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A sound and efficient measure of joint congruence

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Abstract

In the medical world, the term “congruence” is used to describe by visual inspection how the articular surfaces mate each other, evaluating the joint capability to distribute an applied load from a purely geometrical perspective. Congruence is commonly employed for assessing articular physiology and for the comparison between normal and pathological states. A measure of it would thus represent a valuable clinical tool. Several approaches for the quantification of joint congruence have been proposed in the biomechanical literature, differing on how the articular contact is modeled. This makes it difficult to compare different measures. In particular, in previous articles a congruence measure has been presented which proved to be efficient and suitable for the clinical practice, but it was still empirically defined. This article aims at providing a sound theoretical support to this congruence measure by means of the Winkler elastic foundation contact model which, with respect to others, has the advantage to hold also for highly conforming surfaces as most of the human articulations are. First, the geometrical relation between the applied load and the resulting peak of pressure is analytically derived from the elastic foundation contact model, providing a theoretically sound approach to the definition of a congruence measure. Then, the capability of congruence measure to capture the same geometrical relation is shown. Finally, the reliability of congruence measure is discussed.

Keywords

Joint congruence, elastic foundation contact model, articular load distribution

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Introduction

The socioeconomic burden of rheumatic diseases involves 120 million of people in Europe, for an estimated expense of 400 billion dollars per year, according to the European League Against Rheumatism (EULAR). Biomechanical factors and particularly abnormal or excessive articular contact stresses are hypothesized to be among the causes for cartilage degeneration if not at the origin of osteoarthritis.[1–4]

Analytical models of the articular transmission of the load have been provided only for simple, usually two-dimensional and/or axialsymmetric geometries,[5–10] while numerical formulations require long setup and computational time.[11–15] Despite their accuracy, these models are thus difficult to use in the clinical practice. However, clinical experiences[16,17] suggest that the peak stress can be related with the relative position and shape of the articulating surfaces in contact or, referring to the more used nomenclature, with joint congruence. A measure of joint congruence could thus be used as a geometrical indicator of the joint capability to distribute an applied load, providing a simple tool for the characterization of normal physiology of an articulation and conversely for the identification and quantification of pathologies, their progression and possibly their etiology.

In the clinical practice, joint congruence is the geometric similarity of two articulating surfaces evaluated by visual inspection of bioimages, under the assumption that the better the two surfaces would mate each other, the smaller the peak of pressure resulting from an applied load would be. A number of different measures have been presented in the literature differing on how the articular contact is modeled and on the

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techniques employed for their use in the clinical practice. Among these, a congruence measure (CM) has also been presented in Conconi and Parenti-Castelli[18] that proved to be efficient and suitable for clinical purposes but still empirically defined. Unfortunately, the lack of a unique and objective quantification of joint congruence affects the reproducibility of its evaluation, having an impact on the comparability among different studies and on the possibility to perform longitudinal investigations.[19] A sound quantification of joint congruence is thus needed.

The first attempts to provide rigorous measures of congruence were based on the Hertz contact theory, according to which the peak pressure is inversely proportional to the equivalent relative curvature[20] of the touching surfaces, evaluated at the point of contact.[21–24] Within the Hertzian model, it is thus possible to characterize the capability of two conjugate surfaces to distribute an applied load with a single geometrical quantity, namely, the equivalent relative curvature. However, Hertz theory holds for those contacts that begin at a single point and then under load develop small contact areas in comparison with the dimensions of the touching bodies. In other words, it holds for nonconforming contacts.

This is not the case of human joints, which in general develop contact areas comparable in size to the articular surfaces and touch each other at the beginning of the contact on multiple points. For example, the knee and the hip present additional fibrocartilaginous structures that start the contact on finite areas at the periphery of the articulation.[25–27] Moreover, these structures introduce an internal initial gap: in this condition, the equivalent relative curvature of the two articulating surface may become negative, making the application of the Hertz formulae impossible. As a result, in order to apply the Hertz theory to human joint, techniques must be defined to reduce the complex coupling of the articular surfaces to a punctiform contact. This simplification may result in a loss of information and affect the reliability of the CM.

These problems may be overcome by resorting to the elastic foundation contact model (EFCM), originally proposed by Winkler (in ref. 20), which not relying on any assumption on the shape of the contacting bodies may be directly applied to the case of human articulations.

This article has a twofold purpose: on one side, it provides a sound theoretical foundation to the definition of a measure of congruence by relying upon the Winkler's EFCM; on the other side, it shows that the CM can receive a theoretical validation from the results of the EFCM, making CM a sound measure of congruence. In particular, in this article, first the geometrical relation between the contact load and the resulting peak pressure is analytically derived from the Winkler contact model. Then, the capability of CM to capture the same geometrical relation is shown. Finally, the reliability of CM is discussed.

