Appendix B Body dynamic model

B1 Computation of Centre of Mass (CoM) and Inertial Moment of the Deforming Body

The fish model was based on a larval zebrafish (*Danio rerio*, Hamilton, 1822) 5 days post fertilization (dpf) (figure A3*a*), of which the morphological parameters are presented in table A1.

Initially, the model fish was assumed to be of uniform density with a density equal to water. Based on the 33x45 body surface grid (this grid has 45 cross-sections perpendicular to the body axis and the shape of each cross-section was assumed fixed to simplify the geometric modelling), the body was divided into 16x44x10 inner-body cells. The mass of each inner-body cell was calculated when the fish was straight, and it was assumed to not change during swimming regardless of cell deformation as shown in figure A4. This assumption ensured that fish mass remained constant throughout the simulated swimming episode.

The centre of mass (CoM) and moment of inertia were refreshed at each time step during the computation:

$$r_{CoM} = \begin{bmatrix} X_{CoM} \\ Z_{CoM} \end{bmatrix} = \frac{\sum_{i}^{body}(m_i \cdot r_i)}{\sum_{i}^{body} m_i}; \tag{A5}$$

$$I_{XZ} = \sum_{i}^{body}\left(m_i \cdot \left|r_i - r_{CoM}\right|^2\right) \tag{A6}$$

where m_i is the initial mass of the i-th inner-body cell, r_i is the position vector of the centre of the i-th inner-body cell.

B2 Free-swimming

The coupling of flow and body dynamics computations ensured free swimming – the flow forces generated by the fish drive the fish's translational and rotational movement in space, and this movement in turn affects the flow computation.

The body dynamic model was constructed by modelling the zebrafish as a deforming body with a time-varying centre of mass (CoM) and moment of inertia. Although the computational model allows the fish swim freely with six degrees of freedom (DoF), and we locked three DoFs: the vertical component of the swimming speed, pitch and roll were all assumed to be zero because the zebrafish larvae in this study executed swimming mainly in a horizontal plane without obvious pitch and roll.

From the flow solution, the two horizontal force components (F_X, F_Y) exerted on the body surface in the global system were evaluated by a summation of the force vectors of all the cells at the body surface due to the local pressures and shear stresses:

$$\mathrm{F} = \begin{bmatrix} F_X \\ F_Y \end{bmatrix} = -\sum_{i,j}^{surface} (A_{i,j} \cdot P_{i,j} + A_{i,j} \cdot S_{i,j}) \tag{A7}$$

where $A_{i,j}$ is the surface area of the i,j-th cell on the body surface, and $\mathbf{P}_{body,i,j}$ and $\mathbf{S}_{body,i,j}$ are pressure and shear stress vectors on this surface cell, which arose from the solution of NS equations.

The hydrodynamic moment $\boldsymbol{N}_{XY}$ was calculated as the sum of the cross product of the positional vector and the force at each computational cell centre on the body surface, such that:

$$N_{xz} = \sum_{i,j}^{surface} \left(\mathbf{r}_{i,j} \times \mathbf{F}_{i,j} \right) \tag{A8}$$

where $\mathbf{r}_{i,j}$ is the positional vector from the CM to the i,j-th cell centre on the body surface, and $\mathbf{F}_{i,j}$ is the hydrodynamic force vector at the i,j-th cell centre on body surface.

The deformable-body dynamic model was constructed based on the Newton-Euler equations of a 6DoF body motion. By locking three DoFs, we reduced these equations to a set of three coupled nonlinear ordinary differential equations:

$$\begin{aligned} M \frac{d\,\mathrm{u}_{CoM}}{dt} &= M\,\mathrm{a}_{CoM} = \mathrm{F} \\ \frac{d(I_{XY}\omega_{XY})}{dt} &= \frac{dI_{XY}}{dt}\omega_{XY} + I_{XY}\alpha = N_{XY} \end{aligned} \tag{A9}$$

where $\mathbf{F}$ is the fluid force vector acting on the body centre of mass, and $\mathbf{u}_{CM}=(u_{CM,X}, u_{CM,Y})$ is the translational velocity of the centre of mass of the fish, $\mathbf{a}_{CM}$ the translational acceleration of the centre of mass, ω_{XY} its global angular velocity about the Z-axis, α its global angular acceleration about the Z-axis, M the fish mass, I_{XY} the yawing moment of inertia (which depends on the instantaneous body shape) and N_{XY} the hydrodynamic moment about the centre of mass. Fish mass M was assumed constant but the yawing moment of inertia I varied with time and was updated at each time step. The earth frame of reference was used for all dynamic computations to avoid any additional terms that the fish deformations might otherwise introduce into the equations of motion.

