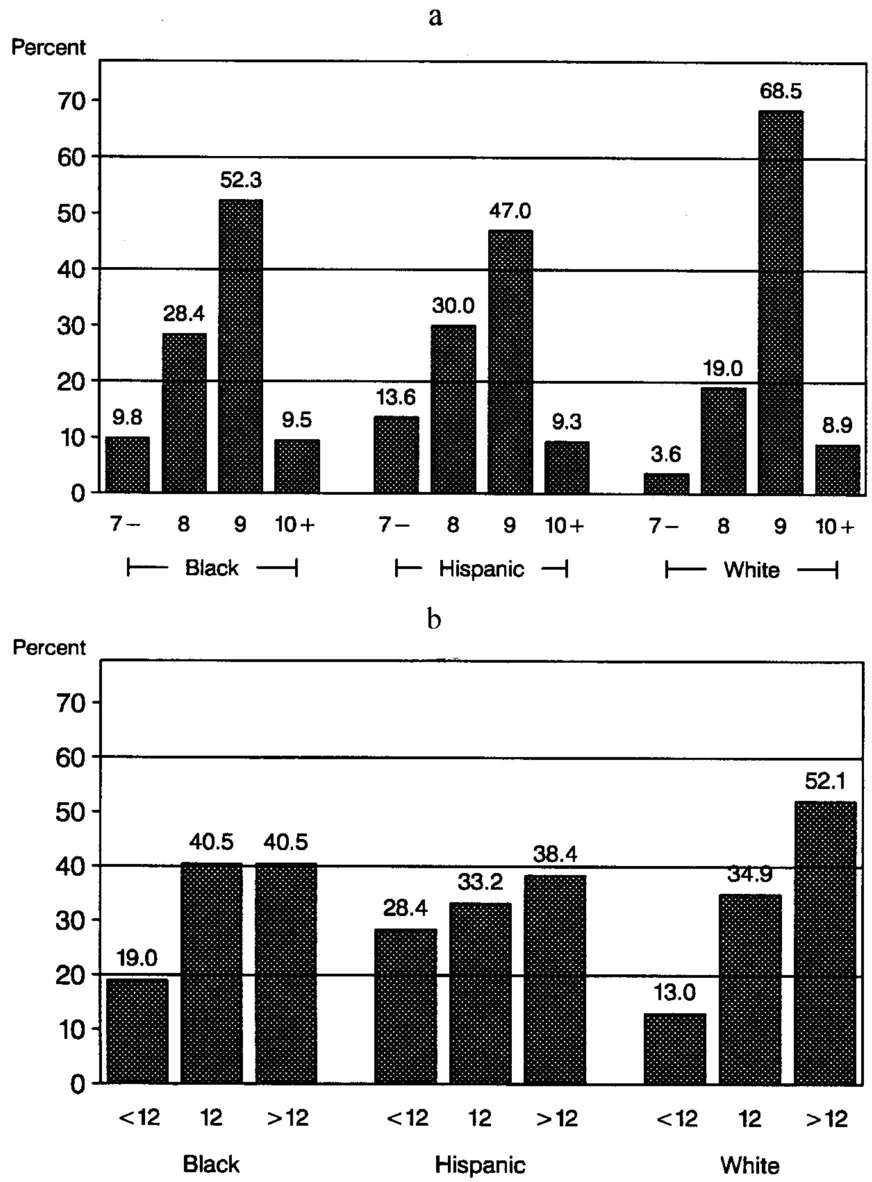
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure E.6: The Effect of Cognitive and Socio-emotional endowments on Trusting People (2008)



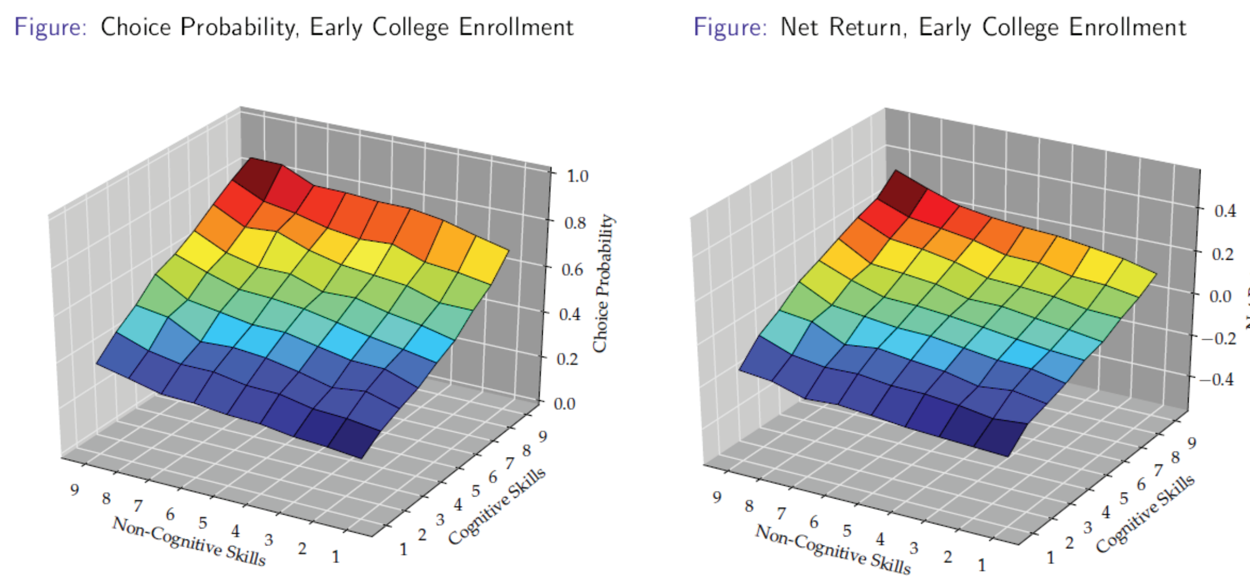
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure E.7: *a*, Highest grade completed at age 15. 7- denotes grade 7 or lower, and 10+ denotes grade 10 or higher. *b*, Highest grade completed at age 24. <12 denotes grade 11 or lower, and 112 denotes college attendance



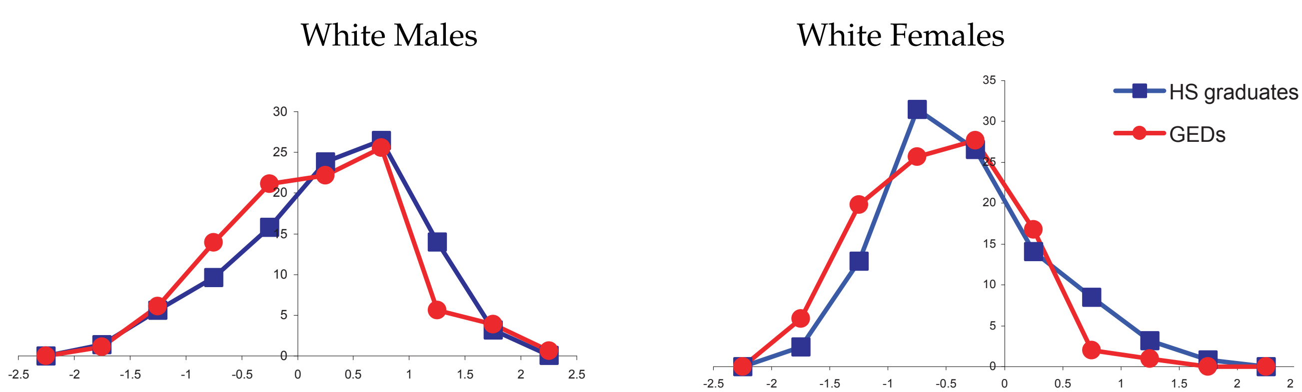
Source: Cameron and Heckman (2001).

Figure E.8: The Probability and Returns of College Enrollment by Endowments Levels



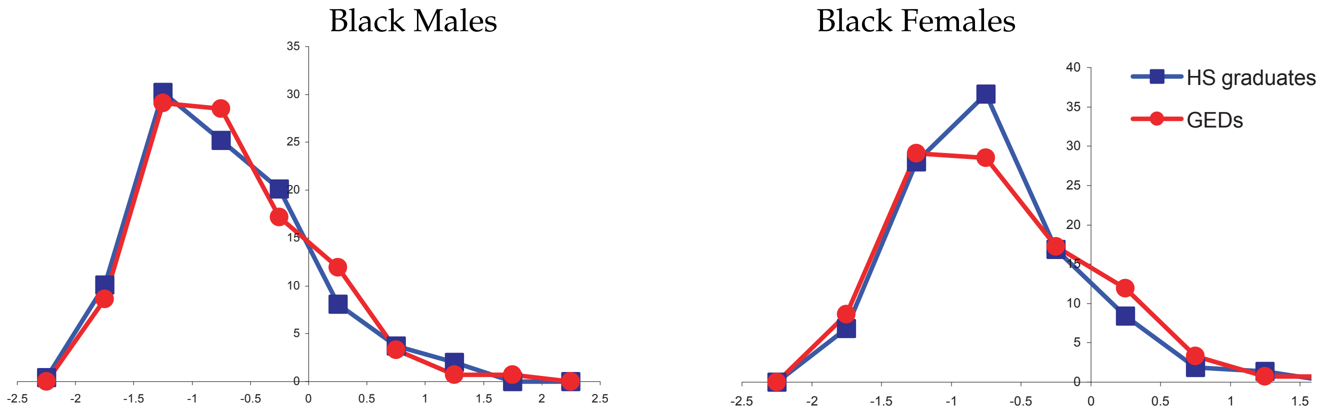
Source: Eisenhauer et al. (2013)
Note: Early college enrollment refer to the individuals who enroll in college immediately after having finished high school. Returns are expressed in units of millions of dollars.

Figure E.9: Density of age adjusted AFQT scores, GED recipients and high school graduates with twelve years of schooling



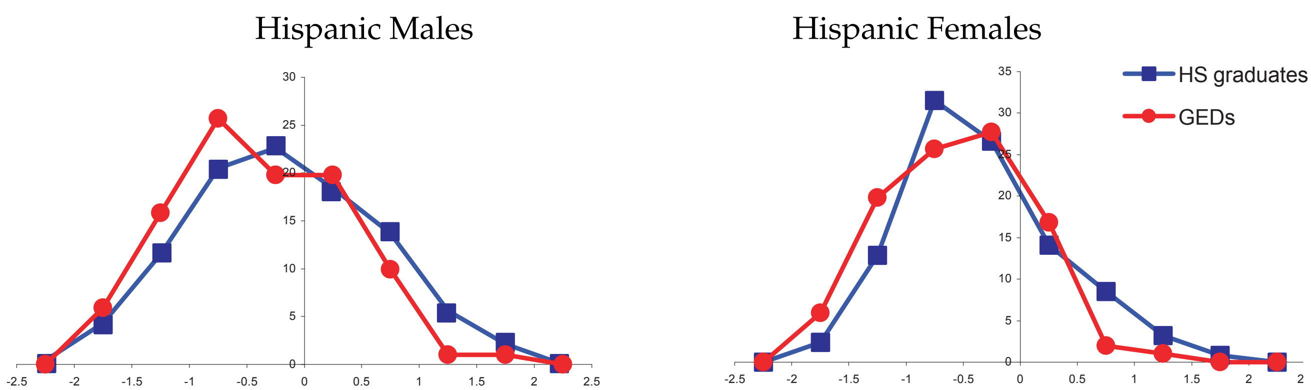
Source: Heckman, Hsee and Rubinstein (2001)

Figure E.10: Density of age adjusted AFQT scores, GED recipients and high school graduates with twelve years of schooling



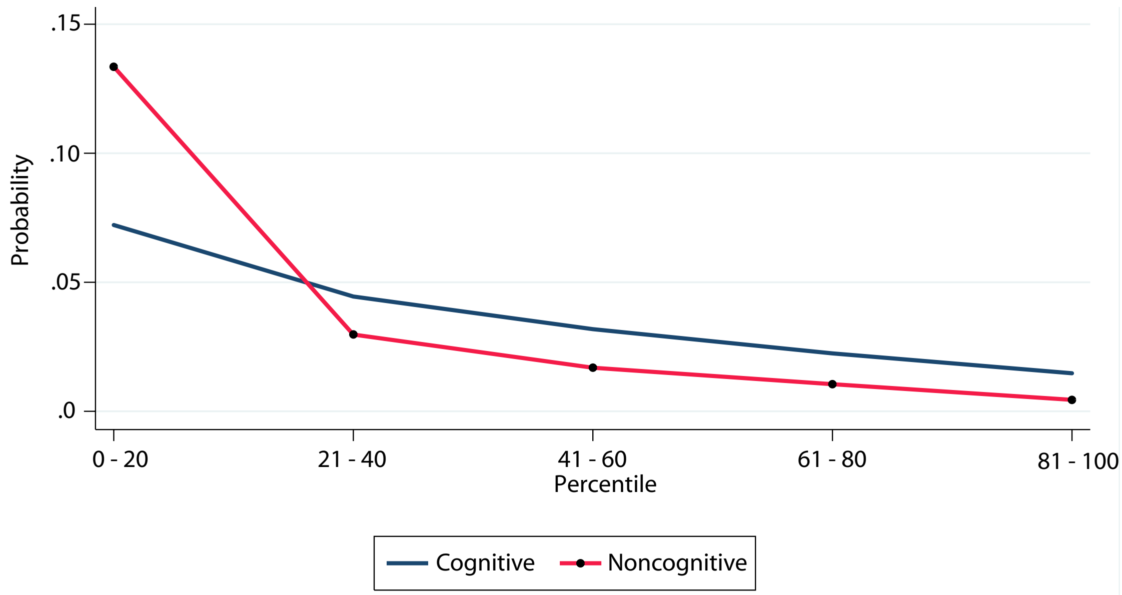
Source: Heckman, Hsee and Rubinstein (2001)

Figure E.11: Density of age adjusted AFQT scores, GED recipients and high school graduates with twelve years of schooling



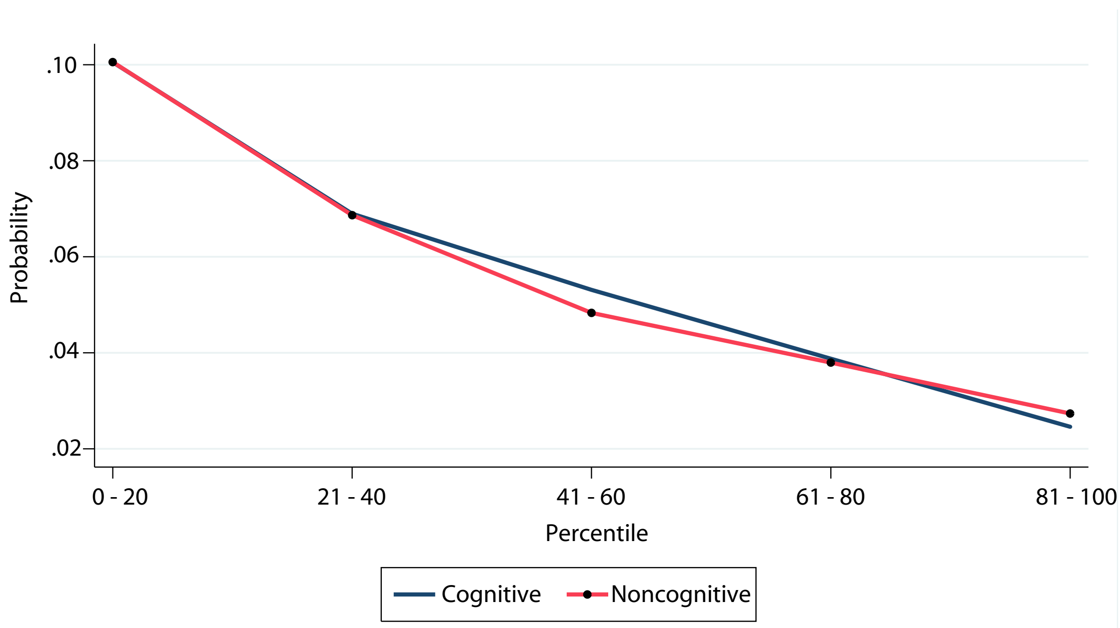
Source: Heckman, Hsee and Rubinstein (2001)

Figure E.12: Ever been in jail by age 30, by ability (males)



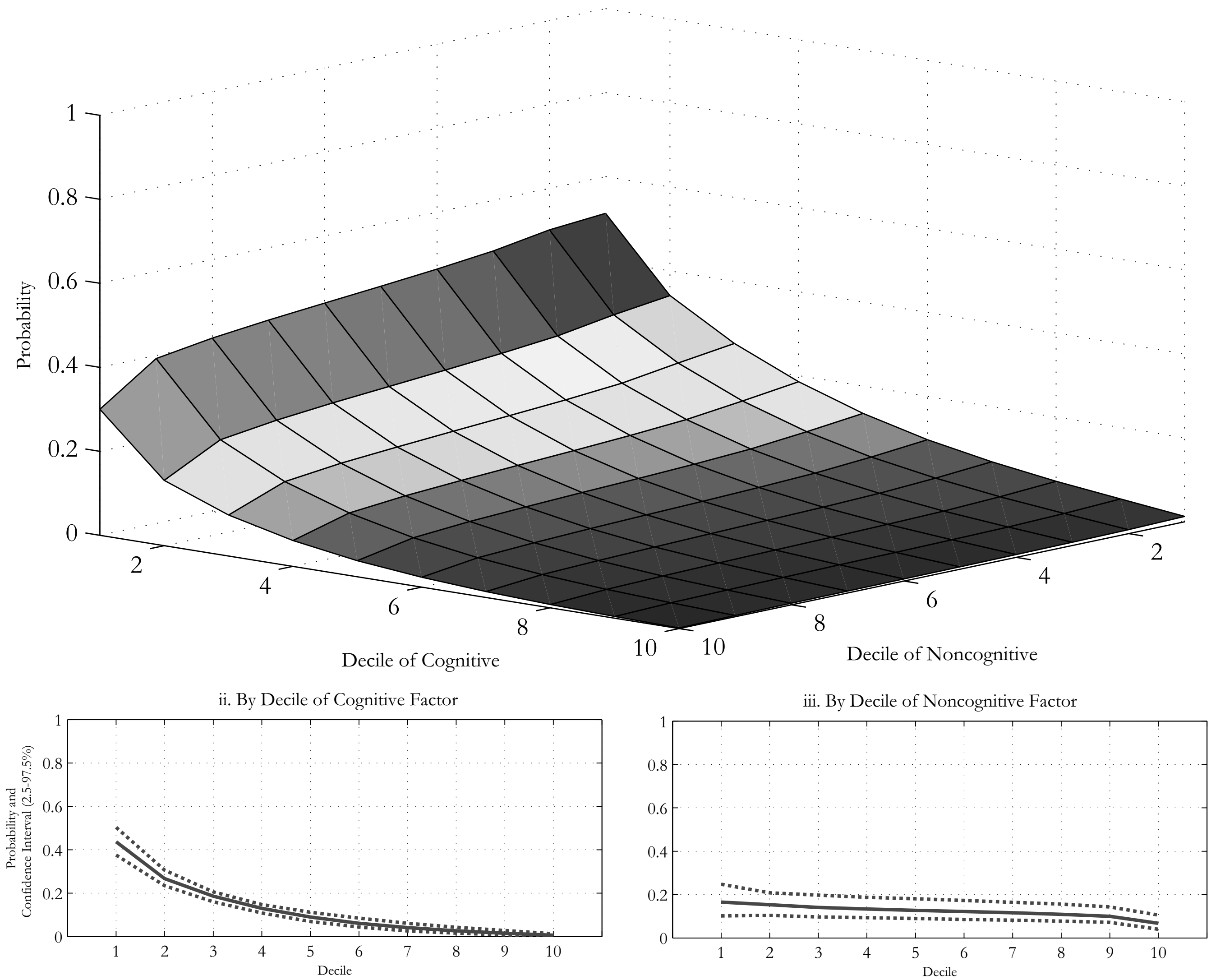
Note: This figure plots the probability of a given behavior associated with moving up in one ability distribution for someone after integrating out the other distribution. For example, the lines with markers show the effect of increasing noncognitive ability after integrating the cognitive ability.
Source: Heckman et al. (2006).

Figure E.13: Probability of being single with children (females)



Note: This figure plots the probability of a given behavior associated with moving up in one ability distribution for someone after integrating out the other distribution. For example, the lines with markers show the effect of increasing noncognitive ability after integrating the cognitive ability.
Source: Heckman et al. (2006).

