

Supporting Information

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SI Text

Parameter Estimations

Eq. 1.

- M_0 : The number of macrophages in the lung is 15×10^5 cells/mL (1) so that their density is $M_0 = 1.5 \times 10^{-3}$ g/mL.
- λ_{MT_α} and λ_{MI_γ} : In experiments reported in ref. 2, macrophages were used to inhibit the growth of *Legionella pneumophila*. The macrophages were then activated either by IFN- γ or by TNF- α . We write the equation for colony-forming unit (CFU) of *L. pneumophila* in the form

$$\frac{d \text{CFU}}{dt} = \lambda \text{CFU} - dM_A \frac{\text{CFU}}{\text{CFU} + K},$$

where M_A is the density of the activated macrophages, and K is a constant. The doubling time of *L. pneumophila* is 2 h (3), so that $\lambda = 8.32 \text{ d}^{-1}$. In the case of inhibition by macrophages that were not activated by either IFN- γ or TNF- α , we have $M_A = M$ and, as reported in ref. 2, the CFU increased from 2×10^4 units (U)/mL to 10^6 U/mL after 1 d and to 5×10^6 U/mL after 2 d ($U = 2 \times 10^{-5}$ mg). Using Newton's method, we get $K = 4.48 \times 10^{-2}$ g/ml and $dM = 0.23 \text{ g} \cdot \text{mL}^{-1} \cdot \text{d}^{-1}$.

In the case of inhibition by macrophage activated by TNF- α , we have

$$M_A = \left(1 + \lambda_{MT_\alpha} \frac{T_\alpha}{T_\alpha + K_{T_\alpha}} \right) M.$$

It was found in ref. 2 that, after 2 d, the CFU, initially at 2×10^4 U/mL, increased to 10^6 U/mL when $T_\alpha = 10^2$ U/mL; to a smaller level, 5×10^5 U/mL when $T_\alpha = 10^3$ U/mL; and to an even smaller level, 4×10^5 U/mL when $T_\alpha = 10^4$ U/mL. Using these three data in the dynamics of CFU and applying Newton's method, we get three slightly different values for λ_{MT_α} , with average $\lambda_{MT_\alpha} = 0.49$.

In the case of inhibition by macrophage-activated by IFN- γ , we have

$$M_A = \left(1 + \lambda_{MI_\gamma} \frac{I_\gamma}{I_\gamma + K_{I_\gamma}} \right) M.$$

In the experimental data in ref. 2, the CFU decreased from 2×10^4 U/mL to 10^4 U/mL when $I_\gamma = 10$ U/mL. Using again Newton's method, we find that $\lambda_{MI_\gamma} = 0.65$.

- λ_{MG} : It is reported in ref. 4 that the inhibition of bacterial growth by macrophages activated by GM-CSF is approximately twice the inhibition by macrophage-activated by IFN- γ . Hence we take $\lambda_{MG} = 2\lambda_{MI_\gamma} = 1.3$.

Eq. 2.

- K_{T_r} : We assume that the inhibition by Treg reduces the production of Th1 by half if Treg is at the "average" density in asthma, reported to be 2×10^{-2} g/cm³ (5), so that $K_{T_r} = 2 \times 10^{-2}$ g/cm³.
- $K_{I_{10}T_1}$: We assume that the inhibition of IL-12 Th1 activation by IL-10 [expressed by $1/(1 + I_{10}/K_{I_{10}T_1})$] is larger than the inhibition of macrophage IL-12 production by IL-10 [expressed

by $1/(1 + I_{10}/K_{I_{10}})$]. By ref. 6, $K_{I_{10}} = 2 \times 10^{-7}$ g/mL, and we take $K_{I_{10}T_1} = (1/10)K_{I_{10}} = 2 \times 10^{-8}$ g/cm³.

- λ_{TI_2} : In ref. 7 it was shown that 10^6 CD25 P14 T cells stimulated by 0.2 ng/mL of IL-2 increased to 2×10^6 cells in 7 d. Using the equation

$$\frac{dT}{dt} = \lambda_{TI_2} \frac{I_2}{K_{I_2} + I_2} T_1 - d_T T$$

with $K_{I_2} = 5 \times 10^{-11}$ g/mL (5, 8), we get

$$T_1(7) = T_1(0) e^{7(\lambda_{TI_2} \times 0.8 - d_T)}.$$

Since $d_T = 7.71 \times 10^{-3} \text{ d}^{-1}$ (9, 10), we get $\lambda_{TI_2} = 0.13 \text{ d}^{-1}$.

