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SUPPLEMENT FOR: "COMBINING ISOTONIC REGRESSION AND EM ALGORITHM TO PREDICT GENETIC RISK UNDER MONOTONICITY CONSTRAINT" BY QIN ET AL

TABLE S.1

Results for Experiment 3 at $t = 2$: bias, empirical standard deviation (emp sd), average estimated standard deviation (est sd), and 95% coverage (95% cov) of estimators at different censoring rates. Results based on 500 simulations with sample size $n = 500$.

	$F_1(t) = 0.4287$				$F_2(t) = 0.6886$			
Estimator	bias	emp sd	est sd	95% cov	bias	emp sd	est sd	95% cov
				Censoring rate = 0%				
EM-PAVA	-0.0018	0.0544	0.0473	0.9460	-0.0017	0.0428	0.0414	0.9740
Oracle EFFAIPW	-0.0019	0.0542	0.0478	0.9400	-0.0014	0.0422	0.0413	0.9760
type I NPMLE	-0.0093	0.0838	0.0594	0.9340	-0.0096	0.0789	0.0634	0.9500
type II NPMLE	0.0503	0.0481	0.0348	0.6720	-0.0635	0.0404	0.0280	0.4600
				Censoring rate = 20%				
EM-PAVA	-0.0019	0.0538	0.0506	0.9280	-0.0016	0.0460	0.0438	0.9460
Oracle EFFAIPW	0.0031	0.0568	0.0503	0.9400	-0.0005	0.0471	0.0436	0.9460
type I NPMLE	-0.0056	0.0781	0.0621	0.9320	-0.0073	0.0844	0.0654	0.9320
type II NPMLE	-0.0110	0.0838	0.0413	0.6320	-0.0045	0.0638	0.0356	0.7700
				Censoring rate = 40%				
EM-PAVA	-0.0028	0.0587	0.0560	0.9320	-0.0017	0.0513	0.0484	0.9340
Oracle EFFAIPW	0.0031	0.0713	0.0558	0.9160	0.0008	0.0572	0.0489	0.9160
type I NPMLE	-0.0084	0.0870	0.0672	0.9260	-0.0118	0.0926	0.0716	0.9100
type II NPMLE	-0.0356	0.0991	0.0432	0.5300	0.0222	0.0795	0.0385	0.7220

TABLE S.2

Results for Experiment 3 across a range of time points: integrated absolute bias, average pointwise variance, and average 95% coverage probabilities of estimators at different censoring rates. Results based on 500 simulations with sample size $n = 500$.

	Censoring rate					
	0%		20%		40%	
Estimator	$F_1(t)$	$F_2(t)$	$F_1(t)$	$F_2(t)$	$F_1(t)$	$F_2(t)$
	Integrated absolute bias*					
EM-PAVA	0.0591	0.0065	0.0755	0.0095	0.1653	0.0249
Oracle EFFAIPW	0.0107	0.0027	0.0865	0.0086	0.1220	0.0525
type I NPMLE	0.0296	0.0093	0.1649	0.0169	0.9244	0.0670
type II NPMLE	0.4574	0.2227	0.1559	0.1301	0.1123	0.0274
	Average pointwise variance*					
EM-PAVA	0.0018	0.0005	0.0026	0.0005	0.0053	0.0009
Oracle EFFAIPW	0.0021	0.0006	0.0036	0.0007	0.0097	0.0021
type I NPMLE	0.0027	0.0013	0.0039	0.0014	0.0108	0.0037
type II NPMLE	0.0009	0.0002	0.0019	0.0003	0.0042	0.0005
	Average 95% coverage probabilities*					
EM-PAVA	0.9425	0.9458	0.9482	0.9457	0.9492	0.9416
Oracle EFFAIPW	0.9477	0.9478	0.9514	0.9483	0.9468	0.9457
type I NPMLE	0.8978	0.9525	0.8996	0.9459	0.8158	0.9357
type II NPMLE	0.6614	0.4011	0.9288	0.7610	0.9459	0.9349

*Computed over (0,10) for $F_1(t)$ and over $(0, 5)$ for $F_2(t)$.

TABLE S.3

Empirical rejection rates for Experiment 3. Test of $F_1(t) = F_2(t)$ over the entire time range was performed using a permutation test with 1000 permutations. Results based on 1000 simulations (for test under H_0) and 200 simulations (for test under H_1), with sample size $n = 500$ and 40% censoring (under H_1).

	Nominal Levels							
Estimator	0.01	0.05	0.10	0.20	0.01	0.05	0.10	0.20
	Under $H_0 : F_1(t) = F_2(t)$				Under $H_1 : F_1(t) \neq F_2(t)$			
EM-PAVA	0.0090	0.0520	0.0970	0.1950	0.6450	0.7950	0.8650	0.9400
Oracle EFFAIPW	0.0100	0.0500	0.0980	0.1860	0.5000	0.6500	0.7450	0.8350
type I NPMLE	0.0080	0.0370	0.0980	0.1930	0.2800	0.4800	0.6200	0.7600
type II NPMLE	0.0080	0.0480	0.0940	0.1980	0.5100	0.7050	0.7850	0.8450

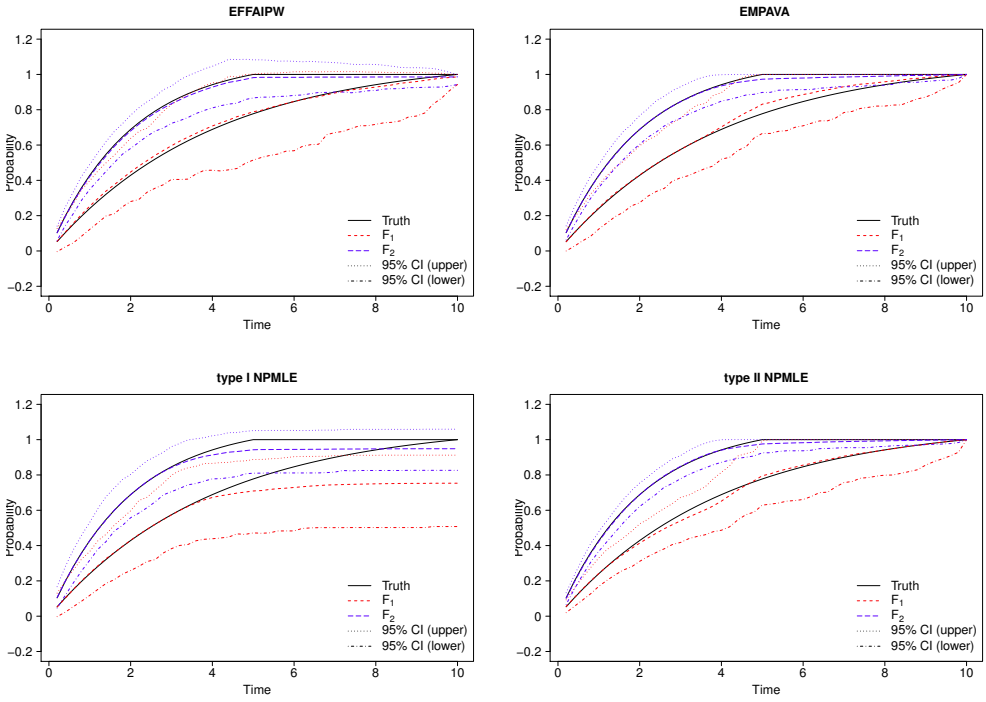

FIG S.1. *Experiment 3. True cumulative distribution function and the mean of 500 simulations along with 95% confidence band (dotted) for the four proposed estimators. Sample size is 500, censoring rate is 40%.*