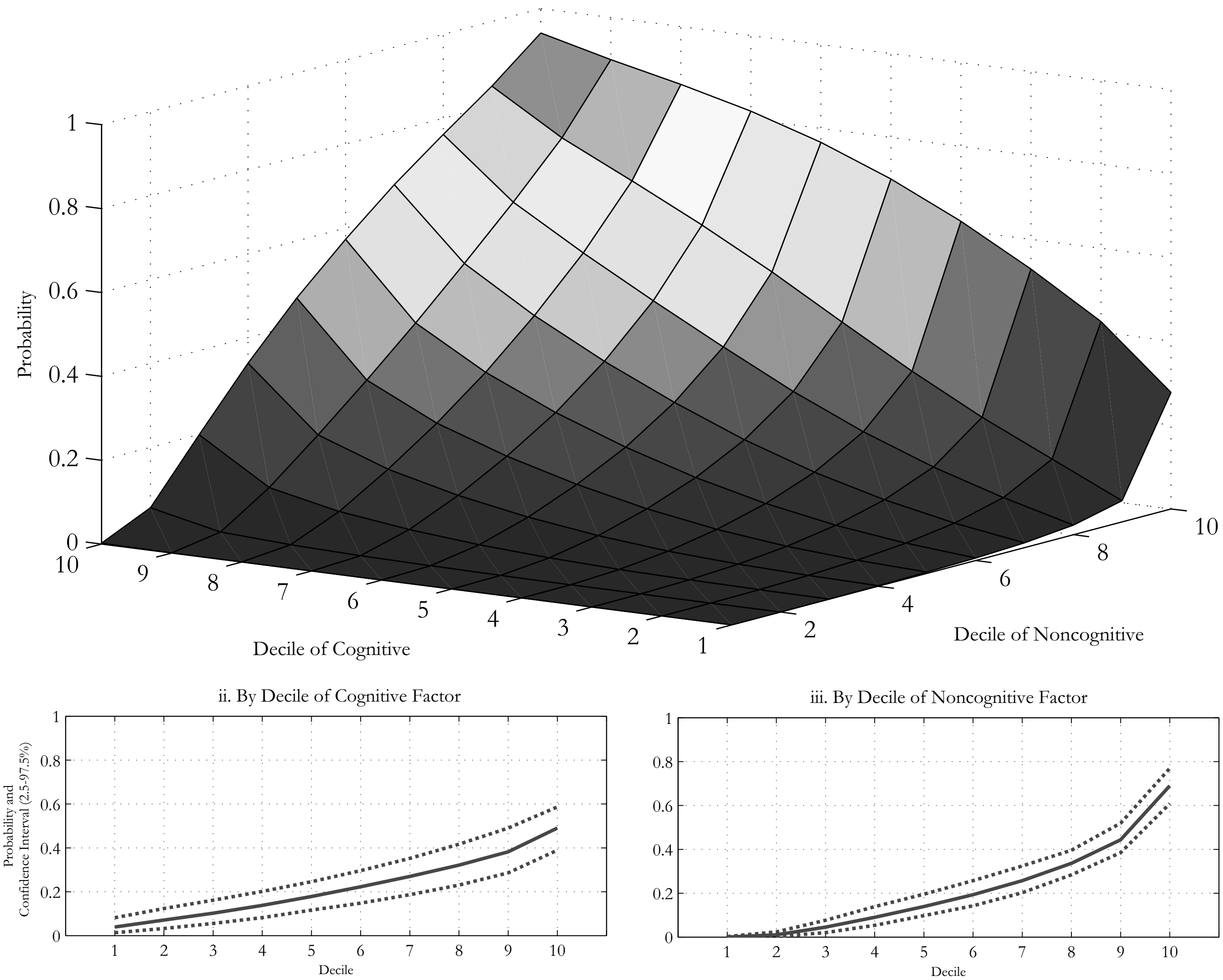
Figure E.14: Probability of being a high school dropout by age 30 (males)



Notes: The data are simulated from the estimates of the model and our NLSY79 sample. We use the standard convention that higher deciles are associated with higher values of the variable. The confidence intervals are computed using bootstrapping (200 draws).

Source: Heckman et al. (2006).

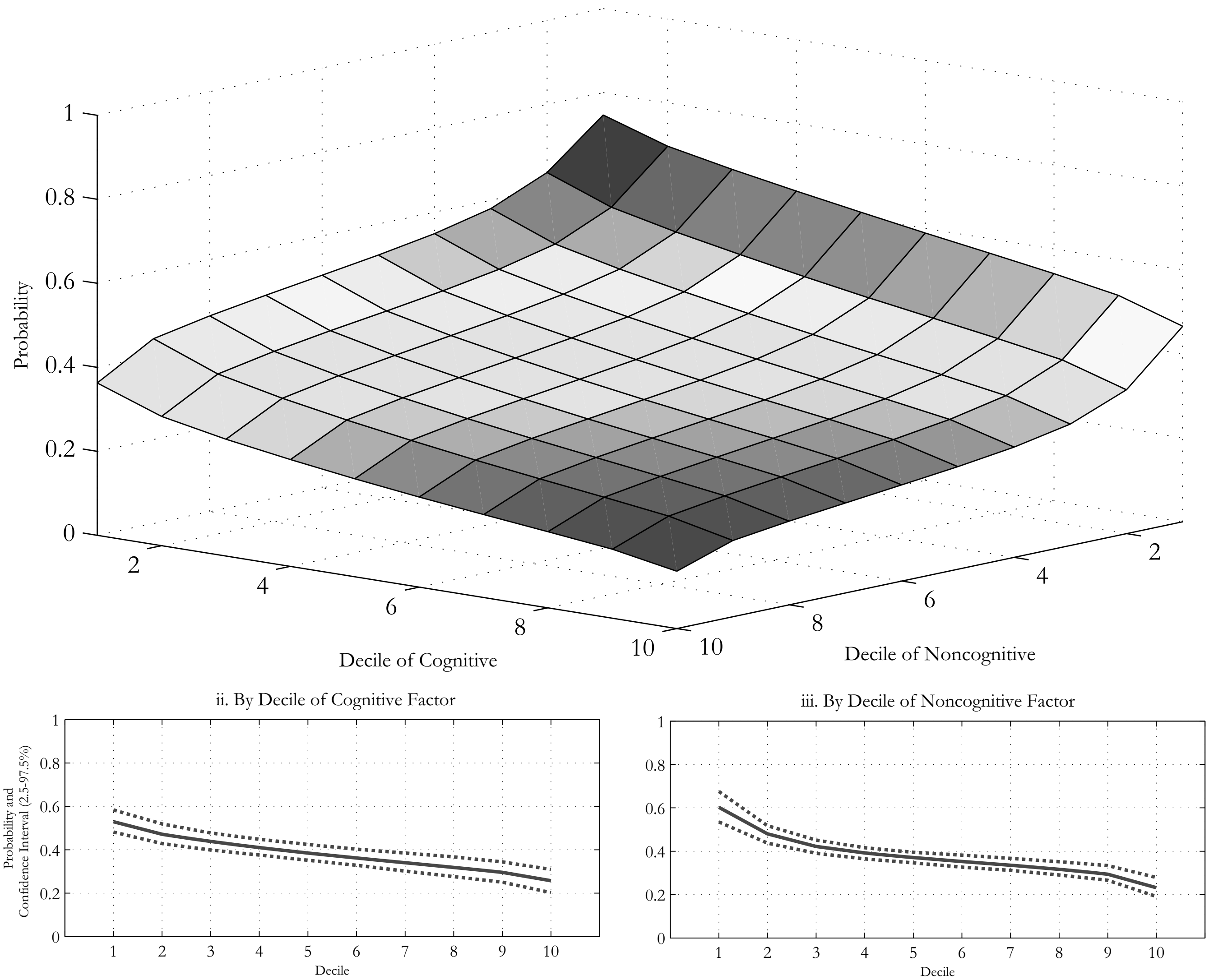
Figure E.15: Probability of being a 4-year college graduate by age 30 (males)



Notes: The data are simulated from the estimates of the model and our NLSY79 sample. We use the standard convention that higher deciles are associated with higher values of the variable. The confidence intervals are computed using bootstrapping (200 draws).

Source: Heckman et al. (2006).

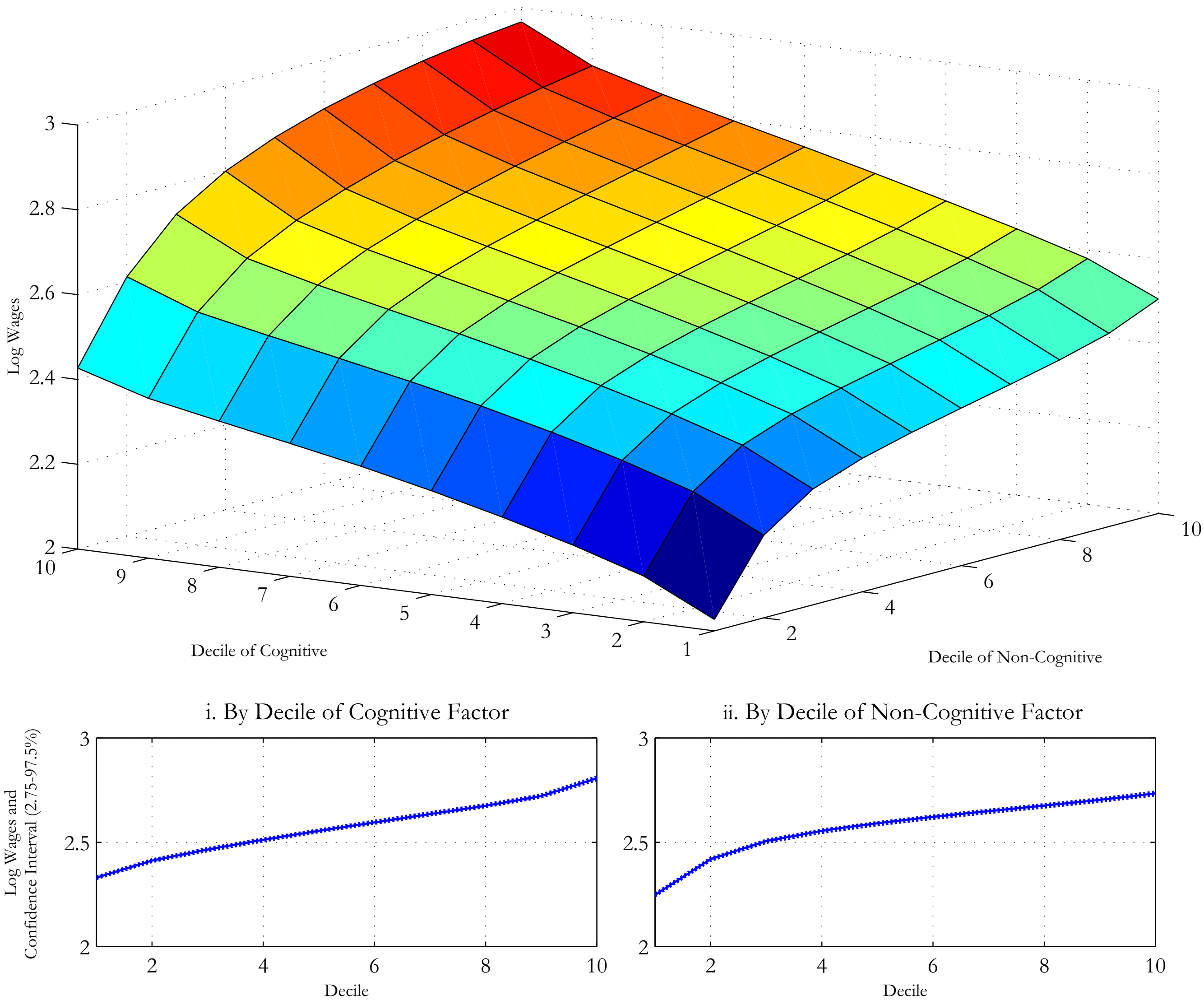
Figure E.16: Probability of daily smoking by age 18 (males)



Notes: The data are simulated from the estimates of the model and our NLSY79 sample. We use the standard convention that higher deciles are associated with higher values of the variable.
The confidence intervals are computed using bootstrapping (200 draws).

Source: Heckman et al. (2006).

Figure E.17: Mean log wages by age 30 (males)



Notes: The data are simulated from the estimates of the model and our NLSY79 sample. We use the standard convention that higher deciles are associated with higher values of the variable. The confidence intervals are computed using bootstrapping (50 draws).

Source: Heckman et al. (2006).

F Estimates of the Technology of Skill Formation

Section F presents a summary of the specifications and estimates of the technology of capability formation. The main features of the empirical models of the technology of capability formation (fully structural or not) are summarized in Table F.1. We focus in particular on whether multiple skills are considered, on the generality of the functional form of the technology of skill formation and on whether capacities are anchored to an observable measure and therefore expressed in economically interpretable units. There we also compare the estimates of self- and cross-productivity effects and discuss whether the empirical findings support the evidence of increasing investments-skill complementarity over stages of development. The main findings are summarized in Section **??** of the paper.

Table F.1: Skill Production Functions

	Skill Output			Functional		Self Productivity		Cross Productivity		Increasing Investments / Skill
	Cognitive	Noncognitive	Health	Form	Anchoring	Cognitive	Noncognitive	Cognitive	Noncognitive	Complementarity over Time[g]
Todd and Wolpin (2003)	✓	X	X	Linear	X	✓ - N/A	X	X		U
Bernal and Keane (2010)	✓	X	X	Linear	X	✓ - N/A	X	X		U
Cunha and Heckman (2008)	✓	✓	X	Linear	✓[a]	0.977	0.884	0.003	0.028	U
Cunha et al. (2010)	✓	✓	X	CES	✓	0.487/0.902[b]	0.649/0.868[b]	0.000/0.008[b]	0.083/0.011[b]	✓
Todd and Wolpin (2007)	✓	X	X	Linear	X	0.21 - 0.34[c]	X	X		U
Cunha (2007)	✓	X	X	CES	✓	0.735/0.799 /0.872[d]	X	X		✓
Del Boca et al. (2014)	✓	X	X	Log-Linear	X	(0.14, 0.503)/(0.172, 0.922)[e]	X	X		N/A
Caucutt and Lochner (2012)	✓	X	X	CES	✓	✓ - N/A	X	X		N/A
Bernal (2008)	✓	X	X	Linear	X	✓ - N/A	N/A	X		U
Gayle et al. (2013)	X[f]	X[f]	X	N/S	X	N/S	N/S	X		N/S
Bernal and Keane (2011)	✓	X	X	Linear	X	✓ - N/A	X	X		U

[a] Anchoring addressed within the class of affine transformations.

[b] The authors consider two stages of skill formation so two estimates are reported.

[c] AFQT Scores on Math and Reading Tests. We report an interval because different estimates are produced for different ages of children.

[d] The author considers three stages of skill formation so three estimates are reported.

[e] The authors reports parameters for one child families (left interval) and two child families (right interval). We report intervals because the estimates change over time. The lower and the upper bounds are for the upper and lower bounds of the time in which parents invest (age 1 and 16 of the children).

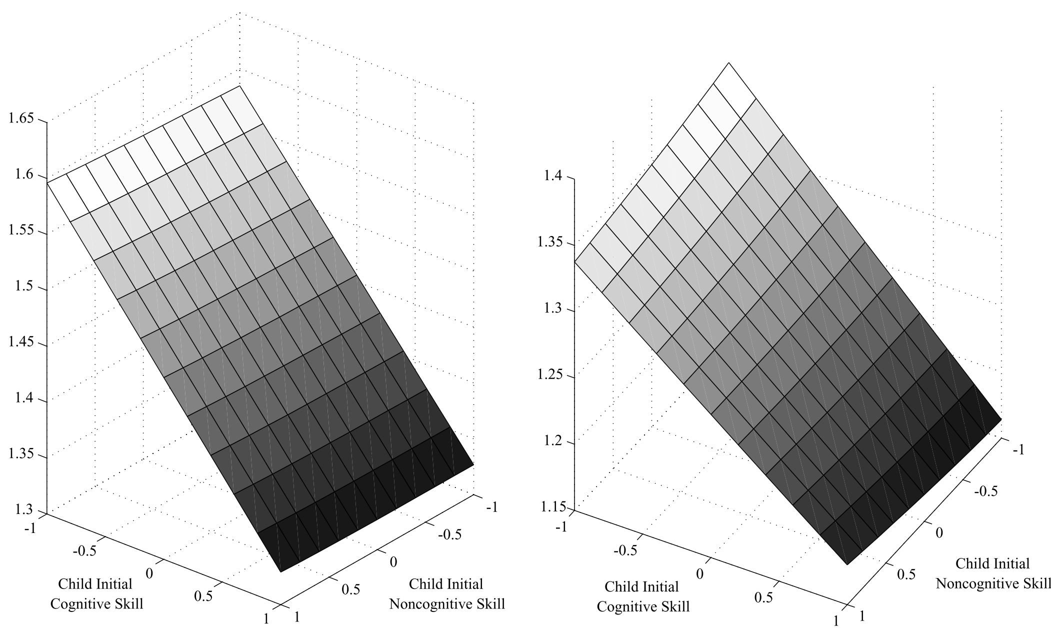
[f] The skill is specified as a general skills that encompasses both cognitive and non-cognitive abilities.

[g] In this column N/A means not available because the authors do not report estimates to address complementarity; U means the technology does not allow for complementarity

[h] N/A means an estimate is not available and N/S means not specified.

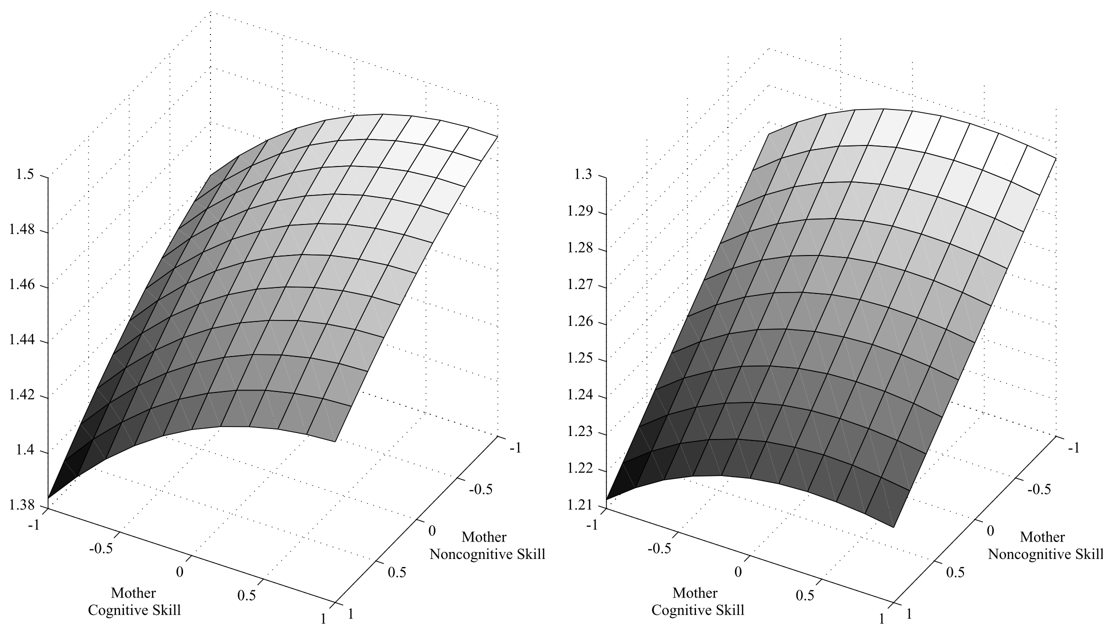
Cunha et al. (2010) simulate their model to examine optimal policies that are for investing in children by initial skill level. They assume that the social planner has full control over the investments made in the child (no parental feedback or response). Their simulations are consistent with the analysis of Sections D.7 and D.8: early investment in disadvantaged is economically productive.

Figure F.1: Ratio of early to late investments by child initial conditions of cognitive and noncognitive skills maximizing aggregate education (left) and minimizing aggregate crime (right) (other endowments held at mean levels). Lightly shaded portions correspond to higher values.



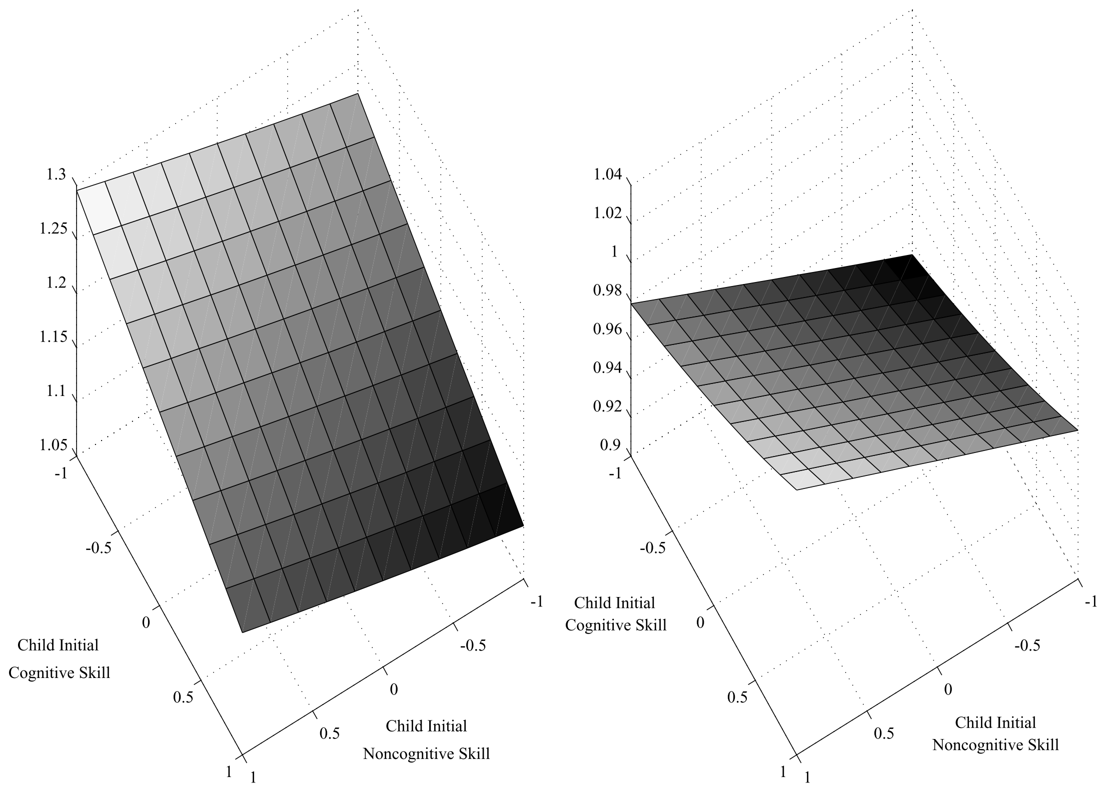
Source: Cunha et al. (2010).