Eq. 5.

- $\lambda_{I_r T_1}$: We assume that T cells produce more IFN- γ than macrophages and take $\lambda_{I_r T_1} = 2\lambda_{I_r M}$.

Eq. 6.

- $\lambda_{T_\beta T_1}$: According to ref. 11, 2×10^6 Th1 cells produced 12 pg/mL of TGF- β . Assuming steady state, we have

$$\lambda_{T_\beta T_1} T_1 - d_{T_\beta} T_\beta = 0.$$

Because $d_{T_\beta} = 3.33 \times 10^2 \text{ d}^{-1}$ (12), we get $\lambda_{T_\beta T_1} = 4.2 \times 10^{-10} \text{ d}^{-1}$.

- $\lambda_{T_\beta T_r}$: According to ref. 11, The production of TGF- β by Treg is 2.2 times more than the production by Th1 cells. Hence we get $\lambda_{T_\beta T_r} = 2.2\lambda_{T_\beta T_1} = 9.24 \times 10^{-10} \text{ d}^{-1}$.
- $\lambda_{T_\beta M}$: According to ref. 13, 10^6 alveolar macrophages produced 40 pg/mL of TGF- β . Assuming steady state, we have

$$\lambda_{T_\beta M} M_A - d_{T_\beta} T_\beta = 0.$$

Because $d_{T_\beta} = 3.33 \times 10^2 \text{ d}^{-1}$ (12), we get $\lambda_{T_\beta M} = 3.86 \times 10^{-7} \text{ d}^{-1}$.

Eq. 7.

- $\lambda_{I_{12} I_\gamma}$: According to ref. 6, alveolar macrophages incubated with IFN- γ produced three times more IL-12 p40 than without IFN- γ . Hence $\lambda_{I_{12} I_\gamma} = 3$.

Eq. 8.

- $\lambda_{T_\alpha M}$: In our model, the equation for TNF- α secretion by (in-activated) macrophages is

$$\frac{dT_\alpha(t)}{dt} = \lambda_{T_\alpha M} M - d_{T_\alpha} T_\alpha,$$

and, if M is fixed, then

$$T_\alpha(t) = \frac{\lambda_{T_\alpha M} M - e^{-d_{T_\alpha} t} (\lambda_{T_\alpha M} M - T_\alpha(0) d_{T_\alpha})}{d_{T_\alpha}}.$$

Productions of TNF- α by macrophages are reported in ref. 14 with 10^6 macrophages. After 1 d, T_α increased from 0 to 5×10^{-8} g/mL. Taking $M = 10^{-3}$ g/mL, $T_\alpha(0) = 0$, and

$d_{T_a} = 55.45 \text{ d}^{-1}$ (15), we get $\lambda_{T_a M} = 2.72 \times 10^{-3} \text{ d}^{-1}$. Alternatively, in ref. 16, 10^6 alveolar macrophages produced 66 ng/mL of TNF- α , so that $\lambda_{T_a M} = 3 \times 10^{-3} \text{ d}^{-1}$. By taking the average, we get $\lambda_{T_a M} = 2.86 \times 10^{-3} \text{ d}^{-1}$.

- $K_{I_{13}}$: According to ref. 17, TNF- α production is reduced by 20% when alveolar macrophages are incubated with 5 ng/mL IL-13. Hence we have $1/((1 + 5 \times 10^{-9})/K_{I_{13}}) = 4/5$, which implies that $K_{I_{13}} = 2 \times 10^{-8} \text{ g/cm}^3$. We assume that IL-13 is more active in lung tissue and take $K_{I_{13}} = 2 \times 10^{-7} \text{ g/cm}^3$.

Eq. 9.

- $\lambda_{I_2 T_1}$: Experiments with the production of IL-2 by Th1 are reported in ref. 18: With 3×10^5 cells/mL, IL-2 increased from 0 to 26.7 pg/mL (2 h), 435.3 pg/mL (4 h), 662.2 pg/mL (6 h), and 1,184.2 pg/mL (24 h). In our model, the equation describing the production of IL-2 by Th1 is

$$\frac{dI_2}{dt} = \lambda_{I_2 T_1} T_1 - d_{I_2} I_2.$$

With $d_{I_2} = 2.376 \text{ d}^{-1}$ (19, 20), we find that the choice of $\lambda_{I_2 T_1} = 1.15 \times 10^{-4} \text{ d}^{-1}$ makes the best fit to these experiments.

Eq. 11.

