SUPPLEMENT TEXT

One-sided test modification to avoid discordant effect sizes. Owen (2009) and Pearson (1934) applied a one-sided test form of Fisher's method to address the possible discordance issue. Two Fisher scores are first obtained from left and right one-sided p-values: $S^{Fisher;L} = -2\sum_{k=1}^{K}\log(\tilde{p}_k)$ and $S^{Fisher;R} = -2\sum_{k=1}^{K}\log(1-\tilde{p}_k)$, where $\tilde{p}_k$ is the left-sided p-value of study k. The one-sided corrected Fisher score is defined as $S^{Fisher;C} = \max\left(S^{Fisher;L}, S^{Fisher;R}\right)$. Below we similarly modify the rOP method for a one-sided corrected form. Denote by $S^{rOP;L} = \tilde{p}_{(r)}$, where $\tilde{p}_{(r)}$ is the rth order statistic of left one-sided p-values $\{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_K\}$ from K studies. Similarly, $S^{rOP;R} = \tilde{q}_{(r)}$, where $\tilde{q}_{(r)}$ is the rth order statistic of right one-sided p-values $\{\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_K\} = \{1-\tilde{p}_1, 1-\tilde{p}_2, \ldots, 1-\tilde{p}_K\}$ from K studies. The test statistic is defined as $S^{rOP;C} = \min\left(S^{rOP;L}, S^{rOP;R}\right)$. Under the null hypothesis that the one-sided p-values $\{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_K\}$ are independently and uniformly distributed in $[0,1]$. The null distribution of $S^{rOP;C}$ can be derived using integration by part. Equivalently, the null distribution could also be derived using the following property.

$$\begin{aligned} & \Pr\left(S^{rOP;C} \le p | H_0\right) = \Pr\left(S^{rOP;L} \le p \text{ or } S^{rOP;R} \le p | H_0\right) \\ = \; & \Pr\left(\sum_{k=1}^{K} I(\tilde{p}_k \le p) \ge r \text{ or } \sum_{k=1}^{K} I(\tilde{p}_k \ge 1-p) \ge r \middle| H_0\right) \end{aligned}$$

Because $\sum_{k=1}^{K} I(\tilde{p}_k \le p) \ge r$ and $\sum_{k=1}^{K} I(\tilde{p}_k \ge 1-p) \ge r$ are not mutually exclusive (except when $r \ge [K/2]+1$ and $p \le 0.5$), the above probability should be calculated differently as follows.

1. For $r \ge [K/2]+1$
   (a) If $p \le 0.5$,
   $\Pr\left(S^{rOP;C} \le p|H_0\right) = 2F(K-r; K, 1-p)$, where $F(K-r; K, 1-p) = \sum_{i=0}^{K-r}\binom{K}{i}(1-p)^i p^{K-i}$ is the Binomial CDF for having $K-r$ successes in K Bernoulli trails with success probability $1-p$.
   (b) If $p > 0.5$,
   $\Pr\left(S^{rOP;C} \le p|H_0\right) = 1 - \sum_{i=K-r+1}^{r-1}\sum_{j=K-r+1}^{K-i}\frac{K!}{i!j!(K-i-j)!}(1-p)^{i+j}(2p-1)^{K-i-j}$.
2. For $r \le [K/2]$
   (a) If $p \le 0.5$,
   $\Pr\left(S^{rOP;C} \le p|H_0\right) = 1 - \sum_{i=0}^{r-1}\sum_{j=0}^{r-1}\frac{K!}{i!j!(K-i-j)!}p^{i+j}(1-2p)^{K-i-j}$.
   (b) If $p > 0.5$,
   $\Pr\left(S^{rOP;C} \le p|H_0\right) = 1$.

References.

Owen, A. B. (2009). Karl Pearson's meta-analysis revisited. *The Annals of Statistics* **37** 3867–3892.
Pearson, K. (1934). ON A NEW METHOD OF DETERMINING "GOODNESS OF FIT.". *Biometrika* **26** 425.