Figure F.2: Densities of ratio of early to late investments maximizing aggregate education versus minimizing aggregate crime



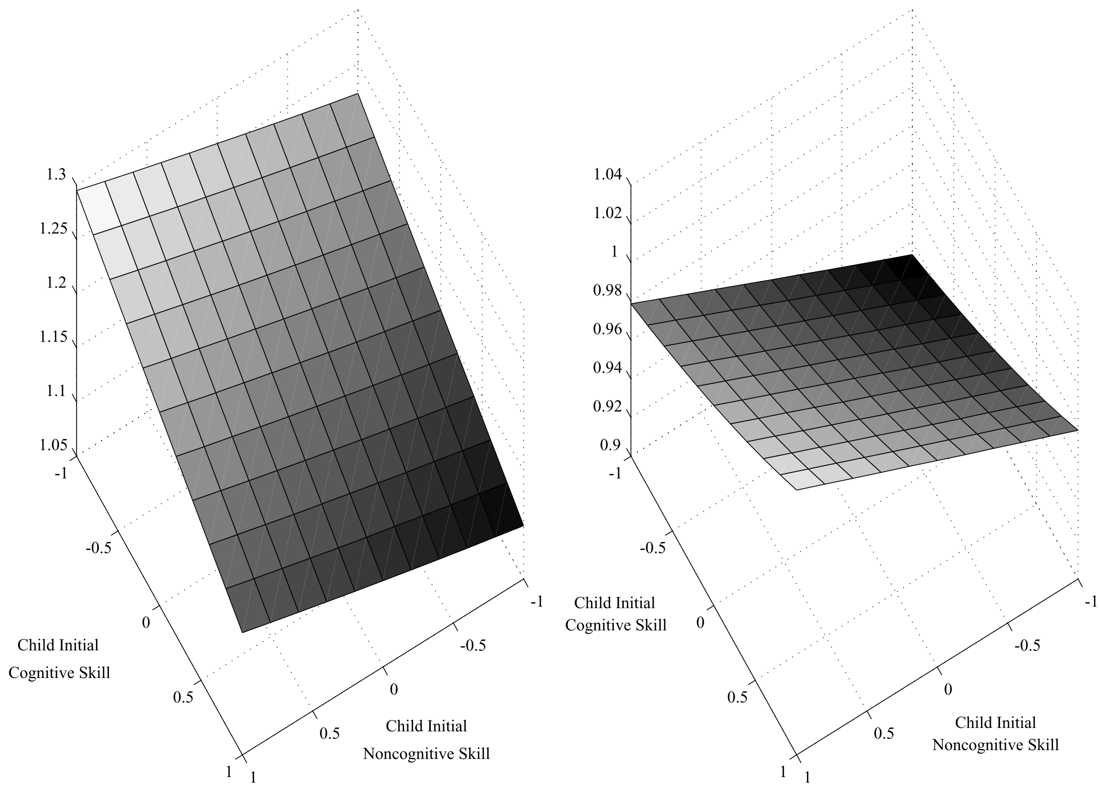
Source: Cunha et al. (2010).

Figure F.3: Optimal early (left) and late (right) investments by child initial conditions of cognitive and noncognitive skills maximizing aggregate education (other endowments held at mean levels)



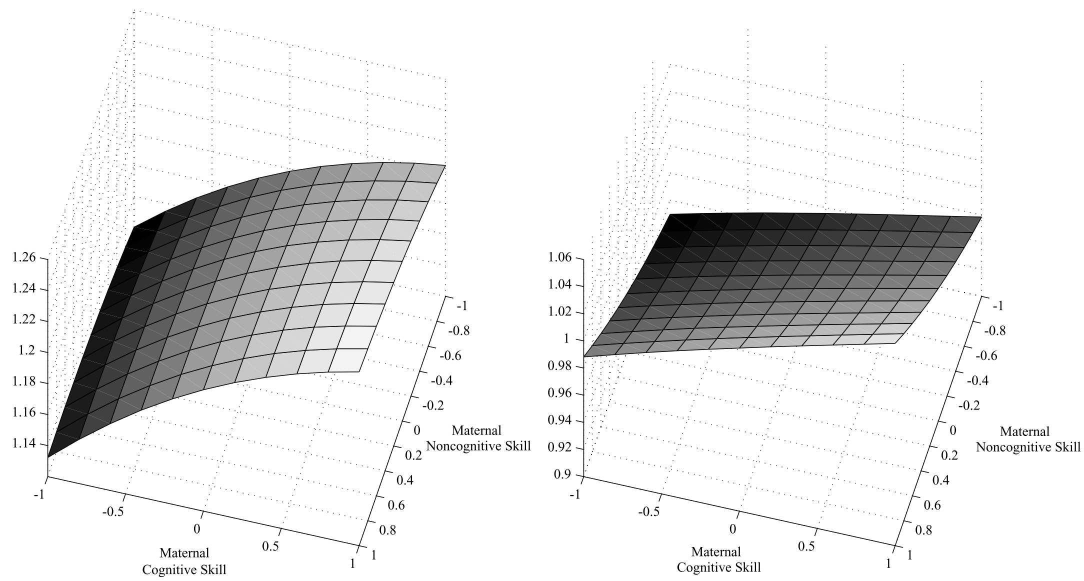
Source: Cunha et al. (2010).

Figure F.4: Optimal early (left) and late (right) investments by child initial conditions of cognitive and noncognitive skills maximizing aggregate education (other endowments held at mean levels)



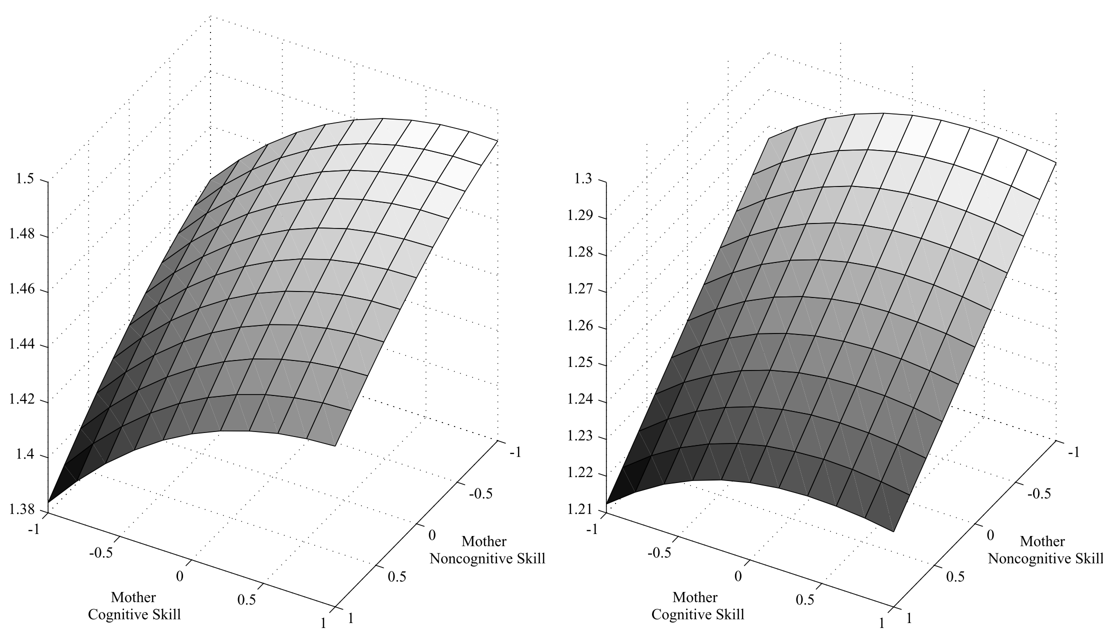
Source: Cunha et al. (2010).

Figure F.5: Optimal early (left) and late (right) investments by maternal cognitive and noncognitive skills maximizing aggregate education (other endowments held at mean levels)



Source: Cunha et al. (2010).

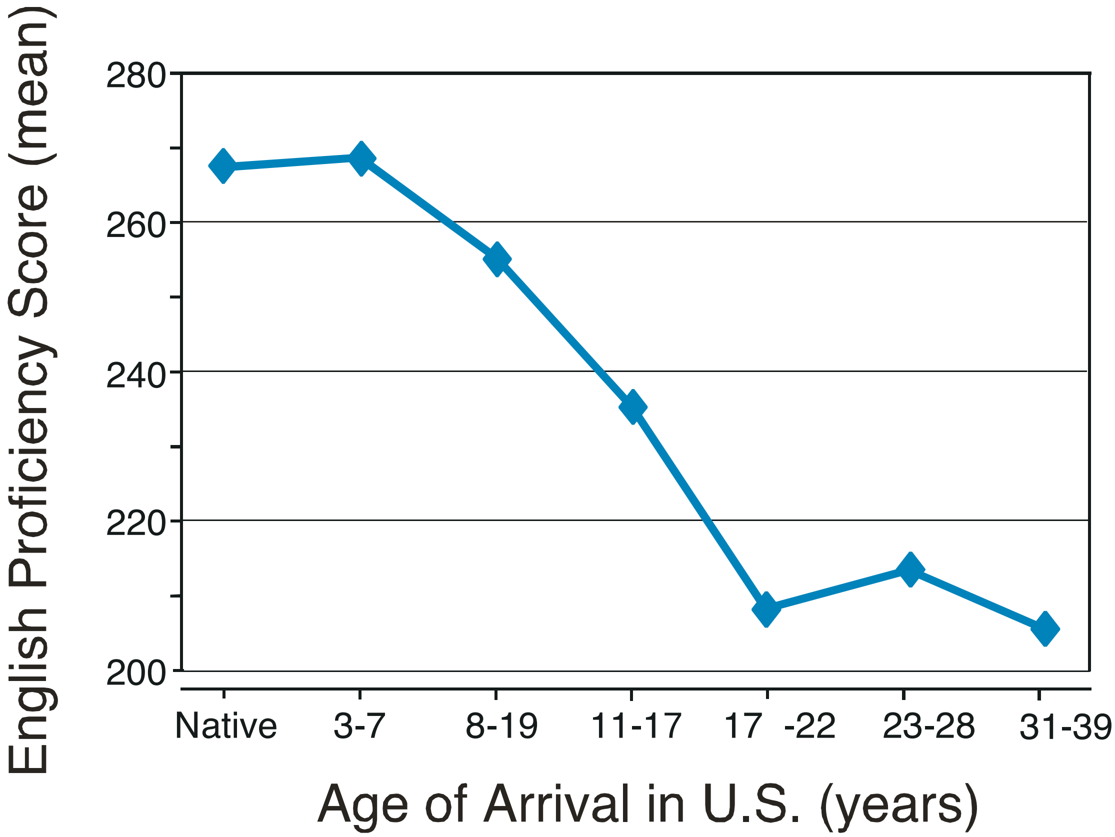
Figure F.6: Ratio of early to late investments by maternal cognitive and noncognitive skills maximizing aggregate education (left) and minimizing aggregate crime (right) (other endowments held at mean levels)



Source: Cunha et al. (2010).

G Evidence of Critical and Sensitive Periods and of Dynamic Complementarities

Figure G.1: Second language learning



Source: Johnson and Newport (1989).

Table G.1: Return to one year of college for individuals at different percentiles of the math test score distribution
White males from high school and beyond

	5%	25%	50%	75%	95%
Average return in the population	0.1121 (0.0400)	0.1374 (0.0328)	0.1606 (0.0357)	0.1831 (0.0458)	0.2101 (0.0622)
Return for those who attend college	0.1640 (0.0503)	0.1893 (0.0582)	0.2125 (0.0676)	0.2350 (0.0801)	0.2621 (0.0962)
Return for those who do not attend college	0.0702 (0.0536)	0.0954 (0.0385)	0.1187 (0.0298)	0.1411 (0.0305)	0.1682 (0.0425)
Return for those at the margin	0.1203 (0.0364)	0.1456 (0.0300)	0.1689 (0.0345)	0.1913 (0.0453)	0.2184 (0.0631)

Source: Carneiro and Heckman (2003).

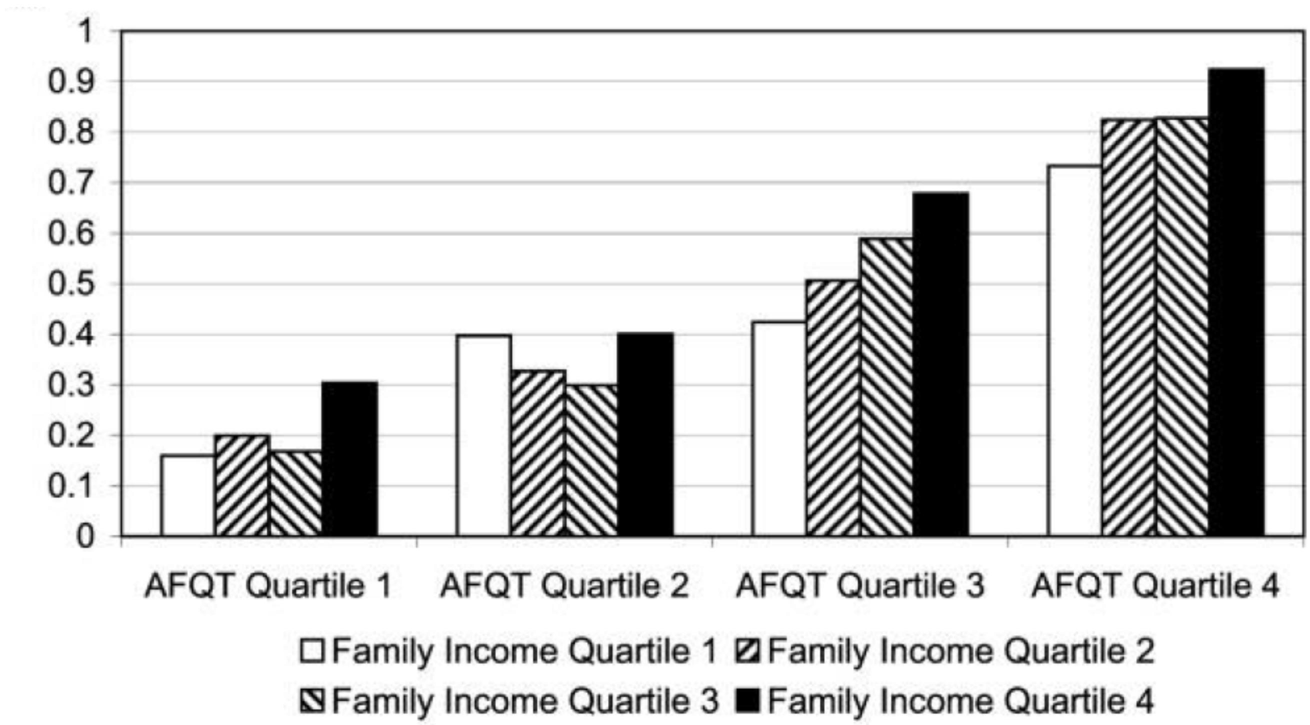
Notes: Wages are measured in 1991 by dividing annual earnings by hours worked per week multiplied by 52. The math test score is an average of two 10th grade math test scores. There are no dropouts in the sample and the schooling variable is binary (high schoolâĂŞcollege). The gross returns to college are divided by 3.5 (this is the average difference in years of schooling between high school graduates who go to college and high school graduates who do not in a sample of white males in the similar NLSY data). To construct the numbers in the table, we proceed in two steps. First we compute the marginal treatment effect using the method of local instrumental variables as in Carneiro, Heckman, and Vytlacil (2001). The parameters in the table are different weighted averages of the marginal treatment effect. Therefore, in the second step we compute the appropriate weight for each parameter and use it to construct a weighted average of the marginal treatment effect (see also Carneiro 2002). Individuals at the margin are indifferent between attending college or not. Standard errors are in parentheses.

For additional evidence see Knudsen et al. (2006) and Cunha et al. (2006).

H Some Recent Evidence on the Importance of Credit Constraints and Family Income

What is the effect of family income on college going? Belley and Lochner present some interesting updates of the study by Carneiro and Heckman (2002).

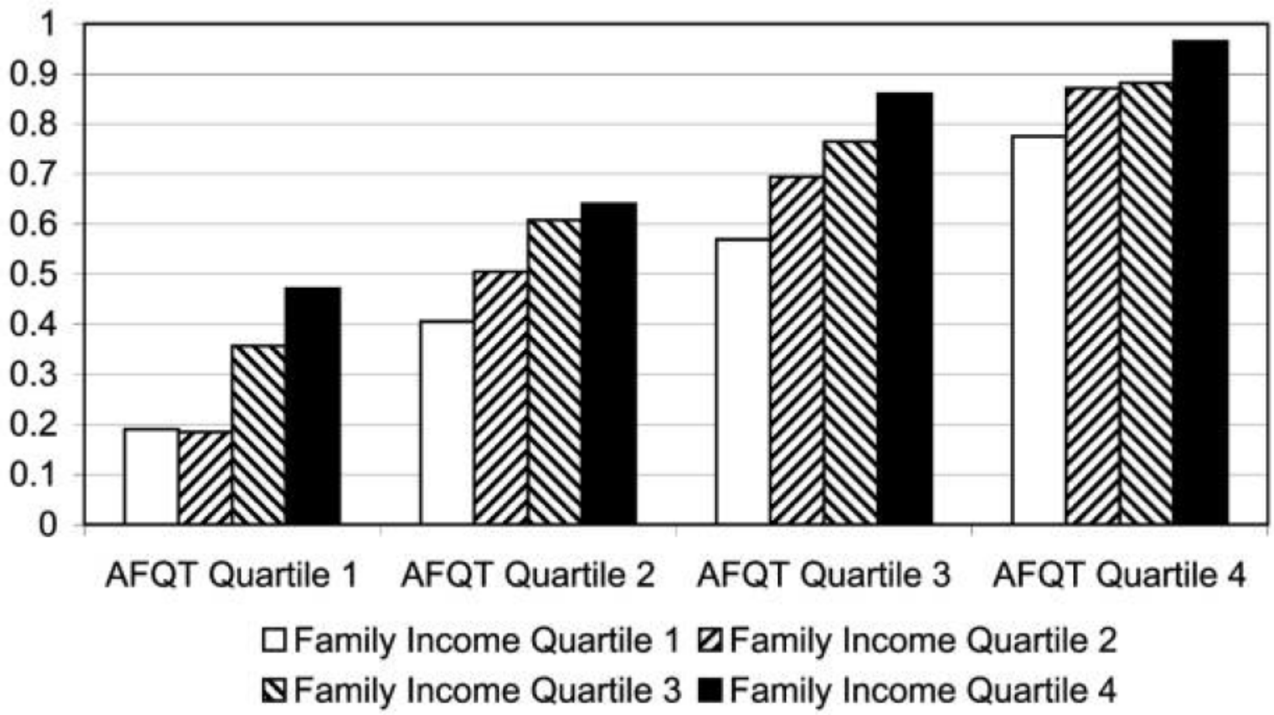
Figure H.1: College attendance by AFQT and Family Income Quartiles (1979)



Source: Belley and Lochner (2007).

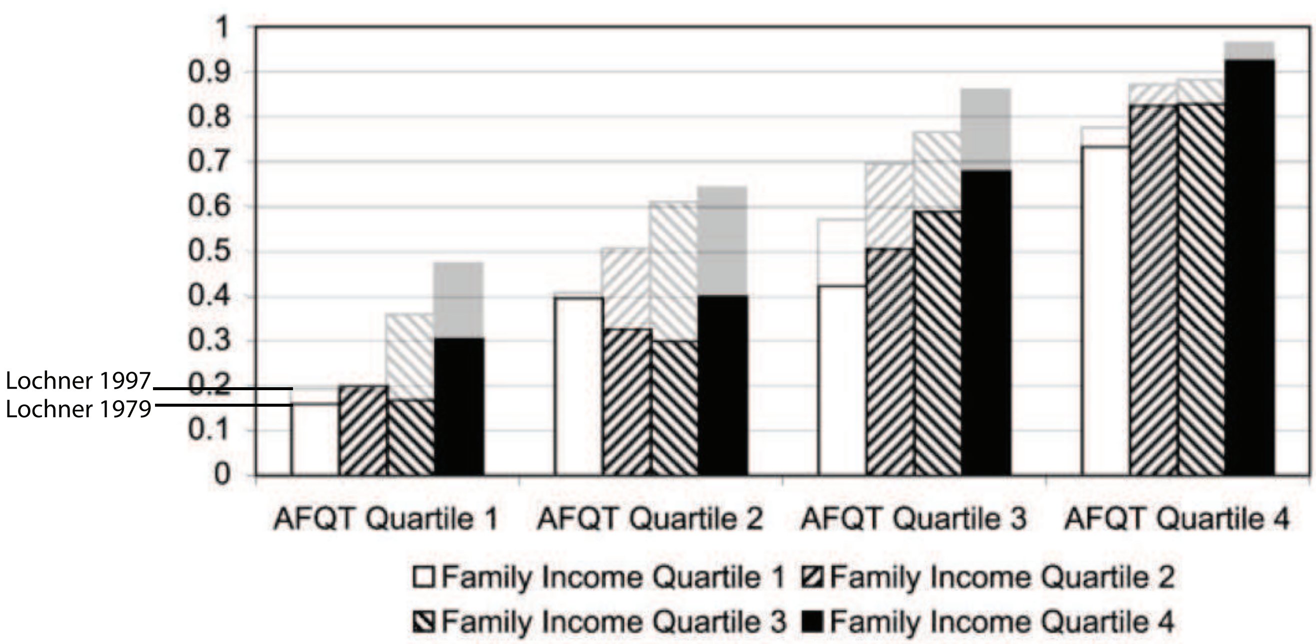
This reproduces the graph in Carneiro and Heckman (2002). Belley and Lochner update that paper using 1997 data (NLSY97).

Figure H.2: College attendance by AFQT and Family Income Quartiles (1997)



Source: Belley and Lochner (2007).

Figure H.3: College attendance by AFQT and Family Income Quartiles (1979 and 1997 on one graph)



Source: Belley and Lochner (2007).