- $\lambda_{I_{10} M}$: According to ref. 21, 10^6 alveolar macrophages produce (in vitro) 3,200 pg/mL of IL-10. Using the steady-state equation

$$\lambda_{I_{10} M} M_A - d_{I_{10}} I_{10} = 0,$$

with $d_{I_{10}} = 16.64 \text{ d}^{-1}$ (22), we get $\lambda_{I_{10} M} = 5.32 \times 10^{-5} \text{ d}^{-1}$. We assume that in lung tissue, alveolar macrophages are more active than in vitro and take $\lambda_{I_{10} M} = 2 \times 10^{-3} \text{ d}^{-1}$.

Eq. 12.

- $\lambda_{I_{13} M}$: In ref. 23, 10^6 alveolar macrophages under LPS in systemic sclerosis produce 1.25 ng/mL of IL-13. However, the concentration of IL-13 in BALF was twice as much in sarcoidosis as in systemic sclerosis. Using the steady-state equation

$$\lambda_{I_{13} M} M - d_{I_{13}} I_{13} = 0,$$

with $I_{13} = 2.5 \times 10^{-9} \text{ g/mL}$, $d_{I_{13}} = 12.47 \text{ d}^{-1}$ (24), we get $\lambda_{I_{13} M} = 3.98 \times 10^{-5} \text{ d}^{-1}$. We assume that in lung tissue, alveolar macrophages are more active and take $\lambda_{I_{13} M} = 3.98 \times 10^{-4} \text{ d}^{-1}$.

Eq. 13.

- d_{CR} and d_{CH} : We assume that internalization rates of CCL20, d_{CR} and d_{CH} are equal. We further assume that when C , T_r , and T_{17} are sufficiently large, namely,

$$C = K_C, T_r = K_{T_r}, \text{ and } T_{17} = K_{T_r},$$

the loss by internalization is the same as the loss due to degradation, so that

$$2d_{CR} \frac{K_C}{K_C + K_C} K_{T_r} = d_C K_C.$$

Then with $d_C = 1.73 \text{ d}^{-1}$ (19), we get $d_{CR} = d_{CH} = 1.73 \times 10^{-6} \text{ d}^{-1}$.

- $\lambda_{CT_{17}}$: We assume that the production of CCL20 by Th17 is less than the production by activated macrophage and take $\lambda_{CT_{17}} = \frac{1}{5} \lambda_{CM}$.

Other Parameters. To determine all of the remaining parameters, we use the expression of cytokines in healthy lung tissue reported by Crouser et al. (25) and summarized in Table S1. We also use the following facts in steady-state lung tissue: The ratio of Th1 to Th17 in the healthy case is 10 (26), i.e., $T_{17} = T_1/10$, and the ratio of Treg to Th17 is ~ 5 (27), i.e., $T_r = T_1/2$; furthermore, the production of IFN- γ by macrophage is approximately half of that by Th1 cells (28), so that $\lambda_{I_r M} = (1/2)\lambda_{I_r T_1}$. Using these relations and inserting the data from Table S1 into the steady-state equations of the model for the healthy case (namely, when $f = 0$), we obtain 14 equations for concentrations of Th1 cells (T_1), for T_β , and for the 12 unknown parameters ($\lambda_{I_r T_1}$, $\lambda_{T_{17}}$, $\lambda_{I_{12} M_A}$, $\lambda_{I_{12} M}$, $\lambda_{T_a I_r}$, λ_{GM} , $\lambda_{I_{13}}$, $d_{I_{10} M}$, $\lambda_{T_r I_2}$, λ_{CM} , d_T , and d_M). Solving for these unknowns, we find the concentration of the T cells to be

$$T_1 = 1.074 \times 10^{-2}, T_r = 5.372 \times 10^{-3}, T_{17} = 1.074 \times 10^{-3}, \\ T_\beta = 1.55 \times 10^{-10}.$$

The values of the 12 unknown parameters listed above are given in Tables S2 and S3.

Sensitivity Analysis. The parameters chosen are those whose baseline was somewhat crudely estimated whereas at the same time they seem to play an important role in the development of the granuloma. Specifically, we chose all of the 15 production rate parameters from the third column of Table S2.

