SUPPLEMENT THEOREMS

Theorem 1. *When all K studies have equal effect sizes ($\theta_1 = \cdots = \theta_K = \theta \neq 0$) and the effect sizes are moderate (so that the single study power* $\Pr(p_k < \alpha|\theta_k = \theta) < \pi_0$*), the power of vote counting converges to 0 when $K \to \infty$.*

Proof. Denote $\pi = \Pr(p_k < \alpha|\theta_k = \theta)$. Under the alternative hypothesis $\theta_1 = \cdots = \theta_K = \theta$, $r \sim BIN(K, \pi)$ and $r/K \to \pi$ as $K \to \infty$. Since $\pi < \pi_0$, $\Pr(\text{reject } H_0|H_a) \to 0$. □

Theorem 2. *When all K studies have equal effect sizes ($\theta_1 = \cdots = \theta_K = \theta \neq 0$) and the effect sizes are moderate but informative (so that the single study power satisfies* $\Pr(p_k < a|\theta_k = \theta \neq 0) > \Pr(p_k < a|\theta_k = 0) = a$, $\forall 0 < a < 1$*), the power of rOP for r under significance level α converges to 1 when $K \to \infty$ and $r/K = c < 1$.*

Proof. Denote $\alpha_0 = B_\alpha(r, K - r + 1)$ (quantile of $Beta(r, K - r + 1)$). α_0 is the critical value for a single study. When $K \to \infty$, $Beta(r, K - r + 1)$ has mean converges to $r/K = c$ and variance converges to 0. As a result, $\alpha_0 \to c$ as $K \to \infty$. Assume $\Pr(p_k < \alpha_0|\theta_k = \theta) = \alpha_0 + \epsilon$, and $\epsilon > 0$. Denote $m = \sum_{k=1}^{K} I(p_k < \alpha_0|\theta_k = \theta)$. For $K \to \infty$, by the law of large numbers, $(m - r)/K = m/K - r/K \xrightarrow{p} \alpha_0 + \epsilon - c \xrightarrow{p} \epsilon > 0$. Therefore $\Pr(m \geq r) \to 1$ and the power of rOP $= \Pr(\text{reject } H_0|H_a) \to 1$. □