Geometrical relation between contact load and peak pressure within the EFCM

For the sake of clarity and without loss of generality, the geometrical relation between applied load and peak pressure will be derived for a simple nonconforming contact as depicted in Figure 1. However, the EFCM and the relations derived from it, not depending on the shape of the contacting surfaces, hold also in the case of multiple contacts or full conforming surfaces, as shown in Figure 5.

Both the contacting bodies are taken as deformable and represented by an elastic foundation of independent springs of constant height h_i, $i = 1, 2$, resting on a rigid base (Figure 1), where interaction among adjacent elements of the foundation is ignored, according to the EFCM. Friction at the interface is also considered negligible, thanks to the lubrication provided by synovial fluid.[28] As a consequence, a set of contacting springs can be in equilibrium only if the two are aligned to each other. As an additional hypothesis, all the set of contacting springs within the contact region are considered

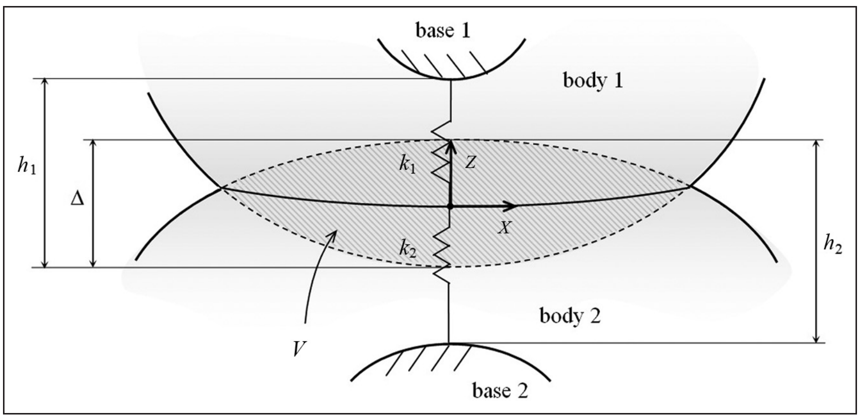

Figure 1. Cross-sectional schematization of the Winkler EFCM: dotted lines represent the undeformed profiles of the contacting bodies, while continuous lines show the final deformed configuration.

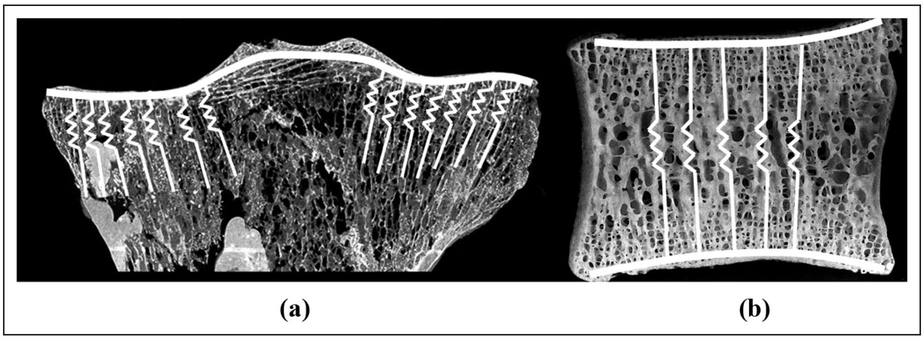

Figure 2. Coronal cross section of (a) a tibia and (b) a vertebra. Bold white represents springs that overlap with trabecular distribution, aligned almost parallel to each other and perpendicular to the articular surface, in analogy with the EFCM.
Source: reproduced from Mazurier et al.[29]

aligned with the loading direction, parallel to z in Figure 1. This assumption provides a good representation of the trabecular bone supporting the articular surfaces, as illustrated in Figure 2. In fact, trabeculae are almost perpendicular to the articular surface[30–32] and, where the curvature variation of the latter is small as within normal contact regions, they may be considered as parallel to each other. Furthermore, several studies have shown how trabeculae align with the principal directions of deformation,[33–35] thus working mainly in tension and compression, similar to springs.