Following the sensitivity analysis method described in ref. 29, we performed Latin hypercube sampling and generated 100 samples to calculate the partial rank correlation coefficients (PRCC) and P values with respect to the radius of granuloma after 100 d. The PRCCs are shown in Fig. S1, and all of the P values (not shown here) are less than 0.01. We see that the production rates of IFN- γ by Th1 cells ($\lambda_{I_r T_1}$), IL-2 by Th1 cells ($\lambda_{I_2 T_1}$), and IL-12 p40 by macrophages ($\lambda_{I_{12} M}$) are highly positively correlated with the growth of the granulomas, whereas the production rates of TGF- β by Treg ($\lambda_{T_\beta T_r}$), TGF- β by Th1 cells ($\lambda_{T_\beta T_1}$), IL-10 by macrophages ($\lambda_{I_{10} M}$), and IL-13 by macrophages ($\lambda_{I_{13} M}$) are highly negatively correlated with the growth of the granulomas.

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Table S2. Parameter descriptions and values

Parameter	Description	Value and source
D_M	Diffusion coefficient of macrophage	$8.64 \times 10^{-7} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (1, 2)
D_{T_1}	Diffusion coefficient of Th1 cell	$8.64 \times 10^{-7} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (1, 2)
D_{T_r}	Diffusion coefficient of Treg cell	$8.64 \times 10^{-7} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (1, 2)
$D_{T_{17}}$	Diffusion coefficient of Th17 cell	$8.64 \times 10^{-7} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (1, 2)
D_{I_γ}	Diffusion coefficient of IFN- γ	$1.08 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
D_{T_β}	Diffusion coefficient of TGF- β	$4.32 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
D_{I_2}	Diffusion coefficient of IL-2	$1.08 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
$D_{I_{10}}$	Diffusion coefficient of IL-10	$1.08 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
$D_{I_{12}}$	Diffusion coefficient of IL-12	$1.08 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
$D_{I_{13}}$	Diffusion coefficient of IL-13	$1.08 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
D_G	Diffusion coefficient of GM-CSF	$1.728 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
D_C	Diffusion coefficient of CCL20	$1.728 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
D_{T_α}	Diffusion coefficient for TNF- α	$1.29 \times 10^{-2} \text{ cm}^{-2} \cdot \text{d}^{-1}$ (3)
λ_{M, I_γ}	Activation rate of macrophages by IFN- γ	0.69 (4, 5) & estimated
$\lambda_{M, G}$	Activation rate of macrophages by GM-CSF	1.3 (4, 5) & estimated
λ_{M, T_α}	Activation rate of macrophages by TNF- α	0.32 (4, 5) & estimated
$\lambda_{T_1, I_{12}}$	Activation rate of Th1 cells by IL-12	$6 \times 10^{-4} \text{ d}^{-1}$ (3)
λ_{T_1, I_2}	Activation rate of Th1 cells by IL-2	0.13 d^{-1} (6) & estimated
λ_{T_r, I_2}	Activation rate of Treg cells by IL-2	$1.76 \times 10^8 \text{ d}^{-1}$ (7) & estimated
$\lambda_{T_{17}}$	Activation rate of Th17 cells	$3.55 \times 10^7 \text{ d}^{-1}$ (7) & estimated
λ_{I_γ, T_1}	Production rate of IFN- γ by Th1 cells	$2.34 \times 10^{-6} \text{ d}^{-1}$ (7) & estimated
$\lambda_{I_\gamma, M}$	Production rate of IFN- γ by macrophages	$1.17 \times 10^{-6} \text{ d}^{-1}$ (7) & estimated
λ_{T_β, T_1}	Production rate of TGF- β by Th1 cells	$4.2 \times 10^{-10} \text{ d}^{-1}$ (8) & estimated
λ_{T_β, T_r}	Production rate of TGF- β by Treg cells	$9.24 \times 10^{-10} \text{ d}^{-1}$ (8) & estimated
$\lambda_{T_\beta, M}$	Production rate of TGF- β by macrophages	$3.86 \times 10^{-7} \text{ d}^{-1}$ (9) & estimated
$\lambda_{I_{12}^{p40}, M_A}$	Production rate of IL-12 p40 by macrophages	$3.78 \times 10^{-3} \text{ d}^{-1}$ (7) & estimated
$\lambda_{I_{12}^{p70}, M_A}$	Production rate of IL-12 p70 by macrophages	$9.64 \times 10^{-4} \text{ d}^{-1}$ (7) & estimated
$\lambda_{T_\alpha, M}$	Production rate of TNF- α by macrophages	$2.86 \times 10^{-3} \text{ d}^{-1}$ (10, 11) & estimated
$\lambda_{T_\alpha, I_\gamma}$	Production rate of TNF- α by IFN- γ	10.24 (7) & estimated
λ_{I_2, T_1}	Production rate of IL-2 by Th1 cells	$1.15 \times 10^{-4} \text{ d}^{-1}$ (12)
λ_{GM}	Production rate of GM-CSF by macrophages	$8.65 \times 10^{-4} \text{ d}^{-1}$ (7) & estimated
$\lambda_{I_{13}}$	Production rate of IL-13 by Th2 cells	$2.12 \times 10^{-7} \text{ g} \cdot \text{mL}^{-1} \cdot \text{d}^{-1}$ (7) & estimated
$\lambda_{I_{13}, M}$	Production rate of IL-13 by macrophages	$3.98 \times 10^{-4} \text{ d}^{-1}$ (13) & estimated
$\lambda_{I_{10}, M}$	Production rate of IL-10 by macrophages	$2 \times 10^{-3} \text{ d}^{-1}$ (9) & estimated
λ_{CM}	Production rate of CCL20 by macrophages	$4.86 \times 10^{-3} \text{ d}^{-1}$ (7) & estimated
$\lambda_{CT_{17}}$	Production rate of CCL20 by Th17	$9.71 \times 10^{-4} \text{ d}^{-1}$ (7) & estimated