I Summary of Empirical Evidence on the Efficacy of Interventions

This section summarizes the empirical evidence from a variety of interventions ranging from targeting prenatal infants to targeting young adults. In Heckman and Kautz (2014), we discuss these programs in great detail. They focus on programs that have been well studied, have long-term follow-ups, have been widely adopted, or offer unique insights.

For three reasons, evaluating and comparing the evidence from intervention programs is challenging. First, many interventions are only evaluated with short-term follow-ups, which could lead to upward-biased estimates of returns if the benefits eventually dissipate or to downward-biased estimates of the returns if the effects of the programs appear later in life. Second, not all studies measure the same outcomes. Ideally, all studies would report outcomes in terms of the rate of return of the program. Reported outcomes often differ across studies. Many studies only consider the effect of an intervention on a few outcomes. Without knowing the range of outcomes affected, it is difficult to calculate a rate of return. Third, many programs target specific demographic groups. Applying the findings from one group to another might be problematic if groups differentially benefit from programs.

Table I.1 (taken from Heckman and Kautz, 2014)) summarizes the effects of each intervention discussed in this section. The table displays information about the nature of the intervention, the quality of the evaluation, the effects on later life outcomes, and estimates of the rate of return and cost-benefit ratio when available. The squares in the "Components" columns indicate the extent to which the program and the evaluation of it have the features defined in the table. The dots in the "Effects on Outcomes" columns indicate the extent to which the program influenced skills and outcomes. (The notes at the bottom of the table define the symbols and abbreviations used.)

Three striking patterns emerge about the nature of the programs and the quality of the available evaluations of them. First, as a group, early childhood and elementary school programs have longer follow-ups. All of the early childhood or elementary school programs in Table I.1

have evaluations that follow participants for at least 10 years and many follow them more than 20 years, whereas only two evaluations of adolescent programs follow participants for at least 10 years (the longest is 12).

Second, early childhood programs tend to measure cognitive and character skills in addition to a variety of later-life outcomes, whereas many of the adolescent evaluations focus solely on labor market outcomes. Because of these features of data availability, we can better understand the sources of the effects on adult outcomes of early childhood programs by considering how these interventions produce skills. Due to the absence of measures of skills for many adolescent interventions, understanding these programs requires examining the curricula of the programs themselves, for example, whether the program seeks to foster cognitive or character skills.

Third, selection into programs differs by the age of intervention. In most early childhood evaluations, the programs first contact parents to participate and then parents opt into the program. In contrast, in most adolescent evaluations, participants themselves chose to enter the program.

Table I.1 also suggests certain features of effective programs. Only very early interventions (before age 3) improve IQ in a lasting way, consistent with the evidence that early childhood is a critical period for cognitive development (see Knudsen et al., 2006). The most successful interventions target preschoolers and primary school children. They improve later-life outcomes by developing character skills.

Programs that target adolescents have not been established to be as effective as programs that target earlier ages, in part because there have been fewer long-term evaluations of them. Several of the successful adolescent mentoring or residential programs improve labor market and social outcomes, but have relatively short follow-ups. The two programs with the longest follow-ups improve outcomes in the short run, but the benefits fade after a few years. These programs alter participants' environments and incentives during the intervention, which could influence their behavior in the short term without having a lasting effect.

The most promising adolescent programs integrate aspects of work into traditional education. Such programs break down the rigid separation between school and work that characterizes the American high school.

High schools create an adolescent society with values distinct from those of the larger society and removed from the workplace.[8] Even in affluent communities, the adolescent society has an anti-academic, anti-achievement bias. It was not until 1940 that more than half of each birth cohort graduated from high school.[9] In earlier times, adolescents took apprenticeships and jobs where they were supervised and mentored by adults. Mentoring involved teaching valuable character skills—showing up for work, cooperating with others, and persevering on tasks. These skills could be fostered in high schools, but with the relaxation of discipline in the schools, it is more difficult to do so.[10]

The apparent success of apprenticeship programs might arise in part from their cultivation of character skills. The attachment of a supervisor to an apprentice helps create character in a version of the attachment bond between parent and child.[11]

[8]See Coleman (1961).
[9]See Goldin and Katz (2008).
[10]See Arum (2005).
[11]See Bowlby (1951); Sroufe (1997); Sroufe et al. (2005).

Table I.1: Summary of Effects for Main Interventions

Program	Age	Duration	Target	Selection	Follow-Up	Sample	Home	Health	Parental	On Site	Group	IQ	School	Character	Education	Health	Crime	Earnings	Return	Benefit/Cost
	Participant/Evaluation Characteristics						Components					Effects on Outcomes							Return/Benefits	
Early																				
NFP	< 0	2Y	SES	Prgrm	19Y	640	☒	☒	☒	☐	☐	●	◐	●	○	◍	◐	.		2.9
ABC	0	5Y	SES	Refer	30Y	90	☐	☒	☒	☒	☒	●	●	◐	◐	◍	◐	◐		3.8
IHDP	0	3Y	Health	Prgrm	18Y	640	☒	☐	☒	☒	☒	◐	◐	◐	○	○	○	.		
FDRP	0	5Y	SES	Prgrm	15Y	110	☒	☒	☒	☒	☒	○	◐	◍	.	.	●	.		
PCDC	1	2Y	SES	Prgrm	15Y	170	☒	☐	☒	☒	☒	◍	◍	●	.	.	.	.		
JSS	1–2	2Y	Health	Prgrm	22Y	160	☒	☒	☒	☐	☐	●	●	●	◍	●	.	●		
Perry	3	2Y	SES, IQ	Prgrm	37Y	120	☒	☐	☒	☒	☒	◍	●	●	◐	○	●	◐	7–10	7.1–12.2
Head Start	3	2Y	SES	Prnt	23Y	4,170	☒	☒	☒	☒	☒	◍	◍	○	●	●	◐	●		
CPC	3–4	2Y	SES	Prnt	25Y	1,290	☐	☒	☒	☒	☒	.	●	●	●	●	●	●	18	10.8
TEEP	3,5	2Y	SES	Prgrm	22Y	260	☒	☒	☒	☒	☐	◍	●	●	●	.	.	.		
STAR	5–6	4Y	SES	Prgrm	22Y	11,000	☐	☐	☐	☒	☒	.	◍	●	●	.	.	●	6.2	
Elementary																				
LA's Best	5–6	6Y	SES	Schl	12Y	19,320	☐	☒	☒	☒	☒	.	◍	.	●	.	○	.		0.9
CSP	5–13	5Y	Behav	Refer	35Y	510	☒	☒	☒	☒	☒	.	.	○	.	⊗	○	.		
SSDP	6–7	6Y	Crime	Prgrm	21Y	610	☐	☒	☒	☒	☒	.	●	●	●	●	◍	○		3.1
Adolescence																				
BBBS	10-16	1Y	SES	Self	1Y	960	☐	☐	☐	☐	☐	.	◐	○	.	◐	○	.		1.0
IHAD	11–12	7Y	SES	Prgrm	8Y	180	☐	☒	☒	☒	☒	.	.	.	●	.	.	.		
EPIS	13–15	3Y	Schl	Schl	2Y	45,070	☐	☐	☐	☒	☒	.	●	.	.	.	.	.		0.9–3.0
xl club	14	2Y	Schl	Schl	2Y	261,420	☐	☐	☐	☒	☒	.	○	.	.	.	.	.		
SAS	14–15	5Y	Schl, SES	Schl	6Y	430	☐	☐	☐	☐	☐	.	◍	○	●	.	.	.		
STEP	14–15	2Y	Schl, SES	Self	4Y	4,800	☐	☒	☐	☒	☒	.	○	.	○	.	.	○		
QOP	14–15	5Y	Schl	Prgrm	10Y	1,070	☐	☒	☐	☒	☒	.	○	.	◍	○	⊗	○		0.42
Academies	13–16	4Y	Schl, SES	Self	12Y	1,460	☐	☐	☐	☒	☒	.	◍	○	○	○	◍	◐		
ChalleNGe	16–18	1Y	Dropout	Self	3Y	1,200	☐	☒	☐	☒	☒	.	.	◍	●	◐	◍	●	6.4	2.66
Job Corps	16–24	1Y	SES	Self	9Y	15,300	☐	☒	☐	☒	☒	.	.	.	○	◍	◍	◍		0.22
Year-Up	18–24	1Y	SES	Self	2Y	200	☐	☐	☐	☒	☒	.	.	.	.	.	.	●		

Notes: ☐ – Does not include intervention component. ☒ – Includes intervention component. ○ – No effects. ● – Positive effects. ◍ – Weakly positive effects. ◐ – Mixed effects (either different studies find different results or only particular sub-populations benefited). ⊗ – Negative effects.". " – Not measured. "Age" – The age at which participants entered the program. For programs that targeted grades, rather than ages, it was assumed that children entered kindergarten at ages 5–6 and the age range advanced one year for each subsequent grade. "Duration" – Length of the treatment. In cases where the treatment length varied for participants, the longest duration was presented. "Target" – Population that was targeted by the program. SES – socioeconomic status or disadvantage. Behav – Behavior. Schl – School Performance. Crime – local crime rates. IQ – low IQ. "Selection" – The party that acted first in joining the sample. Prgrm – Evaluation program contacted participants. Refer – Other party referred participants to program. Prnt – Parent applied to program. Self – Participant applied to program. Schl – School selected participants. "Follow-Up" – Duration of longest follow-up evaluation in years. "Sample" – Largest sample size from the studies examined (rounded to nearest 10). "Home" – Included home visits. "Health" – Included a nutritional component. "Parental" – Involved parents. "On Site" – Took place at an on site location. "Group" – Whether the intervention combined participants in groups. "IQ" – IQ score. "School" – school performance. "Character" – measured character skills. "Education" – educational attainment. "Health" – health (including drug use). "Crime" – crime. "Earnings" – earnings or related outcomes. "Return" – Annual rate of return. "Benefit/Cost" – Estimated benefits divided by costs.

Sources: **NFP** – Eckenrode et al. (2010); Kitzman et al. (2010); Olds (2006); Olds et al. (2004, 2010, 2007, 2004). **ABC** – Breitmayer and Ramey (1986); Heckman et al. (2014); Temple and Reynolds (2007). **IHDP** – McCormick et al. (2006). **FDRP** – Lally et al. (1988). **PCDC** – Besharov et al. (2011); Bridgeman et al. (1981); Johnson and Walker (1991). **JSS** – Gertler et al. (2013); Grantham-McGregor et al. (1991); Walker et al. (2005, 2007). **Perry** – Heckman et al. (2010a,b, 2013). **Head Start** – Carneiro and Ginja (2012); Currie and Thomas (1995); Deming (2009); Garces et al. (2002); Ludwig and Miller (2007); U.S. Department of Health and Human Services (2010). **CPC** – Niles et al. (2006); Reynolds (1994); Reynolds and Temple (1998); Reynolds et al. (2011, 2002, 2011). **TEEP** – Kagitcibasi et al. (2001, 2009). **STAR** – Chetty et al. (2011); Krueger (2003). **LA's BEST** – Goldschmidt et al. (2007); Huang et al. (2000, 2005). **CSP** – McCord (1978). **SSDP** – Hawkins et al. (1999, 2005, 2008). **BBBS** – Tierney et al. (1995). **IHAD** – Kahne and Bailey (1999). **EPIS** – Martins (2010). **XL Club** – Holmlund and Silva (2009). **SAS** – Johnson (1999). **STEP** – Walker and Vilella-Velez (1992). **QOP** – Rodríguez-Planas (2010, 2012). **Academies** – Kemple and Snipes (2000); Kemple and Willner (2008). **ChalleNGe** – Bloom et al. (2009); Millenky et al. (2010, 2011). **Job Corps** – Schochet et al. (2001, 2008). **Year Up** – Roder and Elliot (2011).

First note that more children are going to college at virtually all quartiles of ability and income. Increases in college going are strongest for the lowest ability group, especially less able children with richer parents. However, this provides no firm evidence for or against credit constraints. Also note that the absolute income gap is widening across income quartiles over time. The trend could simply be a consequence of wealth elasticity of child education by parents. Rich families can afford to spend their money on dumb kids' education. Education is an income elastic merit good. This is consistent with work on targeted family transfers Keane and Wolpin (2001), Johnson (2013). Targeted (tied) transfers promote college going and explain much of their estimated effect of parental income on college going. More educated parents have a greater marginal propensity to transfer income (in a tied fashion to children). We don't know (but would like to) how this marginal propensity is affected by information and parenting supplements. But drawing on Carneiro et al. (2011) there is no efficiency argument for investing in less able adolescents. Carneiro et al. (2011) show that the returns to college are negative for low ability students. Interpretations in this literature confuse its finding that income is "more relevant" today than in the past with the claim that it has somehow become dominant—which it has not. Recent "evidence" claiming to show that early life income matters more in fact shows what Carneiro and Heckman show.

I.1 Some Evidence on Early Life Interventions

We focus on the evidence regarding interventions which have a long-term follow-up, which have been extensively studied or widely adopted, or that offer unique insights.[12]

I.1.1 Nurse Family Partnership

The Nurse-Family Partnership (NFP) is a program targeted at low-income, unmarried, and/or adolescent mothers. It consists of nurse visits to young mothers from the first or second trimester of the mother's first pregnancy until the second birthday of her first child. The program en-

[12] We draw on the analysis of Heckman and Kautz (2014) where a more comprehensive discussion of each program is presented.

courages mothers to reduce smoking, teaches them how to take care of their children and helps them to pursue education and find jobs. Evaluated exploiting the random assignment, the program benefits children. The treated group exhibits persistent higher IQ scores through age 6 (Olds et al., 2007), lower rate of substance abuse and lower levels of internalizing behavior (e.g. anxiety, depression and, withdrawal) by age 12 (Kitzman et al., 2010) and lower propensity to engage in crime by age 19 (Eckenrode et al., 2010). The program also benefits mothers by reducing their dependence on welfare. The effects are at best weak on grades and achievement scores, suggesting that the program was most effective in promoting non-cognitive, character skills in the child and improving maternal income and employment prospects.

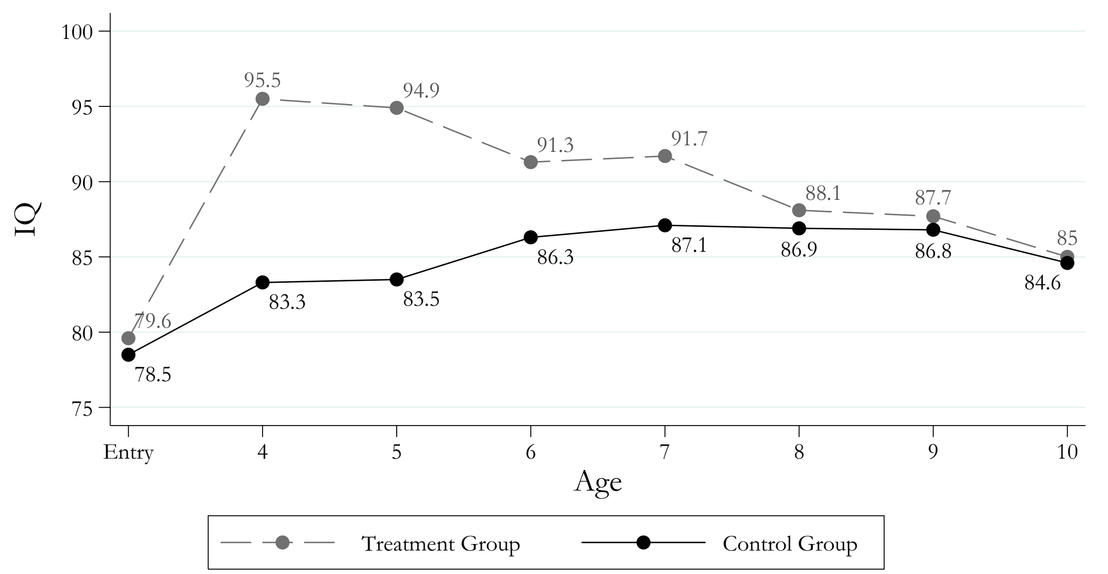
I.1.2 Perry Preschool Program

The Perry Preschool program targeted 3- and 4-years old low income black children with initial IQ below 85 at age 3. Selection into the program was based on random assignment. Children attended 2.5 hours of center-based preschool five days a week for two years. Teachers were also involved in home visits during which they interacted, played and talked with the child. The program focused on building organizational and social skills and was designed to cultivate independence and a sense of responsibility in the children (Schweinhart et al., 1993). The daily routing was understood as a key component of teaching children temporal relations (Weikart et al., 1971). Children where first planning an activity to execute and then would go to the art, large motor, doll or quiet center to complete their planned activity. The program ended after two years of enrollment and then children from both treatment and control group attended the same school.

While it appears that the program did not have a lasting effect on IQ scores (Figure I.1 and Figure I.2), it improved adult outcomes including academic achivement, employment, earnings, marriage, health and crime (Table H.9), resulting in a statistically significant rate of return of around 6-10% per annum (Campbell et al., 2013; Heckman et al., 2010a,b). These returns are above the post- World War II, pre-2008 meltdown, stock market returns to equity in U.S. labor market that are estimated to be 5.8% per annum.[13]

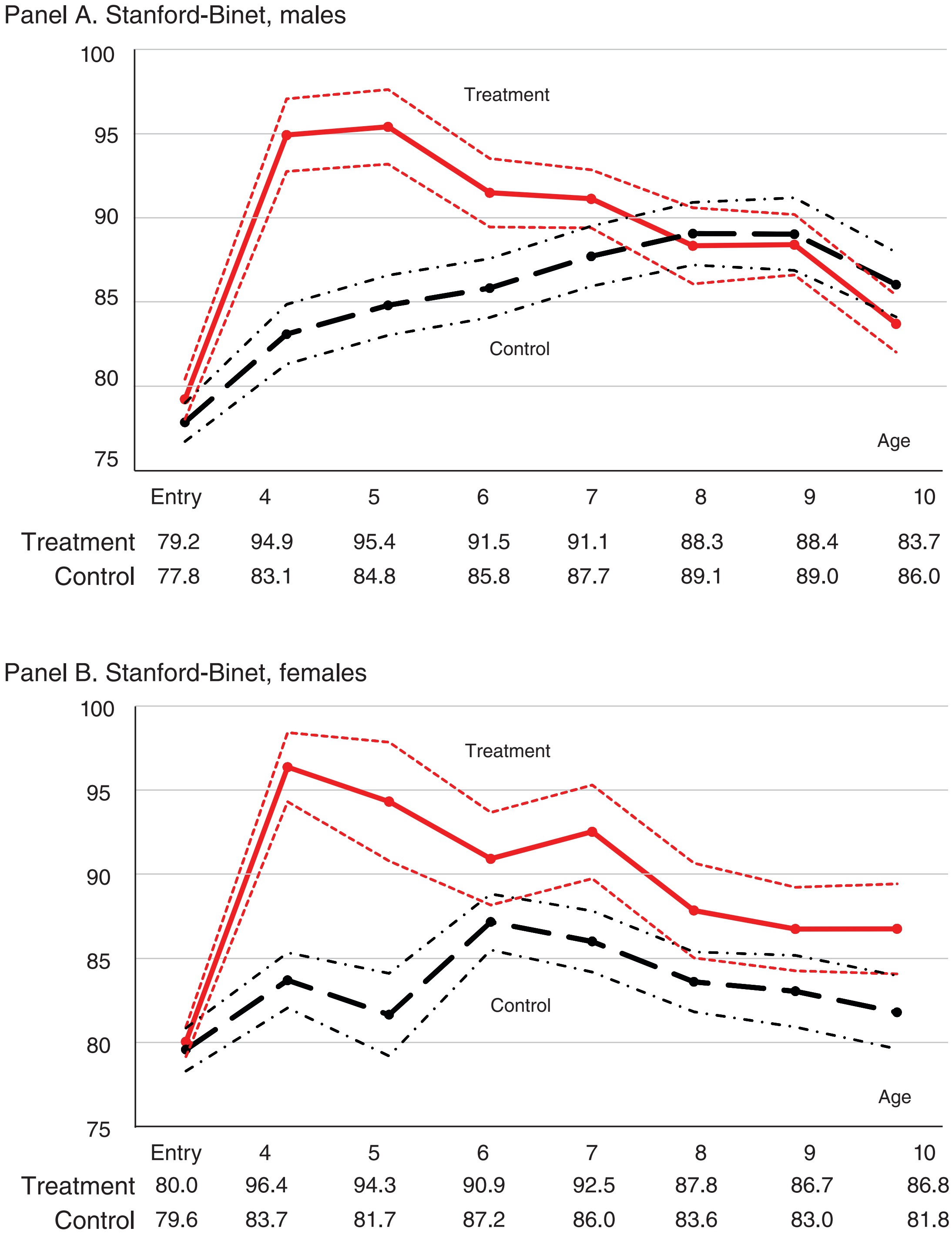
The Perry Preschool Program worked primarily through improving character traits which, in turn, improved labor market outcomes, health behavior and reduced crime. Figure I.3 shows that the treatment groups of both genders improved their teacher-reported externalizing behavior, a trait related to Agreeableness and Conscientiousness. For girls, the program improved Openness to Experience (proxied by academic motivation). Heckman et al. (2013) decompose the treatment effects on adult outcomes and shows that most of the Perry treatment effects arise from lasting changes in character traits not from changes in IQ. (Tables J.7 and J.8).

Figure I.1: Perry Preschool Program: IQ, by age and treatment group



[13]See DeLong and Magin (2009).