By coupling the fish dynamic model with the NS solver, we directly solved the nonlinear equations of body motion numerically to obtain translational and angular acceleration variables ($a_{CM,X}$, $a_{CM,Y}$, α) at each time step. Then the second-order Runge-Kutta integration method was used to obtain the three time-varying velocies ($u_{CM,X}$, $u_{CM,Y}$, ω). We neglected the internal force during the fish's deformation and assumed that the fish's rotation was identical to the heading angle. Within the allowed three DoFs, the body-fitted grid system was updated at each time step to take into account both the swimming dynamics and the prescribed fish-body deformation: the fish model was rotated to its updated heading angle, and translated to its updated CoM position. The body-fitted grid also deformed according to the fish model and the updated grid system was then applied to flow computation in the next physical step. The relationship between the fish frame of reference and the earth frame of reference is expressed as:

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \left(\begin{bmatrix} X_i^F \\ Y_i^F \end{bmatrix} - \begin{bmatrix} X_{CoM}^F \\ Y_{CoM}^F \end{bmatrix} \right) \cdot \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} + \begin{bmatrix} X_{CoM} \\ Y_{CoM} \end{bmatrix} \tag{A10}$$

where X_i, Y_i is the position of point in earth frame of reference, X_{COM}, Y_{COM} is the position of CoM in the earth frame of reference Φ is the heading angle of the fish, also as the angle between fish frame of reference and earth frame of reference. Superscript ^{F} denotes coordinates in the fish frame of reference.

B3 Power

The total power output was defined as the sum of the powers acting on the fish and water, respectively:

$$P_{total} = P_{body} + P_{hydro} \tag{A11}$$

Hydrodynamic power was calculated as the sum of the dot products of the velocity and the hydrodynamic forces on the body, such that:

$$P_{hydro} = -\sum_{i,j}^{surface} (\mathbf{F}_{hydro,i,j} \cdot \mathbf{u}_{i,j}) \tag{A12}$$

where P_{hydro} is the hydrodynamic power; $\mathbf{F}_{hydro,i,j}$ is hydrodynamic force acting on the i, j-th surface element; $\mathbf{u}_{i,j}$ is the velocity of the i, j-th surface element.

Body power was computed as:

$$P_{iner} = \sum_{i,j,k}^{body} (\mathbf{F}_{iner,i,j,k} \cdot \mathbf{u}_{body,i,j,k}) = \sum_{i,j,k}^{body} (m_{i,j,k} \mathbf{a}_{body,i,j,k} \cdot \mathbf{u}_{body,i,j,k}) \tag{A13}$$

where $\mathbf{F}_{iner,i,j,k}$ are the inertial forces acting on the i,j,k-th inner body element cell of the body; $\mathbf{a}_{body,i,j,k}$ and $\mathbf{u}_{body,i,j,k}$ are the acceleration and velocity of the i,j,k-th inner body element cell of the body; $m_{i,j,k}$ is the mass of the i,j,k-th inner body element cell.