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Table S3. Parameters' description and value

Parameter	Description	Value and source
d_M	Death rate of macrophage	0.49 d^{-1} (1) & estimated
d_T	Death rate of Th1 cell	$1.97 \times 10^{-1} \text{ d}^{-1}$ (1) & estimated
d_{T_r}	Death rate of Treg cell	$1.97 \times 10^{-1} \text{ d}^{-1}$ (1) & estimated
$d_{T_{17}}$	Death rate of Th17 cell	$1.97 \times 10^{-1} \text{ d}^{-1}$ (1) & estimated
d_{I_γ}	Degradation rate of IFN- γ	0.69 d^{-1} (2)
d_{T_β}	Degradation rate of TGF- β	$3.33 \times 10^2 \text{ d}^{-1}$ (3)
$d_{I_{12}}$	Degradation rate of IL-12	1.188 d^{-1} (2)
d_{T_α}	Degradation rate of TNF- α	55.45 d^{-1} (4)
d_{I_2}	Degradation rate of IL-2	2.376 d^{-1} (2, 5)
d_G	Degradation rate of GM-CSF	4.16 d^{-1} (6)
$d_{I_{13}}$	Degradation rate of IL-13	12.47 d^{-1} (7)
$d_{I_{10}}$	Degradation rate of IL-10	16.64 d^{-1} (8)
d_C	Degradation rate of CCL20	1.73 d^{-1} (2)
d_{CR}	Degradation rate of CCL20 by chemotaxis of Treg	$1.73 \times 10^{-6} \text{ d}^{-1}$ estimated
d_{CH}	Degradation rate of CCL20 by chemotaxis of Th17	$1.73 \times 10^{-6} \text{ d}^{-1}$ estimated
χ_C	Chemotactic sensitivity parameter	$10 \text{ cm}^{-5} \cdot \text{g}^{-1} \cdot \text{d}^{-1}$ (2, 9)
M_0	Alveolar macrophage density	$1.5 \times 10^{-3} \text{ g} \cdot \text{ml}^{-1} \cdot \text{d}^{-1}$ (10) & estimated
K_M	Alveolar macrophage density	$3 \times 10^{-3} \text{ g} \cdot \text{cm}^{-3} \cdot \text{d}^{-1}$ (10) & estimated
K_{I_γ}	IFN- γ saturation	$2 \times 10^{-7} \text{ g} \cdot \text{cm}^{-3}$ (11)
K_{T_α}	TNF- α saturation	$5 \times 10^{-7} \text{ g/cm}^3$ (11)
K_G	GM-CSF saturation	$1 \times 10^{-6} \text{ g/cm}^3$ (11)
$K_{I_{12}}$	IL-12 saturation	$1.5 \times 10^{-5} \text{ g/cm}^3$ (1)
$K_{I_{10}}$	IL-10 saturation	$2 \times 10^{-7} \text{ g/cm}^3$ (12)
$K_{I_{10}T_1}$	IL-10 saturation by Th1 cells	$2 \times 10^{-8} \text{ g/cm}^3$ (12) & estimated
K_{I_2}	IL-2 saturation	$5 \times 10^{-7} \text{ g/cm}^3$ (11)
K_{T_r}	Treg cells saturation	$3 \times 10^{-2} \text{ g/cm}^3$ (13)
$K_{I_{13}}$	IL-13 saturation	$2 \times 10^{-7} \text{ g/cm}^3$ (14) & estimated
K_C	CCL20 saturation	$2 \times 10^{-6} \text{ g/cm}^3$ (15)

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