Figure I.2: Perry Preschool Program: Stanford-Binet IQ Test Scores by Gender and Treatment Status



Panel A. Stanford-Binet, males

	Entry	4	5	6	7	8	9	10
Treatment	79.2	94.9	95.4	91.5	91.1	88.3	88.4	83.7
Control	77.8	83.1	84.8	85.8	87.7	89.1	89.0	86.0

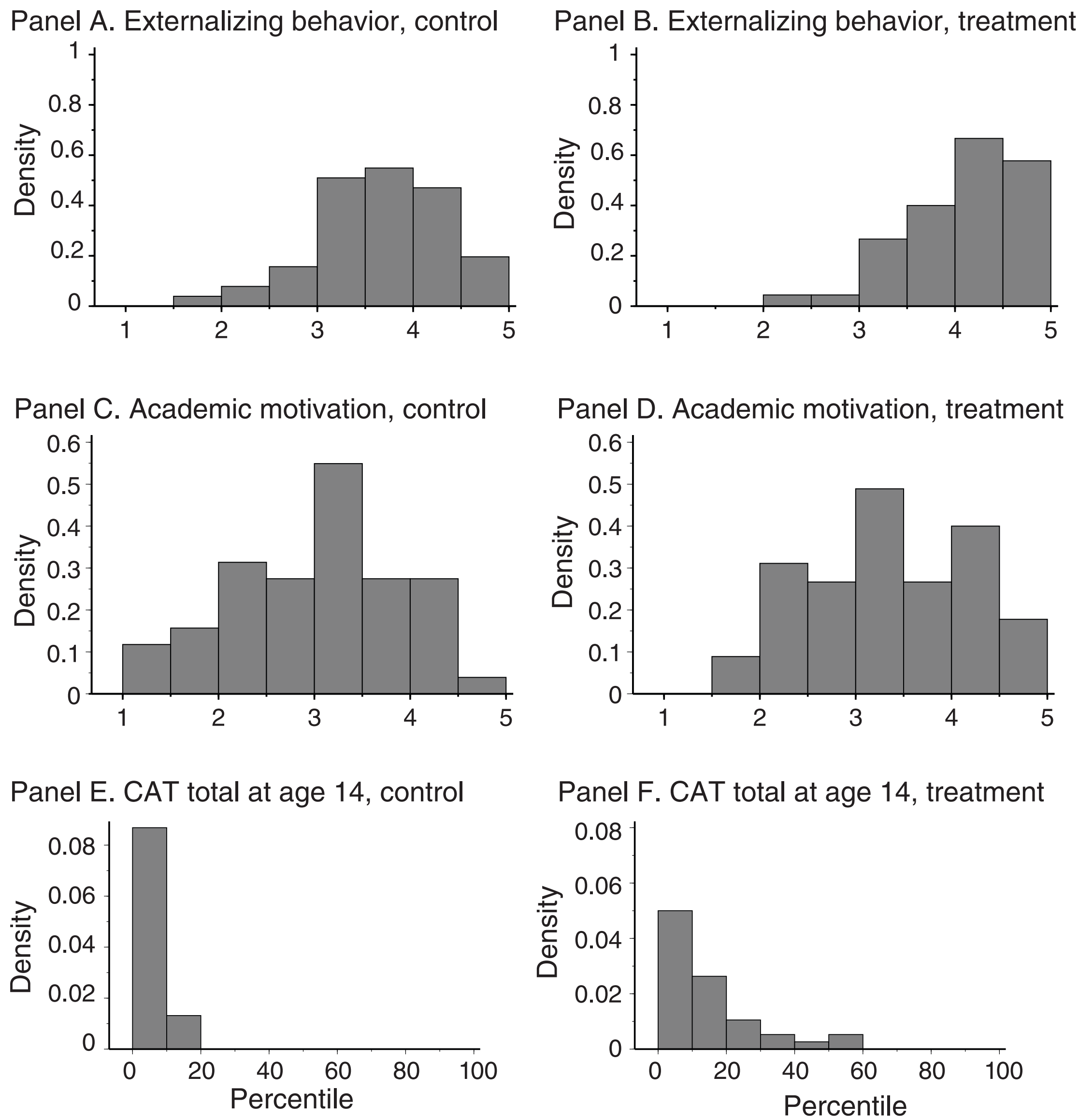
Panel B. Stanford-Binet, females

	Entry	4	5	6	7	8	9	10
Treatment	80.0	96.4	94.3	90.9	92.5	87.8	86.7	86.8
Control	79.6	83.7	81.7	87.2	86.0	83.6	83.0	81.8

FIGURE 1. STANFORD-BINET IQ TEST SCORES BY GENDER AND TREATMENT STATUS

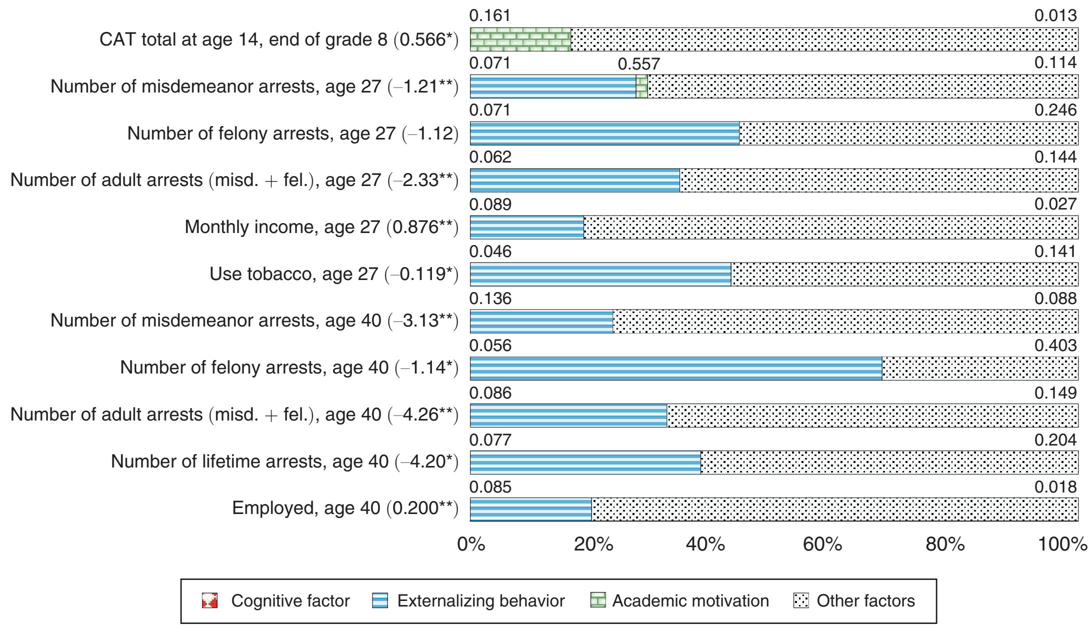
Source: Heckman et al. (2013).

Figure I.3: Perry Preschool Program: Histograms of Indices of Personality Skills and CAT Scores



Source: Heckman et al. (2013).

Figure I.4: Perry Preschool Program: Decompositions of Treatment Effects on Outcomes, Males



Source: Heckman et al. (2013).

Figure I.5: Perry Preschool Program: Decompositions of Treatment Effects on Outcomes, Females

Source: Heckman et al. (2013).

Table I.2: Perry Preschool Program: Program Treatment Effects

	Treatment effect			Control group		Treatment group	
Variable	Effect	Effect size	*p*-value	Mean	Standard error	Mean	Standard error
Panel A. Males							
CAT total at age 14, end of grade 8	0.566*	0.652	(0.060)	0.000	(0.164)	0.566	(0.204)
Number of misdemeanor arrests, age 27	−1.21**	−0.363	(0.036)	3.03	(0.533)	1.82	(0.445)
Number of felony arrests, age 27	−1.12	−0.324	(0.101)	2.33	(0.554)	1.21	(0.342)
Number of adult arrests (misd.+fel.), age 27	−2.33**	−0.402	(0.024)	5.36	(0.927)	3.03	(0.734)
Monthly income, age 27	0.876**	0.607	(0.018)	1.43	(0.231)	2.31	(0.352)
Use tobacco, age 27	−0.119*	−0.236	(0.093)	0.538	(0.081)	0.419	(0.090)
Number of misdemeanor arrests, age 40	−3.13**	−0.372	(0.039)	8.46	(1.348)	5.33	(1.042)
Number of felony arrests, age 40	−1.14*	−0.266	(0.092)	3.26	(0.684)	2.12	(0.598)
Number of adult arrests (misd.+fel.), age 40	−4.26**	−0.373	(0.041)	11.7	(1.831)	7.46	(1.515)
Number of lifetime arrests, age 40	−4.20*	−0.346	(0.053)	12.4	(1.945)	8.21	(1.778)
Employed, age 40	0.200**	0.394	(0.024)	0.500	(0.085)	0.700	(0.085)
Sample size	72			39		33	
Panel B. Females							
CAT total, age 8	0.565*	0.614	(0.062)	0.000	(0.196)	0.565	(0.223)
CAT total, age 14	0.806**	0.909	(0.014)	0.000	(0.209)	0.806	(0.204)
Any special education, age 14	−0.262**	−0.514	(0.025)	0.462	(0.100)	0.200	(0.082)
Mentally impaired at least once, age 19	−0.280**	−0.569	(0.017)	0.364	(0.105)	0.083	(0.058)
Number of misdemeanor violent crimes, age 27	−0.423**	−0.292	(0.032)	0.423	(0.284)	0.000	(0.000)
Number of felony arrests, age 27	−0.269**	−0.325	(0.021)	0.269	(0.162)	0.000	(0.000)
Jobless for more than 1 year, age 27	−0.292*	−0.573	(0.071)	0.542	(0.104)	0.250	(0.090)
Ever tried drugs other than alcohol or weed, age 27	−0.227**	−0.530	(0.045)	0.227	(0.091)	0.000	(0.000)
Number of misdemeanor violent crimes, age 40	−0.537**	−0.364	(0.016)	0.577	(0.289)	0.040	(0.040)
Number of felony arrests, age 40	−0.383**	−0.425	(0.028)	0.423	(0.177)	0.040	(0.040)
Number of lifetime violent crimes, age 40	−0.574**	−0.384	(0.019)	0.654	(0.293)	0.080	(0.055)
Months in all marriages, age 40	39.6*	0.539	(0.076)	47.8	(15.015)	87.5	(18.853)
Sample size	51			26		25	

Source: Heckman et al. (2013).

Notes: Statistics are shown for the outcomes analyzed in this paper. There are differences in treatment effects by gender although strong effects are found for both. "CAT total" denotes the California Achievement Test total score normalized to control mean zero and variance of one. Test statistics are corrected for the effect of multiple hypothesis testing and threats to validity (see Heckman et al., 2010b, Campbell et al., 2013). The reported effect is the difference in means between treatment and control groups. The effect size is the ratio of the effect to the standard deviation of the control group. Stars denote statistical significance: *** - 1 percent level, ** - 5 percent level, * - 10 percent level. Monthly income is adjusted to thousands of year-2006 dollars using annual national CPI.

*** Significant at the 1 percent level.

** Significant at the 5 percent level.

* Significant at the 10 percent level.

I.1.3 Abecedarian Program

Similarly to Perry, the Abecedarian program was also designed to promote self-reinforcement among the children and reduce dependence on adult feedback (Ramey et al., 1982). It was more intense than Perry combining a preschool component starting as early as at 6 weeks old and a school-age treatment through grade three. The curriculum focused on "educational games" to build cognitive abilities (language, math, reading, writing), behavioral skills (attending behavior, task orientation, listening, task completion), and creativity and motor skills (through action songs, rhymes, story telling, fingerplays). It also had a medical and nutritional component. The program produced lasting improvements in IQ (mostly for girls) because the interventions started very early in life (Campbell et al., 2001). Evidence suggests that IQ is more malleable in the very early childhood (National Research Council and Institute of Medicine, 2000). Girls also showed a greater educational attainment, reduced participation in crime, decrease in substance abuse, and improved internalizing and externalizing behavior. Boys showed better health conditions and improvements in non-cognitive skills (Campbell et al., 2013).

I.1.4 Jamaican Study

The Jamaican Supplementation study is an example of a childhood program offered in a less developed country with a long-term follow-up. It consists of two years of nutritional supplementation (milk formula) or stimulation (encouraged the mother to play with children in an effective manner) or both. The stimulation intervention appeared more effective. Both interventions stimulated short-term cognitive development, but only stimulation improved cognitive and character skills (in particular internalizing behavior) in the long run. Stimulation also improved earnings and educational attainment (Gertler et al., 2013; Grantham-McGregor et al., 1991).

I.2 Large Scale Programs

The success of early interventions such as Perry and Abecedarian incentivized policymakers to propose similar programs on larger scale. Head Start is one of them with children eligible for enrollment from age three to five. It combines center-based preschool interventions with medical services and parental assistance. The program largely vary by site making an overall evaluation difficult (Deming, 2009). The empirical evidence on its effectiveness is mixed. IQ and achievement test scores are improved only in the short run, but some studies find that educational attainment are improved and criminality is reduced in particular for blacks (Deming, 2009; Garces et al., 2002). These effects are likely underestimated as many members of the control group joined the program in a different site than where they originally applied or enrolled in other more intensive early childhood programs. The Chicago Child-Parent Center program is targeted at 3- and 4- years old disadvantaged children. It offers half- or full-day of preschool intervention, but parents are encouraged to be involved, visit the center, receive advice on good parenting behavior and are assisted in pursuing further education and seeking jobs. The program appears to have improved education, criminal behavior, reduced substance abuse and also increased annual earnings at age 28 (Niles et al., 2006; Reynolds, 1995; Reynolds et al., 2011, 2001).

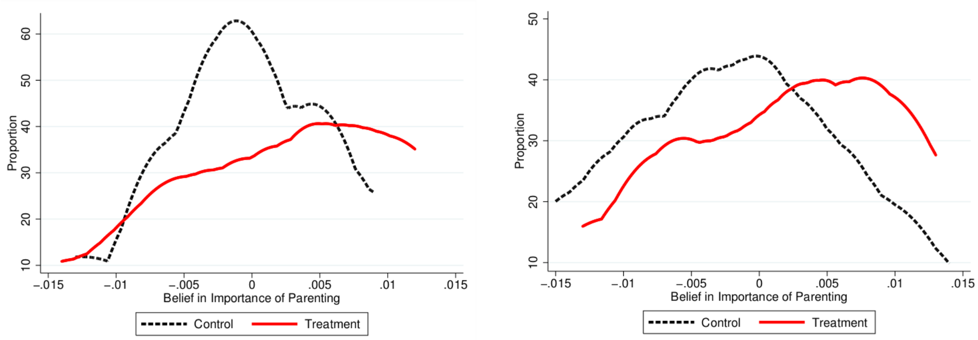
I.3 Interventions in Kindergarten and Elementary School

Many programs have been proposed to promote moral an character education in school. The subject, however, raises controversies as scholars disagree about the origins of character and morality (see Lapsley and Yeager, 2012). The Seattle Social Development Project focused on classroom management, interactive teaching, and cooperative learning and aimed at fostering the attachment between children and their parents and teachers. It does not have strong effects when evaluated in terms of achievement tests, but it appears successful when life outcomes such as earnings, participation in crime or health status are considered (Hawkins et al., 1999, 2005, 2008). The Cambridge-Somerville Program, targeted at five to thirteen years old boys with behavioral problems, is an example of an harmful program as the treated group performed worse than the control in terms of drinking habits, health and participation in crime. A possible explanation can be attributed to the creation of dependence on outside assistance (McCord, 1978). In project Star children and teachers were randomly assigned to kindergarten classrooms of different class sizes. The effects on achievement scores fade over time, but children placed in better classes shown better fourth- and eight-grade behavior according to teacher based ratings and higher earnings in early adulthood (Chetty et al., 2011). This evidence shows, as in the case of the Perry program, the importance of long-term follow-ups to properly assess the outcomes of an early intervention.

J Parental Responses to Intervention Programs

This appendix presents evidence on parental responses from the NFP, Perry preschool and ABC intervention programs surveyed in Appendix I.1. The NFP program provided home visits to first time teenage mothers, advising them on proper nutrition and care of young children, including the importance of cognitive stimulation. The Perry program had home visits on average once a week. The ABC program did not have home visits, but interacted with parents at the ABC center. The evidence generally supports positive (complementary) responses of parents to interventions.

Figure J.1: Parental Response to Perry Preschool Program After 1-year experience of treatment



(a) Girls **(b)** Boys

Notes:
(a) Parental response is measured by a factor score obtained from 10 items of Parental Attitude Research Instrument (PARI) administered at child's age 4 or 5 after 1-year experience of Perry Preschool intervention. (b) The factor model was estimated by a maximum likelihood factor estimation with categorical indicators. A higher value indicates that a mother has a stronger belief in importance of warm parenting. (c) 10 items used in this estimation are a mother's 4-point scale response to the following questions : "One of the worst things about taking care of a home is a woman feels that she can't get out"; "Children would be happier and better behaved if parents would show an interest in their affairs"; "A mother should do her best to avoid any disappointment for her child"; "Mothers very often feel that they can't stand their children a moment longer"; "Having to be with the children all the time gives a woman the feeling that her wings have been clipped"; "Parents must earn the respect of their children by the way they act." "Parents who are interested in hearing about their children's parties, dates, and fun help them grow up right"; "A childs' ideas should be seriously considered in making family decisions"; "Parents should know better than to allow their children to be exposed to difficult situations"; and "When a child is in trouble, he ought to know he won't be punished for talking about it with his parents."
Source: Moon (2013)

Table J.1: NFP Memphis, Parental Responses (Females)

Outcome	**Age (years)**	Sample Size # C	# T	Conditional Effect Size	Asymptotic *p*-values	Permutation Single *p*-val	Freedman-Lane Stepdown
Home Observation Measurement of the Environment (HOME)	1	220	104	0.354	**0.003**	**0.004**	**0.007**
Non-Abusive Parenting Attitudes (Bavolek)	1	227	105	0.288	**0.012**	**0.005**	**0.005**
Home Observation Measurement of the Environment (HOME)	2	222	101	0.301	**0.010**	**0.003**	**0.006**
Non-Abusive Parenting Attitudes (Bavolek)	2	222	102	0.370	**0.003**	**0.006**	**0.006**

Source: Moon (2013)

Table J.2: NFP Memphis, Parental Responses (Males)

Outcome	**Age (years)**	Sample Size # C	# T	Conditional Effect Size	Asymptotic *p*-values	Permutation Single *p*-val	Freedman-Lane Stepdown
Home Observation Measurement of the Environment (HOME)	1	221	95	0.208	**0.051**	**0.041**	**0.041**
Non-Abusive Parenting Attitudes (Bavolek)	1	225	100	0.273	**0.015**	**0.003**	**0.006**
Home Observation Measurement of the Environment (HOME)	2	224	98	0.169	**0.092**	**0.075**	**0.075**
Non-Abusive Parenting Attitudes (Bavolek)	2	228	99	0.316	**0.006**	**0.003**	**0.006**

Source: Moon (2013)

Table J.3: Abecedarian Intervention, Attachment (Videotapes)

Variable	Age (In Months)	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Gen. Diff.
		Males				*Females*				
Mutual reading	6	35.322	30.678	**0.066**	**0.017**	30.079	34.281	**0.002**	**0.005**	0.917
Mutual reading, 20m	20	50.327	44.157	**0.024**	**0.033**	20.089	34.663	**0.019**	**0.092**	0.754
Mutual reading	36	37.762	148.430	**0.003**	**0.000**	46.308	20.484	0.141	0.497	**0.010**
Mutual reading	60	97.200	55.300	**0.070**	**0.002**	44.174	-3.947	0.602	**0.014**	0.219
Mutual playing with toy	6	382.409	2.022	0.353	0.889	308.236	134.748	**0.040**	0.134	**0.055**
Mutual playing with toy	20	397.764	-51.479	0.894	0.872	302.274	178.659	**0.008**	**0.003**	**0.001**
Mutual playing with toy	36	381.429	112.456	**0.063**	**0.019**	297.808	188.192	**0.014**	**0.002**	0.471
Mutual playing with toy	60	618.350	-79.619	0.598	0.119	341.957	212.589	**0.030**	**0.000**	**0.014**
Child plays alone	6	-411.678	-19.906	0.511	0.871	-565.800	132.776	0.140	**0.092**	**0.056**
Child plays alone	20	-595.291	-45.509	0.868	0.910	-723.348	149.177	**0.044**	**0.017**	**0.006**
Child plays alone	36	-815.286	115.978	**0.068**	**0.014**	-899.962	204.837	**0.007**	**0.001**	0.401
Child plays alone	60	-552.350	-94.150	0.615	0.185	-853.130	216.721	**0.029**	**0.000**	**0.011**

Notes:

(a) Ctr. Mean denotes mean value for control group

(b) Diff. Means denotes the difference in the mean values between treatment and control groups

(c) Blk. *p*-value denotes the block *p*-value for the the male block

(d) IPW P. Co. Co. denotes the inverse probability weighting correlation coefficient

(e) Gen. Diff. denotes the *p*-value for the mean values of the two genders being equal

Source: Moon (2013)

Table J.4: Abecedarian Intervention, Parental Investment (HOME)

Variable	Age (In Months)	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Gen. Diff.
		Males				*Females*				
Maternal warmth	6	7.043	-0.599	0.805	0.957	6.700	0.420	**0.070**	**0.044**	**0.068**
Maternal warmth	18	7.619	0.122	0.209	**0.058**	6.714	1.112	**0.040**	**0.001**	**0.091**
Maternal warmth	30	7.286	-0.206	0.635	**0.001**	7.111	0.472	**0.057**	**0.006**	0.309
Organization of environment	6	4.652	0.422	**0.076**	**0.001**	4.633	0.007	0.641	0.439	0.145
Organization of environment	18	5.238	0.021	0.361	**0.069**	4.964	0.340	0.311	**0.017**	0.394
Organization of environment	30	5.238	0.070	0.676	**0.088**	5.148	0.102	0.286	**0.065**	0.934
Avoidance of restrict./punish.	42	5.619	-0.219	0.708	0.553	5.808	0.109	0.185	**0.045**	0.575
Avoidance of restrict./punish.	54	5.571	0.081	0.241	**0.045**	5.917	0.447	**0.044**	**0.000**	0.400
Stimulation of mature behavior	42	8.286	0.114	0.654	0.333	8.385	0.574	0.660	0.227	0.548
Stimulation of mature behavior	54	8.857	0.882	**0.051**	**0.001**	9.000	1.000	**0.045**	**0.000**	0.885