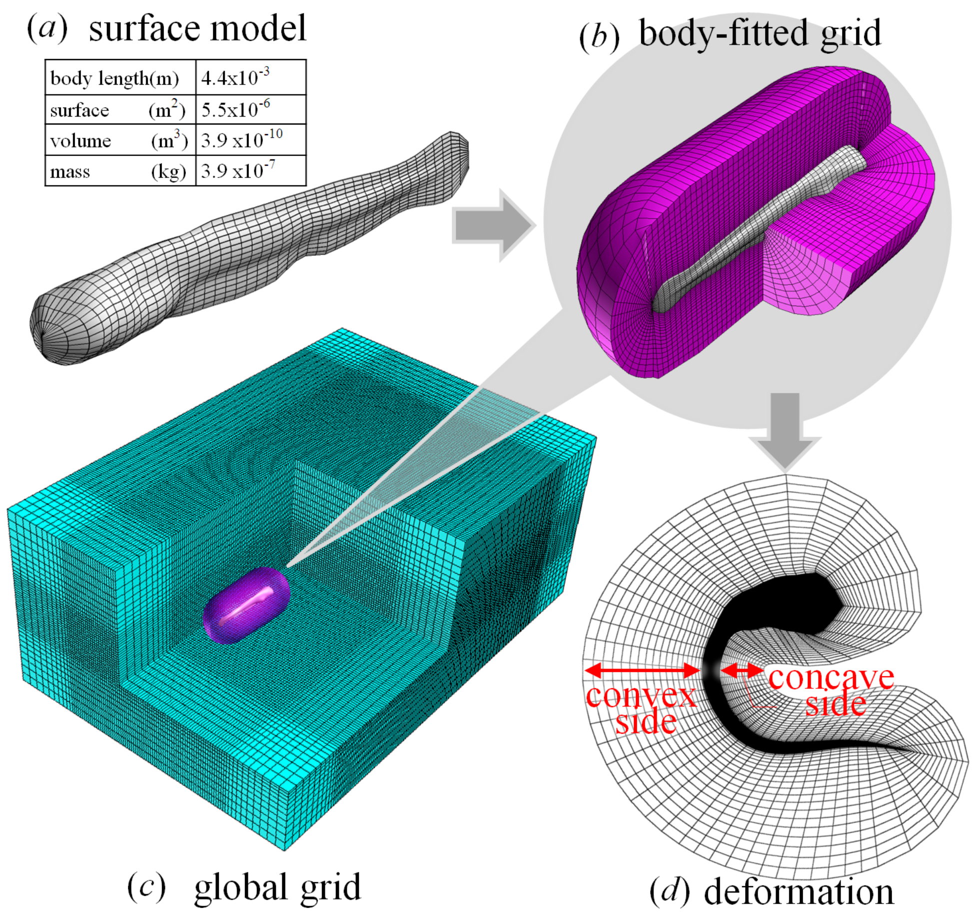

Figure A3. (*a*) Surface model; (*b*) Body-fitted grid; (*c*) Global grid; (*d*) Deformation of body-fitted grid to solve geometric problems during large body deformations.

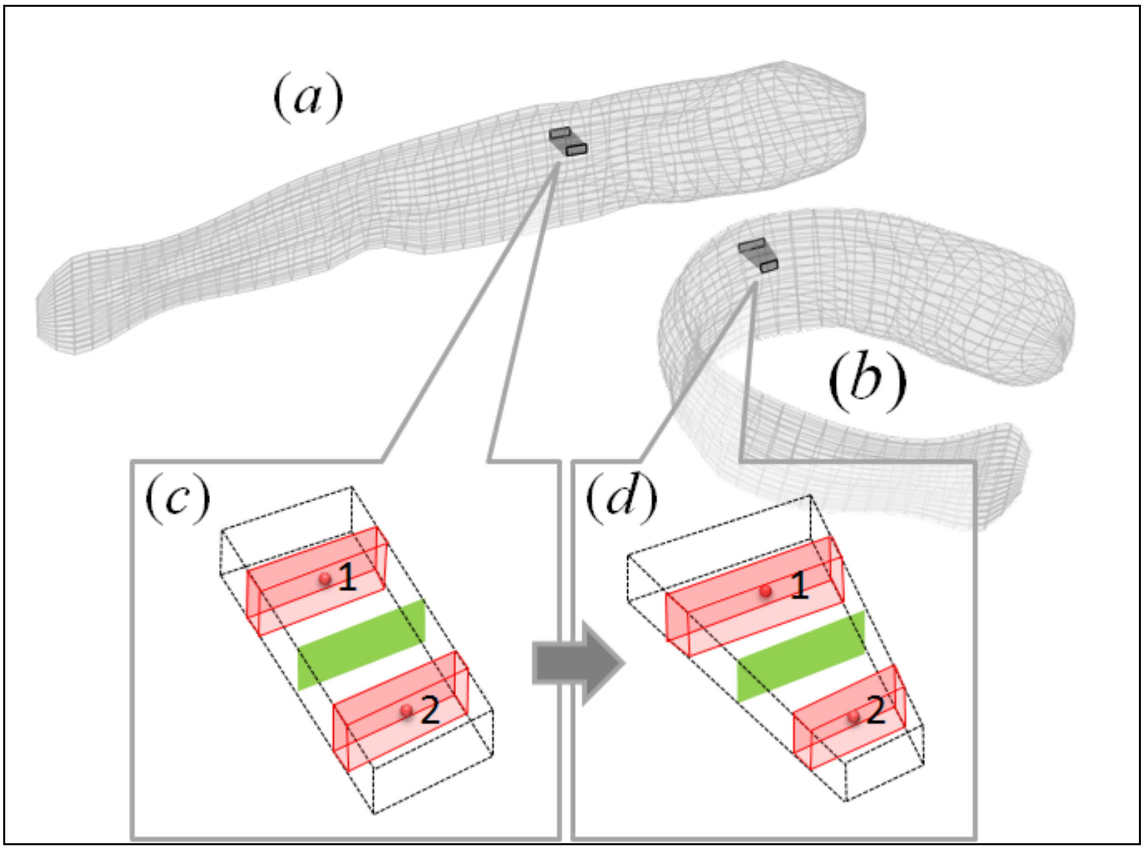

Figure A4. Body grid to calculate centre of mass and moment of inertia. (*a*) and (*b*) Based on the body surface grid, we defined rectangular columns that cut transversely through the body (example column in dark grey). These columns deform as the fish bends. Each column was divided into 10 body cells. We made the simplifying assumption that the mass of each cell remains constant despite the changes in cell volume occurring during body bending. (*c*) The mass of each body cell was calculated when the fish was straight (green: neutral plane of bending). (*d*) Although the cells change shape and size as the fish bends, cell mass was conserved.

length of the fish	4.4×10^{-3} m
Surface of fish (one side)	2.75×10^{-6} m^3
volume of body (straight fish)	3.65×10^{-10} m^3
mass of body	3.65×10^{-7} kg
axial position of CoM along the fish from the snout (straight fish)	1.21×10^{-3} m
inertial moment (straight fish)	3.16×10^{-13} kg·m^2

Table A1. Physical parameters of modeled fish.