Notes:

(a) Ctr. Mean denotes mean value for control group

(b) Diff. Means denotes the difference in the mean values between treatment and control groups

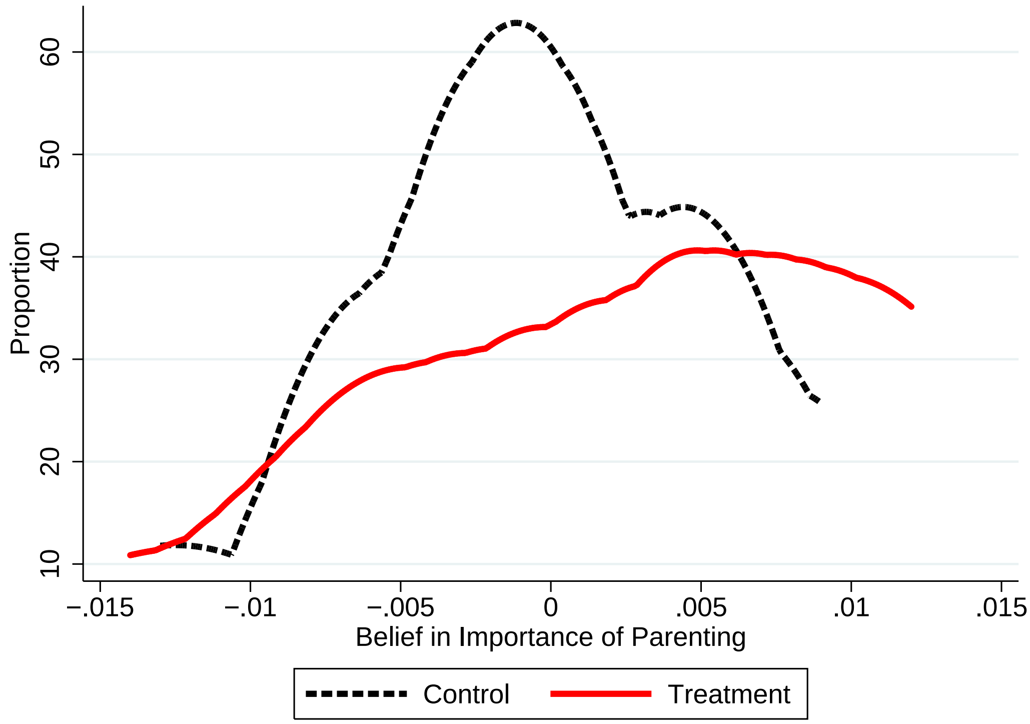
(c) Blk. *p*-value denotes the block *p*-value for the the male block

(d) IPW P. Co. Co. denotes the inverse probability weighting correlation coefficient

(e) Gen. Diff. denotes the *p*-value for the mean values of the two genders being equal

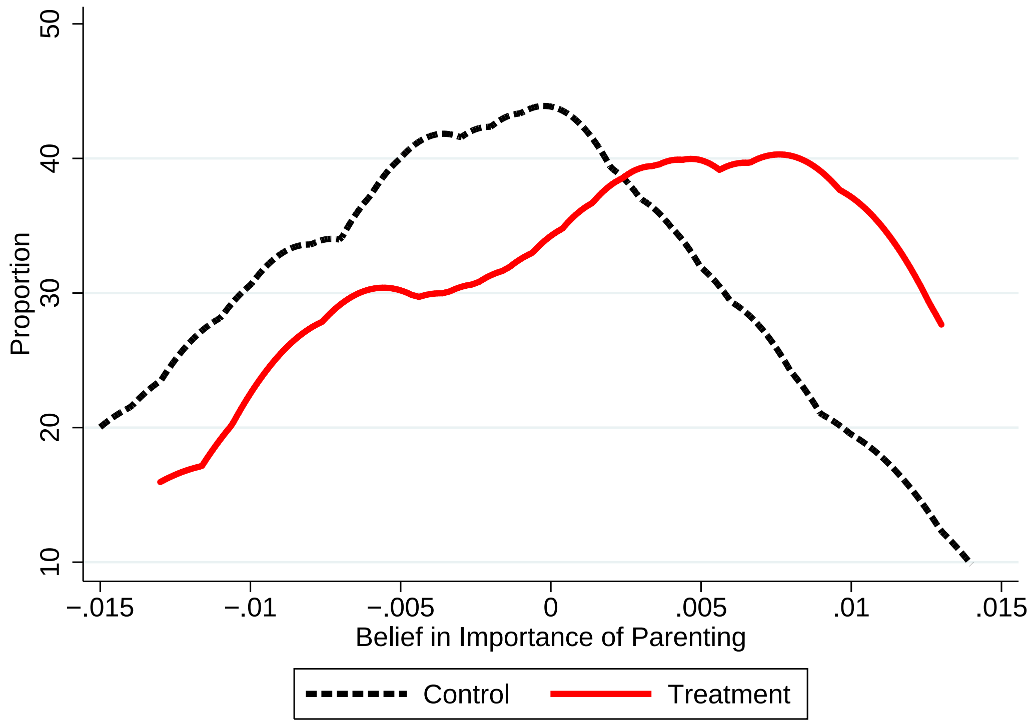
Source: Moon (2013)

Figure J.2: Parental response to Perry Preschool Program after 1 year experience of treatment: Girls



Source: Moon (2013)

Figure J.3: Parental response to Perry Preschool Program after 1 year experience of treatment: Boys



Source: Moon (2013)

Table J.5: NFP Memphis, Parental Responses (Females)

Outcome	**Age (years)**	Sample Size # C	# T	Conditional Effect Size	Asymptotic *p*-values	Permutation Single *p*-val	Freedman-Lane Stepdown
Home Observation Measurement of the Environment (HOME)	1	220	104	0.354	**0.003**	**0.004**	**0.007**
Non-Abusive Parenting Attitudes (Bavolek)	1	227	105	0.288	**0.012**	**0.005**	**0.005**
Home Observation Measurement of the Environment (HOME)	2	222	101	0.301	**0.010**	**0.003**	**0.006**
Non-Abusive Parenting Attitudes (Bavolek)	2	222	102	0.370	**0.003**	**0.006**	**0.006**

Source: Moon (2014).

Table J.6: NFP Memphis, Parental Responses (Males)

Outcome	**Age (years)**	Sample Size # C	# T	Conditional Effect Size	Asymptotic *p*-values	Permutation Single *p*-val	Freedman-Lane Stepdown
Home Observation Measurement of the Environment (HOME)	1	221	95	0.208	**0.051**	**0.041**	**0.041**
Non-Abusive Parenting Attitudes (Bavolek)	1	225	100	0.273	**0.015**	**0.003**	**0.006**
Home Observation Measurement of the Environment (HOME)	2	224	98	0.169	**0.092**	**0.075**	**0.075**
Non-Abusive Parenting Attitudes (Bavolek)	2	228	99	0.316	**0.006**	**0.003**	**0.006**

Source: Moon (2014).

Table J.7: Abecedarian Intervention, Attachment (Videotapes)

Variable	Age (In Months)	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Gen. Diff.
		Males				*Females*				
Mutual reading	6	35.322	30.678	**0.066**	**0.017**	30.079	34.281	**0.002**	**0.005**	0.917
Mutual reading, 20m	20	50.327	44.157	**0.024**	**0.033**	20.089	34.663	**0.019**	**0.092**	0.754
Mutual reading	36	37.762	148.430	**0.003**	**0.000**	46.308	20.484	0.141	0.497	**0.010**
Mutual reading	60	97.200	55.300	**0.070**	**0.002**	44.174	-3.947	0.602	**0.014**	0.219
Mutual playing with toy	6	382.409	2.022	0.353	0.889	308.236	134.748	**0.040**	0.134	**0.055**
Mutual playing with toy	20	397.764	-51.479	0.894	0.872	302.274	178.659	**0.008**	**0.003**	**0.001**
Mutual playing with toy	36	381.429	112.456	**0.063**	**0.019**	297.808	188.192	**0.014**	**0.002**	0.471
Mutual playing with toy	60	618.350	-79.619	0.598	0.119	341.957	212.589	**0.030**	**0.000**	**0.014**
Child plays alone	6	-411.678	-19.906	0.511	0.871	-565.800	132.776	0.140	**0.092**	**0.056**
Child plays alone	20	-595.291	-45.509	0.868	0.910	-723.348	149.177	**0.044**	**0.017**	**0.006**
Child plays alone	36	-815.286	115.978	**0.068**	**0.014**	-899.962	204.837	**0.007**	**0.001**	0.401
Child plays alone	60	-552.350	-94.150	0.615	0.185	-853.130	216.721	**0.029**	**0.000**	**0.011**

Source: Moon (2014).

Notes:

(a) Ctr. Mean denotes mean value for control group

(b) Diff. Means denotes the difference in the mean values between treatment and control groups

(c) Blk. *p*-value denotes the block *p*-value for the the male block

(d) IPW P. Co. Co. denotes the inverse probability weighting correlation coefficient

(e) Gen. Diff. denotes the *p*-value for the mean values of the two genders being equal

Table J.8: Abecedarian Intervention, Parental Investment (HOME)

Variable	Age (In Months)	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Ctr. Mean	Diff. Means	Blk. *p*-val	IPW P. Co.Co.	Gen. Diff.
		Males				*Females*				
Maternal warmth	6	7.043	-0.599	0.805	0.957	6.700	0.420	**0.070**	**0.044**	**0.068**
Maternal warmth	18	7.619	0.122	0.209	**0.058**	6.714	1.112	**0.040**	**0.001**	**0.091**
Maternal warmth	30	7.286	-0.206	0.635	**0.001**	7.111	0.472	**0.057**	**0.006**	0.309
Organization of environment	6	4.652	0.422	**0.076**	**0.001**	4.633	0.007	0.641	0.439	0.145
Organization of environment	18	5.238	0.021	0.361	**0.069**	4.964	0.340	0.311	**0.017**	0.394
Organization of environment	30	5.238	0.070	0.676	**0.088**	5.148	0.102	0.286	**0.065**	0.934
Avoidance of restrict./punish.	42	5.619	-0.219	0.708	0.553	5.808	0.109	0.185	**0.045**	0.575
Avoidance of restrict./punish.	54	5.571	0.081	0.241	**0.045**	5.917	0.447	**0.044**	**0.000**	0.400
Stimulation of mature behavior	42	8.286	0.114	0.654	0.333	8.385	0.574	0.660	0.227	0.548
Stimulation of mature behavior	54	8.857	0.882	**0.051**	**0.001**	9.000	1.000	**0.045**	**0.000**	0.885

Source: Moon (2014).

Notes:

(a) Ctr. Mean denotes mean value for control group

(b) Diff. Means denotes the difference in the mean values between treatment and control groups

(c) Blk. *p*-value denotes the block *p*-value for the the male block

(d) IPW P. Co. Co. denotes the inverse probability weighting correlation coefficient

(e) Gen. Diff. denotes the *p*-value for the mean values of the two genders being equal

K A Detailed Review of the Ingredients of the Recent Literature

K.1 Overview of Structural Models of Parental Investments

Section K.1 presents literature review of the leading structural models of parental investment summarized in the Tables K.1, K.2 and K.3. By "structural" we mean models with explicit consideration of the mechanisms of choice and of their outcomes where assumptions about unobservables and their relationship are examined. Table K.1 gives a short summary of the detailed description presented in Table K.2 and Table K.3. Table K.1 is organized by model's main features such as the type of intergenerational links considered, the specifications of parental preferences and of the technology of skill formation, the role of endogenous marriage or fertility decisions. This tabular description is coupled with more in depth analysis developed in Table K.2 and Table K.3 where modeling assumptions are further explained to allow for a precise comparison among the models. The main findings are summarized in Section **??** of the paper.

Table K.4 considers the policy experiments simulated through structural models. Most of the studies of the role of income transfer programs discussed in Section **??** of the paper do not investigate the interactions of public policy interventions and family investments. In order to do so, some authors have employed fully specified structural models and used them to study the effect of various types of policy experiments. The main features of these models are discussed in Tables K.1-K.3. Table K.4 reports the outcomes of these policy experiments discussing the type of policy considered, its financing and effects. The main conclusions that emerge from these studies are presented in Section **??** of the paper.

Table K.1: Structural Models of Parental Investments ("✓" means present; "X" means absent)

	OLG Model	Dynastic Links	Explicit Models of Parental Preferences, Altruism (a) or Paternalism (p)	Model Estimated	Parental Goods Investment	Parental Time Investment	Technology Depends on Parental Skill	Self-productivity
Cunha and Heckman (2007)	✓	A,B,C	✓(a)	X	✓	X	✓	✓
Cunha (2007)	✓	A,B,C	✓(a)	✓	✓	X	✓	✓
Caucutt and Lochner (2012)	✓	B,C	✓(a)	✓	✓	X	X	✓
Del Boca et al. (2014)	X	X	✓(p)	✓	✓	✓	X	✓
Gayle et al. (2013)	✓	A,C	✓(p)	✓	✓	✓	✓	X
Cunha et al. (2013)	X	X	✓(p)	✓	✓	X	✓	✓
Bernal (2008)	X	X	✓(p)	✓	✓	✓	✓	✓
Lee and Seshadri (2014)	✓	A,B,C	✓(a)	✓	✓	✓	✓	✓
Restuccia and Urrutia (2004)	✓	A,C	✓(a)	X	✓	X	X	✓

A: Through parental skills, B: Through asset transfers, C: Through genes (initial conditions), ^{D}Natural borrowing limit, ^{E}Limits can be more stringent than natural limit.

Table K.1: (Cont.) Structural Models of Parental Investments ("$\checkmark$" means present; "X" means absent)

	Parental Learning About Technology	Bequests	Intragenerational Borrowing	Multiple Skills of Children	Multichild Families: Allow for Preferential Treatment Across Children	Endogenous Fertility Decisions	Multiple Parents	Endogenous Mating Decisions
Cunha and Heckman (2007)	X	$\checkmark$	$\checkmark^{E}$	X	X	X	X	X
Cunha (2007)	X	$\checkmark$	$\checkmark^{D}$	X	X	X	X	X
Caucutt and Lochner (2012)	X	X	$\checkmark^{E}$	X	X	X	X	X
Del Boca et al. (2014)	X	X	X	X	$\checkmark$	X	$\checkmark$	X
Gayle et al. (2013)	X	X	X	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
Cunha et al. (2013)	$\checkmark$	X	X	X	X	X	X	X
Bernal (2008)	X	X	X	X	X	X	X	X
Lee and Seshadri (2014)	X	$\checkmark$	$\checkmark^{D}$	X	X	X	X	X
Restuccia and Urrutia (2004)	X	X	X	X	X	X	X	X

A: Through parental skills, B: Through asset transfers, C: Through genes (initial conditions), ^{D}Natural borrowing limit, ^{E}Limits can be more stringent than natural limit.

Table K.2: Structural Models of Parental Investments

Paper	Data	Model Features, Parental Preferences and Constraints	Technology of Human Capital Formation	Results
Cunha (2007)	NLSY79 and CNLSY. 2,233 firstborn white children	*Model Features* The model is in general equilibrium. One sector produces the consumption good with CRS technology and one sector produces the investment good used for cognitive development with linear technology. The model is solved in a stationary steady-state equilibrium. Financing of costly policies is internal to the model: governmental sector running a balanced budget. *Preferences* Dynastic utility function maximized over consumption choices and child's human capital levels at the beginning of adulthood. Per period utility is CRRA over consumption. The parent is altruistic and values the child utility as his own. Each agent lives 38 years, 19 as a child and 19 as an adult. Childhood is divided in 3 developmental stages: first stage is 0 to 4 years old, second is 5 to 9 and third is 10 to 19. In the last period of life as a child the agent choose between enrolling in college (the parent pays his tuition) or stopping at high school working. *Constraints* The parent has access to a single risk-free assets, but cannot leave debt to the child. Therefore savings are bounded below by the natural borrowing limit. *Sources of Uncertainty* Iid shocks to the technology of skill formation. Serially correlated (AR1) productivity shocks affecting labor earnings.	Multistage CES for cognitive skills. Current skills are function of past skills, parental investments, parental human capital, heterogeneity factors of child and parent known to the agents in the model, but not to the econometrician. Child ability is known to the parent at the moment of investments. Allows for complementarity in inputs. The unobserved skills are anchored to the natural logarithm of labor income. It does not consider time inputs, or investment in multiple children. Parental wage is subject to productivity innovations at each period with an AR1 structure.	*Estimation Results* - self-productivity increases with the developmental stages - impact of parental investments is relatively larger in the first developmental stage - the concavity of the function decreases with the developmental stage - complementarity of inputs increases over developmental stages - correlation of child skill and parental skill increases over age; correlation of parental investments and parental skills stabilizes from age 1 on, correlation of child skill and parental investments increases over age stabilizing at age 3. *Policy Experiments* - all policies are financed by a flat income tax rate designed such that the government budget is balanced every period and all effects are evaluated in the new long run steady state - 50% tuition subsidy (tax rate 1,2%): rise in college enrollment from 48.57% to 61.26%. The agents who move are at the high end of the distribution of baseline high school graduates and at the low end of those who enroll in college when the policy is in place - 90% subsidy for early investments (age 3 and 4) for children who are in the bottom quartile of the ability distribution at age 3 and whose family has wealth lower than $40,000 (tax rate is 0.8%): rise in college enrollment from 48.57% to 62.64% - subsidy over all periods of investments. Amount of subsidy is 40% for parents with 0 wealth and decreasing in proportion with parental wealth until it reaches 0 when wealth is $40,000(tax rate is 0.5%): rise in college enrollment from 48.57% to 61.12% - the present value of the equivalent variation for the different policies implies that: the tuition subsidy mostly favors the 6^{th} and 7^{th} deciles of the permanent income distribution, the multi-period subsidy mostly favors agents in the 4^{th} and 5^{th} decile and the early subsidy the agents in the 1^{st} to 4^{th} deciles. The distribution of abilities under the multi-period subsidy first order stochastically dominates the distribution in all other cases.

Table K.2: Structural Models of Parental Investments

Paper	Data	Model Features, Parental Preferences and Constraints	Technology of Human Capital Formation	Results
Cunha et al. (2010)	NLSY79 and CNLSY. 2,207 firstborn white children	Implicit. Endogenous investments are considered to ensure identification and in the estimation.	Multistage CES, different for cognitive (C) and noncognitive (NC) skills. Current skills are function of past skills (C and NC), parental investments, parental skills (C and NC). Allows for complementarity in inputs. The unobserved skills are anchored to educational attainment (years of schooling). It does not consider time inputs, or investment in multiple children.	*Estimation Results* - self productivity of skills - the impact of parental investments on skills (assessed by the magnitude of the estimated share in the CES) is larger in the early stage - NC skills matter (share statistically different from 0) for C development - parental NC skills more relevant (higher share) than C - C skills are gross substitutes (the elasticity of substitution is greater than 1) with investments in first stage, gross complements (elasticity of substitution is less than 1) in the second - NC skills are gross complements (elasticity of substitution is less than 1) with investments in both stages *Policy Experiments* - Social planner maximizes aggregate schooling subject to a budget constraint (2 units of investment per capita): early (first stage) investments are higher for children with low initial conditions for C and NC skills, late (second stage) investments are slightly higher for children with high initial conditions; the optimal ratio of early to late investments is higher for children with low initial conditions. The ratio is lower for children with mothers with very low or very high cognitive skills, then if mother's skills are at the mean. - Social planner minimizes aggregate crime subject to the above budget constraint: optimal ratio of early to late investments is higher for children with low initial conditions, but the ratio is lower than the education maximizing one as crime is more intensive in non-cognitive skills which are less intensive in early investments.

Table K.2: Structural Models of Parental Investments

Paper	Data	Model Features, Parental Preferences and Constraints	Technology of Human Capital Formation	Results
Del Boca et al. (2014)	PSID-CDS, 105 one- and 132 two-child intact families	*Model Features* Partial equilibrium. One generation problem. Financing of costly policies is external to the model. *Preferences* Cobb-Douglas utility function over consumption, leisure of each parent, children quality. Dynamic finite horizon problem with a final value for children quality at age 16. Heterogenous utility parameters. *Constraints* There are no financial markets. In each period the sum of household consumption and expenditure in children needs to be equal to the total family income (labor plus non-labor income). This assumption together with the Cobb-Douglas utility implies that labor supply and investment decisions are independent of expected future income. *Sources of Uncertainty* Stochastic total factor productivity of the child quality production function. Randomness of future wage offers.	Future child quality (C skills) is a Cobb-Douglas function of active time (parents actively participating or engaged with the child) and passive time (parents around the child, but not involved) from both parents, current child quality and parental expenditure. Child ability is known to the parent at the moment of investments. Homogeneous parameters across all households. The model is separately estimated for one-child households and two-children households. Technology parameters for different children are equal. Investments are private goods. Investments are not a function of child quality.	*One-child families estimates* - child quality has the highest weight (measured by the Cobb-Douglas coefficient compared to the ones on leisure and consumption) in the utility function. Greater total value (relative magnitude of the sum of the coefficients) when 2 children are present. The terminal evaluation assigned to age 16 child quality is high and is greater in one-child families (this can be interpreted as the value assigned to the initial conditions of child's future adult life) - maternal active time is the most productive (measured by the Cobb-Douglas coefficient) input in early years, but father's time productivity is also high. Higher productivity of father time in two-child families. After age 12 active and passive time have the same productivity. Increasing importance of goods investments over age. *Simulation exercises* - "child-quality maximizing" preferences (no weights on household leisure and consumption): average latent child quality increases by 43% - "selfish parents" preferences (no weight on child quality): mothers work much more, fathers reduce work hours - "technology optimal allocations" (ignore endogenous investments decisions, equivalent to a social planner using only the estimated technology to derive the optimal allocation of inputs): lower maternal to paternal time ratio and higher maternal time to expenditure ratio. *Policy Analysis* - All proposed transfer scheme have the same cost. Financing is external to the model (i.e. transfers are not financed by imposing taxes on the household) - unconstrained transfer over all development period ($250 per week) : small impact on ability (+1.6%), but fall in parental labor supply (-15% mothers, -12.6% fathers) - constrained transfer over all development period ($250 per week): decrease in labor supply (-10% mothers, -7.5% fathers), small impact on ability (+ 4.6%) - constrained transfer over early or late periods ($500 per week): if received early (age 3-9), child expenditure increases a lot (+51%) especially in families with low weight on children, but no effect on ability, if later (age 10-15) more effective (+9%).

Table K.2: Structural Models of Parental Investments

Paper	Data	Model Features, Parental Preferences and Constraints	Technology of Human Capital Formation	Results
Del Boca et al. (2012)		As in Del Boca et al. (2014)		Analysis of short run one period unanticipated policies *Unrestricted Transfers* - improvements in child quality: 1% for $100 per week, 2.7% for $250, 5% for $500 - labor supply declines, increasing time spent with children and leisure (as expected for normal goods) - the average gain in child's quality is declining in the age at which the transfer is received - the larger the transfer, the more heterogeneous gains from it are *Transfer restricted to child's expenditures* - improvements in child quality: 1.9% for $100 per week, 5.5% for $250, 11% for $500 - labor supply decline less as household cannot use the transfer as desired (the constraint in the use of the transfer distorts the marginal conditions). Lower increase in time with children and in leisure - the average gain in child's quality is declining in the age at which the transfer is received - larger right tail in the gains distribution and gains are also on average larger than for the unrestricted transfer *Transfer conditional to have improved child quality (optimal combination of target and reward)* - perform much better in terms of child quality improvement (analysis done at age 10) - at low per capita costs ($50 each) conditional transfers are around 10 times more effective than restricted and 20 times more than unrestricted - at high costs ($150 each) they are 5 times more effective than restricted transfers and 10 times more than unrestricted - the reason why conditional cash transfer are more effective is that they allow parents to choose the optimal combination of time and good inputs to achieve the target. Possible problem: it might be hard to monitor child quality.

Table K.2: Structural Models of Parental Investments

Paper	Data	Model Features, Parental Preferences and Constraints	Technology of Human Capital Formation	Results
Caucutt and Lochner (2012)	NLSY79 and CNLSY, 2006 March CPS.	*Model Features* Partial equilibrium. Financing of costly policies is external to the model. *Preferences* Dynastic utility function maximized over consumption choices and child's cognitive skills at the beginning of adulthood. The parents is altruistic. Per period utility is CRRA over consumption. Agents live six periods of life: 2 as child, 2 as adult with child and 2 as adult with a grandchild. Utility function is homogeneous across households. *Constraints* The parent has access to a single risk free assets, but cannot leave debt to the child. Borrowing is bounded below by the a borrowing limit which can be stricter than the natural one. Income is exogenous. *Sources of Uncertainty* Iid shocks to adult earnings. Two-state Markov process on initial conditions for ability.	Focus on cognitive skills only. 1^{st} period public and private investments are perfect substitutes and map directly into 2^{nd} period child's cognitive skills. 3^{rd} period cognitive skill is a CES function of 2^{nd} period skill, public and private investments (perfect substitutes). Child ability is known to the parents at the moment of investments. Productivity depends on genetic ability represented by a two-stage Markov process over generations. Late private investments are subsidized. It does not consider time inputs, or investment in multiple children. Parental earnings increase exogenously.	*Estimates* - elasticity of substitution between early and late investments is 0.48: strong dynamic complementarity. The estimate is very similar estimate to the one in Cunha et al. (2010) - 50% of young and 12% of old parents are borrowing constrained - non monotonic relationship of parental education and constraints of young parents: high school dropouts (50% constrained), high school graduate (38%), college dropouts (60%) and college graduates (68%) - constraints of old parents are monotonic in education *Policy Experiments* - $2,500 increase in borrowing limits for young parents (exogenously imposed): effect in the short run (SR), but almost vanishes in the long run (LR). Increase in both early and late investments. SR effects: college educated parents +18.8% early investments, high school drop outs +0%. LR effects: reduction in investments by all (hs dropouts -16.4%), but for college graduates (+10.5%). Entry wages: +0%. The modest LR effects are due to the increase in the overall debt level of the dynasties and in particular of least educated young adults who consequently reduce financial transfers and human capital investments in children - 10% subsidy to early investment: increase in both early (+28% in SR, +34% in LR) and late investments (+12% in SR, +17% in LR), increase in college graduation rate (+30% in SR, +40% in LR), no changes in hs graduation rate, wages grow by 3.1% in SR and 3.9% in LR - increase from 50% to 53% in subsidy of later investments (same cost as above policy): moderate effects, but for an increase in hs graduation rate, effectiveness limited by parents being still constrained when young. Important to consider early investment responses to late subsidies: if ignored effects underestimated by 60% - $322 extra public investments (same costs as policies above): crowds out $116 private investments. Increases high school completion rates by 12%, college attendance by 25%, but graduation only by 1%. Effect only at the bottom end of the early investment distribution, while early subsidy affects the high end - $2,500 income transfer to young parents: more effective if the policy is put in place only once (early investments +7.8%, late +5%, entry wages +1.1%), than if implemented permanently (early investments +5.7%, late +4%, entry wages +0.8%).

Table K.2: Structural Models of Parental Investments

Paper	Data	Model Features, Parental Preferences and Constraints	Technology of Human Capital Formation	Results
Gayle et al. (2013)	PSID Family Individual File. 12,318 individual observations.	*Model Features* Partial equilibrium. Non-cooperative game between spouses. Stationary Markov Perfect Equilibrium whose solution can be represented as a single agent optimization problem. *Preferences* Dynastic utility with Barro-Becker (1989) weights on children. Utility is linear in consumption, number of children, leisure and a stochastic preference shock. Consumption depends on labor supply decisions, accumulated work experience, personal time-constant characteristics of both spouses. The agents choose: labor market time and time with children. Females also choose birth. Structural utility parameters are gender-specific. Utility from leisure is choice specific where choices depend on how much time the agent works and how much time she spend with the children. *Constraints* There are no financial markets. In each period the expenditure on consumption needs to be equal to the total family income. *Sources of Uncertainty* Preference shocks. Stochastic production function of children's schooling level and ability. Stochastic determination of spouse's education and ability capturing potential assortative mating.	Children education and ability is a stochastic function of: total parental cumulative time investments (assumed equal across children in the same family), parental education, race, individual specific effects. Only total parental time is considered.	*Estimation Results* - loss of human capital from spending time out of work is higher for men than for women - mother's time increases the probability of college graduation, father's time increase the probability of high school graduation and college attendance - time invested with children strongly increases their educational outcomes; parental education increases the productivity of time investments - maternal time investments 1.67 times more valuable than paternal time investments. Maternal time increases the likelihood of graduating college which is the most valuable educational outcome in the labor and marriage market. - there is no independent effect of parental labor income on children's education - return to maternal time investment is higher for blacks than for white, while there are no differences in the return of paternal time - maternal time returns are higher for boys. Boys have lower ability and a lower likelihood of high education outcomes compared to girls. Mothers invest more in boys in a compensatory manner. - quantity quality trade-off more pronounced for boys then for girls and twice as large for blacks than whites (as more black single mothers) - do not perform policy experiments.

Table K.3: Structural Models of Parental Investments in Recent Literature

Paper	Data	Maximization Problem, Preferences and Constraints	Technology of Human Capital Formation
Cunha (2007)	NLSY79 and CNLSY. 2,233 firstborn white children	$V(1,h,\theta_1,\varepsilon_1,a_1,j_1) = \max_{\{c_t,I_t,a_{t+1}\}_{t=1}^{T_c}} \underbrace{\sum_{t=1}^{T_c} \beta^{t-1}\mathbb{E}_{t-1}U(c_t)}_{\text{Lifetime utility of parent}} + \underbrace{\beta^{T_c}\mathbb{E}_{T_c}V'(1,h',\theta_1',\varepsilon_1',a_1',j_1')}_{\text{Value of adult child}}$ s.t. $c_t + I_t + a_{t+1} = wh\varepsilon_t + (1+r)a_t$ $a_{t+1} \geq -wh\varepsilon_{min}$ $\ln\epsilon_{t+1} = \rho\ln\varepsilon_t + \nu_t$ Specification/Calibration Parameters: $T_c = 19 \quad h = \theta_{T_c} \quad \rho = .7910$ $U_(c_t) = \frac{c_t^{1-\lambda}-1}{1-\lambda} \quad \lambda = 2$	$\theta_{C,t+1} = f^{j_t}(\theta_{C,t}, I_t, h_C, \pi_t, \eta_t)$ $j_t = j_{d1}$ for $t = 1,\ldots,4$ $j_t = j_{d2}$ for $t = 5,\ldots,9,$ $j_t = j_{d3}$ for $t = 10,\ldots,14$ Estimation: Functional Form : $\ln\theta_{t+1} = \frac{\rho_{n_t}}{\phi_{n_t}}\ln[\gamma_{1,n_t}e^{\phi_{n_t}\ln\theta_t} + \gamma_{2,n_t}e^{\phi_{n_t}\ln x_t} + \gamma_{3,n_t}e^{\phi_{n_t}\ln h}] + \eta_t{}^{\theta}$ Estimates: Stage: 1, 2, 3 γ_{1,n_t}: .6546, .7561, .8504 γ_{2,n_t}: .1438, .0821, .0925 γ_{3,n_t}: .2016, .1617, .0570 ρ_{n_t}: .6192, .8833, .9142 ϕ_{n_t}: -.4643, -.1402, -.212 ${\sigma_{n_t}}^2$: .3775, .3008, .1393 The anchor for θ_t is the natural log of labor income. The anchor for h is the natural log of labor income when the parents are 30 years-old.

T are the total number of periods, T_c are the total number of periods of childhood (i.e. the years for which a child is dependent on the parents), primes indicate the next generation, θ_t represents human capital, h_t represents parental human capital (the subscript C stands for cognitive skills and NC for non-cognitive skills), j_{di} represent developmental stage i ($i = 1,2,3$), π_t is a factor capturing unmodeled heterogeneity, a_t are assets holdings, c_t is consumption, I_t are monetary investment in children, r is a risk free interest rate, β is the discount factor, δ is a parameter measuring altruism, w is the wage rate, $\gamma(\cdot)$ is the growth rate of adult human capital, ε_t and η_t are unobserved shocks, M_t is non labor income, W_t are total labor earnings, l_t is leisure time, τ_t are time investments in children (subscript a indicates active engagement from the parents, while p only a passive presence), p_t are publicly provided investments, su_t are investment subsidies, TI is the total amount of time spent with children over the whole childhood, H is the total amount of time available in a given period, $\mathcal{S}$ is a set of feasible strategy profiles, N_t indicates the number of children, b_t is an indicator for the presence of birth, G_t is a collection of indicators for the genders of all children alive in t, EX_t indicates total labor market experience, $\overline{V}(\cdot)$ indicates the average value function over all children, m stands for male and f for female, α_1 and α_2 are sharing rules indicating the amout of income going to each spouse.

Table K.3: Structural Models of Parental Investments in Recent Literature

Paper	Data	Maximization Problem, Preferences and Constraints	Technology of Human Capital Formation
Cunha and Heckman (2007)	—	$V(1,h,\theta_1,a_1) = \max_{\{c_t,I_t,a_{t+1}\}_{t=1}^{T_c}} \underbrace{\sum_{t=1}^{T_c} \beta^{t-1}\mathbb{E}_{t-1}U(c_t)}_{\text{Lifetime utility of parent}} + \underbrace{\beta^{T_c}\mathbb{E}_{T_c}V'(1,\theta^{P\prime},\theta_1',a_1')}_{\text{Value of adult child}}$ s.t. $c_t + I_t + a_{t+1} = wh_t + (1+r)a_t$ $a_{t+1} \geq \underline{a}$ $a_{T_c} = a_1' \geq 0$ $h_{t+1} = (1+\gamma)h_t$ Specification/Calibration Parameters: $T_c = 2$	$\theta_{t+1} = f(\theta_t, I_t, h_t)$ Estimation: Functional Form: CES, Cobb Douglas, Leontief, Perfect Substitutes. Estimates: No estimation results reported.

T are the total number of periods, T_c are the total number of periods of childhood (i.e. the years for which a child is dependent on the parents), primes indicate the next generation, θ_t represents human capital, h_t represents parental human capital (the subscript C stands for cognitive skills and NC for non-cognitive skills), j_{di} represent developmental stage i ($i = 1,2,3$), π_t is a factor capturing unmodeled heterogeneity, a_t are assets holdings, c_t is consumption, I_t are monetary investment in children, r is a risk free interest rate, β is the discount factor, δ is a parameter measuring altruism, w is the wage rate, $\gamma(\cdot)$ is the growth rate of adult human capital, ε_t and η_t are unobserved shocks, M_t is non labor income, W_t are total labor earnings, l_t is leisure time, τ_t are time investments in children (subscript a indicates active engagement from the parents, while p only a passive presence), p_t are publicly provided investments, su_t are investment subsidies, TI is the total amount of time spent with children over the whole childhood, H is the total amount of time available in a given period, $\mathcal{S}$ is a set of feasible strategy profiles, N_t indicates the number of children, b_t is an indicator for the presence of birth, G_t is a collection of indicators for the genders of all children alive in t, EX_t indicates total labor market experience, $\overline{V}(\cdot)$ indicates the average value function over all children, m stands for male and f for female, α_1 and α_2 are sharing rules indicating the amout of income going to each spouse.

Table K.3: Structural Models of Parental Investments in Recent Literature

Paper	Data	Maximization Problem, Preferences and Constraints	Technology of Human Capital Formation
Del Boca et al. (2014)	PSID-CDS. 105 one- and 132 two-child intact families	$V(1,h,w_{1,1},w_{2,1},M_t) = \max_{\{l_{f,t},l_{m,t},\tau^a_{f,t},\tau^a_{m,t},\tau^p_{f,t},\tau^p_{m,t},I_t\}_{t=1}^T} \underbrace{\sum_{t=1}^{T} \beta^{t-1}\mathbb{E}_{t-1}U(c_t,l_{f,t},l_{m,t},\theta_t \mid h)}_{\text{Lifetime utility of the parent}} + \underbrace{\delta\beta^T\mathbb{E}_T\theta_{T+1}}_{\text{Value of child's ability}}$ s.t. $c_t + I_t = (1+r)a_t + w_{f,t}h_{f,t} + w_{m,t}h_{m,t} + M_t$ $H = l_{i,t} + h_{i,t} + \tau^a_{i,t} + \tau^p_{i,t}$ for $i = f,m$ Specification/Calibration Parameters: $a_t = 0 \quad h = \theta_T \quad T = 16$ $U(c_t,l_{f,t},l_{m,t},\theta_t \mid h) = \alpha_1 \ln l_{1t} + \alpha_2 \ln l_{2t} + \alpha_2 \ln c_t + \alpha_4 k_t$ with $\sum_i \alpha_i = 1$ $\alpha_1 = .196$ $\alpha_2 = .194$ $\alpha_3 = .257$ $\alpha_4 = .353$	$\theta_{t+1} = f(\theta_t, \tau^a_{1,t}, \tau^a_{2,t}, \tau^p_{1,t}, \tau^p_{2,t}, I_t, \eta_t)$ Estimation: Functional Form: Cobb-Douglas Estimates: Family Size: 1-Child / 2-Child Time Moth's Act: .066/ .000 / .077/.003 Fath's Act: .082/.0531 / .071/.008 Moth's Pas: .049/.000 / .076/.003 Fath's Pass: .056/.0472 / .043/.009 Child Exp: .042/.200 / .161/.180 Last Skill: .140/.503 / .172/.992 Estimates for age 1 and 16 reported.

T are the total number of periods, T_c are the total number of periods of childhood (i.e. the years for which a child is dependent on the parents), primes indicate the next generation, θ_t represents human capital, h_t represents parental human capital (the subscript C stands for cognitive skills and NC for non-cognitive skills), j_{di} represent developmental stage i ($i = 1,2,3$), π_t is a factor capturing unmodeled heterogeneity, a_t are assets holdings, c_t is consumption, I_t are monetary investment in children, r is a risk free interest rate, β is the discount factor, δ is a parameter measuring altruism, w is the wage rate, $\gamma(\cdot)$ is the growth rate of adult human capital, ε_t and η_t are unobserved shocks, M_t is non labor income, W_t are total labor earnings, l_t is leisure time, τ_t are time investments in children (subscript a indicates active engagement from the parents, while p only a passive presence), p_t are publicly provided investments, su_t are investment subsidies, TI is the total amount of time spent with children over the whole childhood, H is the total amount of time available in a given period, $\mathcal{S}$ is a set of feasible strategy profiles, N_t indicates the number of children, b_t is an indicator for the presence of birth, G_t is a collection of indicators for the genders of all children alive in t, EX_t indicates total labor market experience, $\overline{V}(\cdot)$ indicates the average value function over all children, m stands for male and f for female, α_1 and α_2 are sharing rules indicating the amount of income going to each spouse.

Table K.3: Structural Models of Parental Investments in Recent Literature

Paper	Data	Maximization Problem, Preferences and Constraints	Technology of Human Capital Formation
Caucutt and Lochner (2012)	NLSY79 and CNLSY, 2006 March CPS.	$V(1,h_1,\varepsilon_1,a_1,\theta_1) = \max_{\{c_t,I_t,c'_t,a_{t+1}\}_{t=1}^{T}} \underbrace{\sum_{t=1}^{T}\beta^{t-1}\mathbb{E}_{t-1}U(c_t)}_{\text{Lifetime utility of parent}} + \delta \underbrace{\sum_{t=1}^{T_c}\beta^{t-1}\mathbb{E}_{t-1}U(c'_t)}_{\text{Utility of child while dependent}}$ $+ \delta\beta^{T_c} \underbrace{\mathbb{E}_{T_c} V'(1,h'_1,\varepsilon'_1,a'_1)}_{\text{Value of adult child}}$ s.t. $c_t + c'_t + I_t(1-su_t) - p_t + a_{t+1} = (1+r)a_t + wh_t + \epsilon_t$ $a_{t+1} \geq \kappa(wh_{t+1})_{min}$ $a_T \geq 0$ $h_{t+1} = (1+\gamma(h_t))h_t$ Specification/Calibration Parameters: $h_1 = \theta_{T_c}$ $T = 6$ $T_c = 2$ $U(c_t) = \frac{c^{1-\lambda}}{1-\lambda}$ $\lambda = 2$ $r = .05$ $\beta = (1+r)^{-1}$ $\kappa = .45$	$\theta_{t+1} = f(\theta_t, I_t)$ $\theta'_1 = g(\theta_1)$ Simulation: Functional Form: CES Results: No results reported.

T are the total number of periods, T_c are the total number of periods of childhood (i.e. the years for which a child is dependent on the parents), primes indicate the next generation, θ_t represents human capital, h_t represents parental human capital (the subscript C stands for cognitive skills and NC for non-cognitive skills), j_{di} represent developmental stage i ($i = 1,2,3$), π_t is a factor capturing unmodeled heterogeneity, a_t are assets holdings, c_t is consumption, I_t are monetary investment in children, r is a risk free interest rate, β is the discount factor, δ is a parameter measuring altruism, w is the wage rate, $\gamma(\cdot)$ is the growth rate of adult human capital, ε_t and η_t are unobserved shocks, M_t is non labor income, W_t are total labor earnings, l_t is leisure time, τ_t are time investments in children (subscript a indicates active engagement from the parents, while p only a passive presence), p_t are publicly provided investments, su_t are investment subsidies, TI is the total amount of time spent with children over the whole childhood, H is the total amount of time available in a given period, $\mathcal{S}$ is a set of feasible strategy profiles, N_t indicates the number of children, b_t is an indicator for the presence of birth, G_t is a collection of indicators for the genders of all children alive in t, EX_t indicates total labor market experience, $\overline{V}(\cdot)$ indicates the average value function over all children, m stands for male and f for female, α_1 and α_2 are sharing rules indicating the amout of income going to each spouse.

Table K.3: Structural Models of Parental Investments in Recent Literature

Paper	Data	Maximization Problem, Preferences and Constraints	Technology of Human Capital Formation
Gayle et al. (2013)	PSID Family Individual File. 12,318 individual observations.	$V_\sigma(X_1) = \max_{\{s\in\mathcal{S}\}_{t=1}^T} \underbrace{\sum_{t=1}^T \beta^{t-1}\mathbb{E}_X\Big[\big(u(s_t)+\mathbb{E}_\varepsilon(\varepsilon_{\sigma,t} \mid s_t)\big)\Big]}_{\text{Lifetime utility of the parent}} + \delta\beta^T \frac{(N_{\sigma,T}+b_T)^{1-v}}{N_{\sigma,t}+b_T} \underbrace{\mathbb{E}_{X,\varepsilon}\overline{V}_{N,\sigma}(X_1')}_{\text{Average value of adult child}}$ where: $X_t = (X_{f,t}, X_{m,t})$ $X_{\sigma,t} \equiv (X_\sigma, N_{\sigma,t}, EX_{\sigma,t}, TI_{\sigma,t}, G_t, \theta)$ $s_t \equiv (c_{\sigma,t}, h_{\sigma,t}, \tau_{\sigma,t}, b_{f,t})$ $c_{\sigma,t} = \alpha_{1,\sigma}W_{m,t} + \alpha_{2,\sigma}W_{f,t}$ $\sigma = m, f$ Specification/Calibration Parameters: $a_t = 0$ $T = 39$ period 0 is age 17 $u(s_t) = \alpha_{1,\sigma}W_{\sigma,t} + \alpha_{2,\sigma}W_{-\sigma,t} + \alpha_{N,\sigma}(N_{\sigma,t}+b_t)$	$\theta_1' = f(X_f, X_m, TI_f, TI_m, \theta_1)$ Estimation: Functional Form: Not Specified. Estimates: Schooling Level Input: HS / S Coll / Coll HS Father: .008 / .023 / .155 S Coll Fath: -.012 / .057 / .162 Coll Fath: -.014 / .021 / .229 HS Moth: .004 / .093 / .083 S Coll Moth: -.016 / .036 / -.089 Coll Moth: -.122 / .003 / .222 Time Fath: .153 / .273 / -.108 Time Moth: -.091 / -.048 / .299 Logit Estimates.

T are the total number of periods, T_c are the total number of periods of childhood (i.e. the years for which a child is dependent on the parents), primes indicate the next generation, θ_t represents human capital, h_t represents parental human capital (the subscript C stands for cognitive skills and NC for non-cognitive skills), j_{di} represent developmental stage i ($i = 1,2,3$), π_t is a factor capturing unmodeled heterogeneity, a_t are assets holdings, c_t is consumption, I_t are monetary investment in children, r is a risk free interest rate, β is the discount factor, δ is a parameter measuring altruism, w is the wage rate, $\gamma(\cdot)$ is the growth rate of adult human capital, ε_t and η_t are unobserved shocks, M_t is non labor income, W_t are total labor earnings, l_t is leisure time, τ_t are time investments in children (subscript a indicates active engagement from the parents, while p only a passive presence), p_t are publicly provided investments, su_t are investment subsidies, TI is the total amount of time spent with children over the whole childhood, H is the total amount of time available in a given period, $\mathcal{S}$ is a set of feasible strategy profiles, N_t indicates the number of children, b_t is an indicator for the presence of birth, G_t is a collection of indicators for the genders of all children alive in t, EX_t indicates total labor market experience, $\overline{V}(\cdot)$ indicates the average value function over all children, m stands for male and f for female, α_1 and α_2 are sharing rules indicating the amout of income going to each spouse.

Table K.4: Policy Experiments from Structural Models

	Financing is Modeled	Welfare Comparison	Policies Considered	Cost to Individual	Cost to Society	Effects
Cunha (2007)	✓	✓[a]	50% college tuition subsidy	Flat tax rate on income: 1,2%	Unknown	- Rise in college enrollment from 48.57% to 61.26%. Marginal individuals in the ability distribution enroll
			90% subsidy for early investments (age 3 and 4) for children in the bottom quartile of the ability distribution with family wealth below $40,000	Flat tax rate on income: 0.8%.	Unknown	- Rise in college enrollment from 48.57% to 62.64%
			90% subsidy over all periods: 40% when family wealth is $0 then proportionally decreasing up to 0 at $40,000	Flat tax rate on income: 0.5%	Unknown	- Rise in college enrollment from 48.57% to 61.12%
Cunha and Heckman (2007)	X	X	Early intervention: moving children to the 4^{th} decile of the distribution of skills through early investments	0	Unknown	- High-school graduation: +24.7%; College enrollment: +8.1%; Conviction: -5.7%; Probation: -6.7%; Welfare: -8.6%
			Adolescent intervention: moving investments at last transition from 1^{st} to 9^{th} decile	0	35% more costly than early intervention	- High-school graduation: +22.8%; College enrollment: +7.1%; Conviction: -5.0%; Probation: -5.9%; Welfare: -8.1%
			Balanced early-late intervention	0	Same cost as adolescent intervention	- High-school graduation: +50.2%; College enrollment: +33.1%; Conviction: -11.5%; Probation: -13.4%; Welfare: -15.1%
Del Boca, Flinn and Wiswall (2013)	X	X	Unrestricted Transfers	0	Three scenarios (1) + $100 per week, (2) +$250 per week, (3) +$500 per week	- Transfer at age 10, changes in child quality: (1) +1%; (2) +2.7%; (3) +5% - Transfer at age 3, changes in child quality: (1) +1.5%; (2) +3.9%; (3) +7% - Larger transfer: more heterogeneous gains - Active time with children increase (+14% for mothers, +16% for fathers in (3) at age 10), so does passive time (+15% for mothers, +16% for fathers)
			Restricted Transfers (buy only child related goods)	0	Three scenarios (1) + $100 per week, (2) +$250 per week, (3) +$500 per week	- Transfer at age 10, changes in child quality: (1) +1.9%; (2) +5.5%; (3) +11% - Transfer at age 3, changes in child quality: (1) +1.5%; (2) +7%; (3) +10% - Larger right tail in the gains distribution and greater gains than for the unrestricted transfer - Active time with children increase (+9% for mothers, +9% for fathers in (3) at age 10), so does passive time (+8% for mothers, +9% for fathers) - Expenditure increases: (1) 6.9%; (2) 35.6%; (3) 111%, great increase for the household who were investing less than the new restricted level
			Conditional transfers for a given improvement in child's quality	0	Optimal combination of targets and rewards to minimize cost. Two scenarios: (1) $50 reward, (2) $150 reward	- (1) 10 times more effective than an equally costly restricted transfer and 20 times more than an equally costly unrestricted - (2) 5 times more effective than an equally costly restricted transfer and 10 times more than an equally costly unrestricted - Allow parents to choose the optimal combination of time and good inputs to achieve the target - Possible problem: hard to monitor child quality
Caucutt and Lochner (2013)	X	X	$2,500 increase in borrowing limits for young parents	0	Unknown	- effect in the short run (SR, one generation ahead), but almost vanishes in the long run (LR, new steady state) - SR effects. Early investments: college graduate parents +18.8%, high school graduates +9.7%, high school dropouts +0%. Average entry wages: +1.5% - LR effects. Early investments: college graduate parents +10.5%, high school graduates -5.2, high school dropouts -16.4%, college graduates +10.5%. Average entry wages: +0%. - The modest LR effects are due to the increase in the overall debt level of the dynasties and in particular of least educated young adults who consequently reduce financial transfers and human capital investments in children

10% subsidy to early investment	0	$900	- Increase in both early (+28.1% in SR, +34% in LR) and late investments (+12.1% in SR, +17% in LR) - Increase in college graduation rate (+30.5% in SR, +39.6% in LR) - Almost no changes in hs graduation rate - Increase in entry wages (+3.1% in SR, +3.9% in LR)
Increase from 50% to 53% in subsidy of later investments	0	Same cost as 10% early investment subsidy	- Increase in both early (+4% in SR, +7.2% in LR) and late investments (+15.5% in SR, +18.1% in LR) - Increase in college graduation rate (+22.9% in SR, +26.8% in LR) - Increase in hs graduation rate (+10.1% in SR, +10.9% in LR) - Increase in entry wages (+1.4% in SR, +1.8% in LR)
$322 extra public first period investments (perfect substitution with privates)	0	Same cost as 10% early investment subsidy	- Crowds out $116 private investments - Increases hs graduation rates +12%, college attendance +25%, college graduation +1% - Effect at low end of the distribution of investments (while investment subsidies have an effect at the high end) - Increase in entry wages +1%
$2,500 income transfer to young parents	0	$2,500	- One time policy: early investments +7.8%, late +5%, entry wages +1.1% - Permanent policy: early investments +5.7%, late +4%, entry wages +0.8%

[a] In terms of equivalent variation.

K.2 Models of Parent-Child Interactions

In Table K.5 we summarize a very few examples of limited literature on parent-child interactions and parental learning by their main features. We specify the precise nature of the interaction, the potential sources of conflicts between parents and children, the information set of parents and children and how it evolves over time. We discuss the role of parental learning and initial beliefs and how they shape the nature of the interaction. A more detailed review of the studies is presented in Section **??** of the paper.

Table K.5: Models of Parent-Child Interaction ("✓" means present, "X" means absent)

	Parental Monetary Investments	Discordance in Preferences between Parent and Child	Multiple Children	Parental Incentives for Child Effort	Effort Produces Greater Skills	Parental Learning About Child Quality	Parental Actions Facilitate Acquisition of Information	Parental Beliefs can Diverge from Truth	Child's Effort Observable	Child's Human Capital Observable	Model is Estimated
Cosconati (2013)	X	✓[a,b]	X	✓[c]	✓	X	X	X	X	✓	✓
Akabayashi (2006)	✓	✓[a]	X	✓[d,e]	✓	✓	✓	✓[f]	✓	X	X
Hao et al. (2008)	✓	✓[b]	✓	✓[e]	X	X	X	X	X	X	✓[g]
Lizzeri and Siniscalchi (2008)	X	✓[d]	X	X	X	X	✓	X	✓	X	X

[a]Difference in discount factors, [b]Differences in utility functions, [c]Restrictions on leisure, [d]Difference in knowledge about proper task execution, [e]Time investments in the child, [f]Monetary investments in the child, [g]The authors analyze the effects of time investments, [h]Implications from the model are empirically tested.

L Dynamic Complementarity for the Vector Case

Consider the following specification for a vector-valued technology mapping a $L \times 1$ vector of parental investments $\boldsymbol{I_t}$, and a $J \times 1$ vector of skills $\boldsymbol{\theta_t}$, into a $J \times 1$ vector of next period capabilities $\boldsymbol{\theta_{t+1}}$:

$$\boldsymbol{\theta_{t+1}} = \boldsymbol{f^t}(\boldsymbol{\theta_t}, \boldsymbol{I_t}).$$

The matrix of second-order partial derivatives of the skill vector $\boldsymbol{\theta_{t+s+1}}$ with respect to the investment vectors $\boldsymbol{I_{t+s}}$ and $\boldsymbol{I_t}$ is given by the $J \times L^2$ matrix:

$$\frac{\partial^2 \boldsymbol{\theta_{t+s+1}}}{\partial \boldsymbol{I_t} \partial \boldsymbol{I_{t+s}}} = \begin{bmatrix} \frac{\partial^2 f^{1,t}(\cdot)}{\partial i_{1,t+s} \partial i_{1,t}} & \cdots & \frac{\partial^2 f^{1,t}(\cdot)}{\partial i_{L,t+s} \partial i_{1,t}} & \frac{\partial^2 f^{1,t}(\cdot)}{\partial i_{1,t+s} \partial i_{2,t}} & \cdots & \cdots & \cdots & \frac{\partial^2 f^{1,t}(\cdot)}{\partial i_{L,t+s} \partial i_{L,t}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & & \ddots & \vdots \\ \frac{\partial^2 f^{J,t}(\cdot)}{\partial i_{1,t+s} \partial i_{1,t}} & \cdots & \frac{\partial^2 f^{J,t}(\cdot)}{\partial i_{L,t+s} \partial i_{1,t}} & \frac{\partial^2 f^{J,t}(\cdot)}{\partial i_{1,t+s} \partial i_{2,t}} & \cdots & \cdots & \cdots & \frac{\partial^2 f^{J,t}(\cdot)}{\partial i_{L,t+s} \partial i_{L,t}} \end{bmatrix}$$

where

$$\frac{\partial^2 f^{j,t}(\cdot)}{\partial i_{l,t+s} \partial i_{l',t}} \quad \text{for} \quad j = 1, \ldots J \quad \text{and} \quad l, l' = 1, \ldots, L$$

is the cross-partial derivative of the entry j of the vector $\boldsymbol{\theta_{t+s+1}}$ with respect to $i_{l,t+s}$, the l^{th} entry of the vector of investments $\boldsymbol{I_{t+s}}$, and $i_{l',t}$, the l' entry of the vector $\boldsymbol{I_t}$.

By analogy with the argument presented in the main text, the sign of each entry is determined by the sign of:

$$\frac{\partial^2 f^{j,t}(\boldsymbol{\theta_t}, \boldsymbol{I_t})}{\partial i_{l,t+s} \partial \boldsymbol{\theta_{t+s}}} \frac{\partial \boldsymbol{\theta_{t+s}}}{\partial i_{l',t}} \quad \text{for} \quad j = 1, \ldots J \quad \text{and} \quad l, l' = 1, \ldots, L.$$

A sufficient condition for the above to be positive is that each cross partial derivative $\frac{\partial^2 f^{j,t}(\boldsymbol{\theta_t}, \boldsymbol{I_t})}{\partial i_{l,t+s} \partial \theta_{j',t+s}}$ is positive for each $j, j' = 1, \ldots J$ and $l = 1 \ldots L$, and each entry in the skill vector is increasing in each type of investment.

M Evidence on Gene Environment Interactions

Tables M.1 and M.2 review the main studies in the behavioral genetics literature on the heritability of capabilities. However, the estimates presented are highly questionable. The first reason of skepticism is that the standard linear additive models (ACE) used in behavioral genetics and social sciences rely on highly questionable assumptions. In particular, they assume that child's genetic inheritance and parenting experience are uncorrelated. For this to hold, parent's genes have to be uncorrelated with the family environment they create. This is internally inconsistent given that the theory postulates that genes affect behavior. A second reason of skepticism is related to the fact that while the transmission of the genotype follows biologically determined mechanisms, the mapping of the genotype into phenotype is unclear and likely affected by the environment through epigenetic forces potentially affecting also future generations (Cole et al., 2012; Jablonka and Raz, 2009; Kuzawa and Quinn, 2009; Youngson and Whitelaw, 2008). We conclude that while genetic influences are likely important, the ways social scientists have developed to measure them fail to provide credible estimates. Table M.1 and M.2 consistently show that whenever the the role of environmental effects in mediating genes expressions is considered, the estimates of heritability are highly impacted (Krueger and Johnson, 2008; Nisbett et al., 2012; Tucker-Drob et al., 2009; Turkheimer et al., 2003).

Table M.1: Heritability of Cognitive Abilities

Study	Data and Method	Genes-Environment Interactions	Findings
Jencks et al. (1972)	Meta-analysis: 18 studies considered on IQ correlations for twins and adoptive siblings and fraternal twins	X	*Correlations:* - siblings raised together: 0.54 - adoptive sibs: 0.42 - MZ twins: 0.86 - DZ twins: 0.58
Golberger (1977)	Meta-analysis: 7 studies considered on IQ correlations for twins and adoptive siblings	X	*Correlations:* - siblings raised together: 0.5 - adoptive sibs: 0.3 - MZ twins: 0.91
Bouchard and McGue (1981)	Meta-analysis: 69 studies considered on IQ correlations for twins and adoptive siblings	X	*Correlations:* - siblings raised together: 0.45 - adoptive sibs: 0.29; - MZ twins: 0.85
Scarr et al. (1993)	426 members of 93 transracial adoptive families. Analysis of IQ correlations parent-child and across siblings measured at age 7 and 17	X	*Correlations at age 7* Transracial adoptees: with adoptive father 0.08, adoptive mother 0.14, adoptive midparent 0.13, birth father 0.42, birth mother 0.29, birth midparent 0.47 Biological offspring: correlation with father 0.25, mother 0.40, midparent 0.48 *Correlations at age 17* Transracial adoptees: with adoptive father 0.21, adoptive mother 0.21, adoptive midparent 0.27, birth father 0.28, birth mother 0.23, birth midparent 0.24 Biological offspring: correlation with father 0.13, mother 0.45, midparent 0.40
Devlin et al. (1997)	Meta-analysis: 212 studies considered on IQ correlations for twins. Model comparison using Bayes factors. Allow for a role of maternal effects.	✓	*Correlations*: - siblings raised together: 0.44 - siblings raised apart: 0.27 - MZ twins raised together: 0.85 - MZ twins raised apart: 0.68 - DZ twins raised together: 0.59 *Variance decomposition*: - narrow sense heritability (additive genetic effects): 34% - broad-sense heritability (include non additive genetic factors): 48% - maternal effect (for twins): 20% - maternal effect (for siblings): 5% - common environment: 17%
Turkheimer et al. (2003)	319 twins pairs from the National Collaborative Perinatal Project sample. Analysis on the relationship between socioeconomic status (SES) and heritability of IQ.	✓	*Variance decomposition*: - genes: 0.1 for low SES, 0.8 for high SES - shared environment: 0.55 for low SES, 0.1 for high SES - non-shared environment: 0.35 for low SES, 0.1 for high SES - parental environments matter more for low SES families often underrepresented in samples
Tucker-Drob et al. (2009)	319 pairs of twins in the National Collaborative Perinatal Project. Nonlinear factor analysis: account for the possibility that correlations in different cognitive abilities is different at different ability levels. Avoid bias in estimating the relationship of SES and heritability of cognitive abilities	✓	*Variance decomposition*: - genes, 0.15 for low SES, 0.6 for high SES - shared environment, 0.55 for low SES, 0.25 for high SES - non-shared environment, 0.3 for low SES, 0.15 for high SES. - SES gradient in heritability (Turkheimer et al., 2003) is less steep but still present when accounting for nonlinear effects

Haworth et al. (2009)	Twins of high ability (> 85th percentile) from samples in United States, Australia, Netherlands and United Kingdom	X	*Variance decomposition*: - genes 50% - shared environment 28% - non-share environment 0.22%
Nisbett et al. (2012)	Meta-analysis: review of recent literature on different aspects of intelligence (IQ, fluid and crystallized) and its relationships with socioeconomic status, interventions and other environmental conditions	✓	IQ and SES: heritability of IQ is higher for higher SES families in the US. Less evident in Europe. IQ and environment: Increase from 12 to 18 points in IQ when children are adopted from working class to middle class homes. IQ and interventions: even if effects on IQ of interventions vanish, there are effects on educational achievements and life outcomes (limits of IQ as the only relevant characteristic)
Briley and Tucker-Drob (2013)	Meta-analysis: 16 articles with 11 unique samples. Total of 11,500 twin and siblings pairs reared together and with cognition measured at least twice between 6 months and 18 years old. Analysis of the changes in the role of genetic heritability over the phases of development.	X	IQ heritability increases over time even when controlling for cross sectional age differences. Innovative genetic influences (activation of new genes because of biological or environmental changes) are predominant until age 8 then genetic amplification (small initial genetic differences are amplified by transactional processes) dominates. Innovative influences are relevant also for the components of variance in IQ due to shared environment, but fades overtime and it is confounded with amplification from age 12.

Table M.2: Heritability of Personality Traits

Study	Data and Method	Genes-Environment Interactions	Findings
Loehlin (2005)	Meta-analisis: correlations in personality measures between parents and children under different scenarios	X	*Biological parents raise children*: extraversion 0.14, agreeableness 0.11, conscientiousness 0.09, neuroticism 0.13, openness 0.17. *Adoptive parents and adopted children*: extraversion 0.03, agreeableness 0.01, conscientiousness 0.02, Neuroticism 0.05, openness 0.07. *Biological parents and adopted children*: extraversion 0.16, agreeableness 0.14, conscientiousness 0.11, neuroticism 0.11, openness 0.14.
Krueger and Johnson (2008)	Twins from Minnesota Twin Family Study. 556 male twin pairs and 604 female pairs. Method: allow for parenting style (measured by regard and conflict) as a form of gene-environment interaction. Parental actions mediate the role of genetic contribution to personality.	✓	*Positive emotionality (PEM)*: proportion of variance explained by genes (heritability) depends on level of parental regard. If low, environmental factors explain 64% of variance, genes 35%, if high, genes explain 76%, environment 23%. Conflict does not mediate genes, but environment. If low environment explains 29%, if high 50%. Il parental actions are ignored (standard ACE model) genes explain 52%. *Negative emotionality (NEM)*: low regard, genes explain 28%, high 56%. Low conflict, genes explain 0.67, high 0.31. If parental actions are ignored 40%. Shared environments explain little, but for high level of conflict 0.56%.
Caprara et al. (2009)	428 Twin Pairs in the Italian Twin Register. Genetic and environmental components of self-esteem, life satisfaction and optimism.	X	Self-esteem: genes explain 73% of the variance Life satisfaction: genes explain 59% of the variance Optimism: genes explain 28% of the variance
Belsky et al. (2012)	1,116 pairs of same sex twins in the E-Risk Longitudinal Twin Study followed from birth to age 12. Analysis of borderline personality related characteristics (BPRCs)	X	BRPCs scale correlation in MZ twins 0.66, in dizygotic (DZ) twins is 0.29. Genes account for 66% of variance in BRPCs. Early childhood physical maltreatment and exposure to maternal negative expressed emotions correlates with BRPCs. Family history of psychiatric disorders increase likelihood of BRPC more in presence of harsh treatment in childhood.

N John Dewey on What Makes for a Successful School

"If we take an example from an ideal home, where the parent is intelligent enough to recognize what is best for the child, and is able to supply what is needed, we find the child learning through the social converse and constitution of the family. There are certain points of interest and value to him, in the conversation carried on: statements are made, inquiries arise, topics are discussed, and the child continually learns. He states his experiences, his misconceptions are corrected. Again the child participates in the household occupations, and thereby gets habits of industry, order, and regard for the rights and ideas of others, and the fundamental habit of subordinating his activities to the general interest of the household. Participation in these household tasks becomes an opportunity for gaining knowledge. The ideal home would naturally have a workshop where the child could work out his constructive instincts. It would have a miniature laboratory in which his inquiries could be directed. The life of the child would extend out of doors to the garden, surrounding fields, and forests. He would have his excursions. His walks and talks, in which the larger world out of doors would open to him.

Now, if we organize and generalize all of this, we have the ideal school. *There is no mystery about it, no wonderful discovery of pedagogy or educational theory. It is simply a question of doing systematically and in a large, intelligent, and competent way what for various reasons can be done in most households only in a comparatively meager and haphazard manner. In the first place, the ideal home has to be enlarged. The child must be brought into contact with more grown people and with more children in order that there may be the freest and richest social life. Moreover, the occupations and relationships of the home environment are not specially selected for the growth of the child; the main object is something else, and what the child can get out of them is incidental. Hence the need of a school."*

Dewey (1915, pp. 35–37)

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