As reported in Perez-Gonzalez et al.,[36] when the effect of the transverse deformations is neglected, the stiffness k_i of a generic spring can be computed as a function of Young's modulus (E_i) and Poisson's coefficient (ν_i) of each elastic foundation by the formula

$$k_i = \frac{1-\nu_i}{(1+\nu_i)(1-2\nu_i)}\frac{E_i}{h_i} \tag{1}$$

while the equivalent stiffness k of the series of two springs on a contacting element is

$$k = \frac{k_1 k_2}{k_1 + k_2} \tag{2}$$

Assessing with $\delta(x, y)$ the deformation of the equivalent spring at the position (x, y), the contact pressure at the same location can be expressed as

$$p(x, y) = k\delta(x, y) \tag{3}$$

It follows that the peak pressure p_0 will take place at the position of maximum indentation Δ, namely

$$p_0 = k\delta_{\max} = k\Delta \tag{4}$$

Considering the equivalent stiffness constant and identifying with A the projection of the contact surface on a plane orthogonal to the springs direction, dA being the infinitesimal area on which acts a single spring, the resultant F of the pressure distribution can be computed as

$$F = \int_A p(x,y)dA = \int_A k\delta(x,y)dA = k\int_A \delta(x,y)dA = kV \tag{5}$$

where V is the volume of the Boolean intersection of the two undeformed bodies (corresponding to the dashed area in the cross-sectional view of the contact depicted in Figure 1).

Within this contact model, the ratio among the resultant force and the peak pressure becomes purely geometrical, that is

$$\frac{F}{p_0} = \frac{kV}{k\Delta} = \frac{V}{\Delta} \tag{6}$$

By means of the Winkler EFCM, it is thus possible to geometrically characterize the relation between peak pressure and applied force even for highly conforming contacts. Hence, equation (6) represents the necessary relation based on which a sound measure of joint congruence may be developed.

It may be worth noting that in theory, equation (6) could be directly used for the evaluation of joint congruence. However, its application would present some clinical difficulties.

In fact, both the indentation Δ and the intersection volume V are difficult to measure in vivo. By referring, for instance, to the knee, it has been reported that regardless of the performed activities, cartilage deformation tends to a plateau of about a 5% of the original thickness.[37] Assuming a mean cartilage thickness of 2.5 mm for both tibia and femur[38,39] will lead to a 0.25 mm of indentation. This value is below the one usual for the in plane resolution of a magnetic resonance imaging (MRI) scan, which can be estimated around 0.3 mm.[39]

Furthermore, in order to observe the effect of a deformation, two pictures for the same joint are needed, one in an unloaded condition and other in a loaded condition. This would increase both the cost and the complexity of the measurements.

Finally, in order to make the measure meaningful when comparing different individuals, one parameter

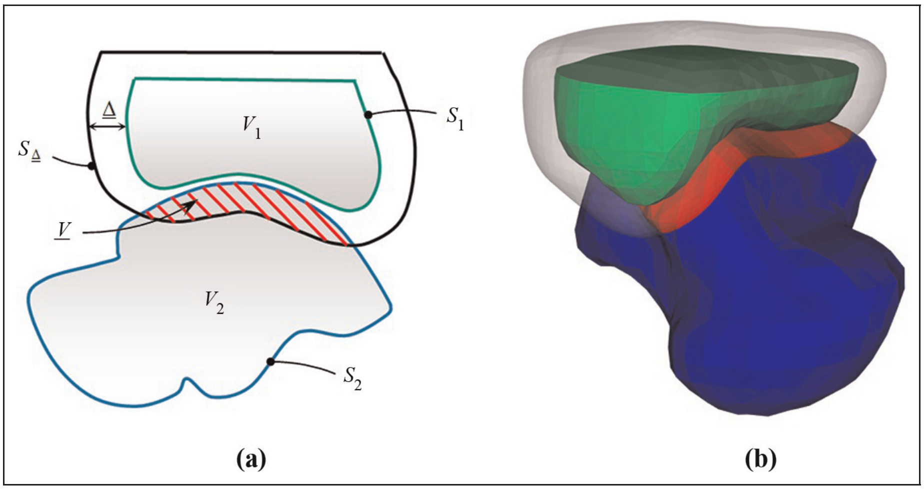

Figure 3. (a) Cross-sectional and (b) 3D representation of the determination of the intersection volume *V* (red dashed area in (a) and red volume in (b)).

among the indentation, the intersecting volume or the articular force should be kept constant. Unfortunately, no one of these quantities can be controlled with the required accuracy in an in vivo application.

Soundness of the CM

In Conconi and Parenti-Castelli,[18] an empirical approach for the evaluation of joint congruence has been proposed, and it is recalled in what follows. Let us consider two articulating surfaces in a given relative position and orientation (denoted as joint configuration). S_1 and S_2 are the undeformed and closed surfaces of the two bones together with their articular cartilage and V_1 and V_2 are the volumes comprised within the two surfaces (Figure 3(a)). An offset surface $S_{\underline{\Delta}}$ is defined as the loci of points whose distance from S_1 is equal to $\underline{\Delta}$, $V_{\underline{\Delta}}$ being the volume in it. The difference between $V_{\underline{\Delta}}$ and V_1 is called control volume V_c, that is

$$V_c = V_{\underline{\Delta}} - V_1 \tag{7}$$

The volume of the second bone laying within V_c is called intersection volume $\underline{V}$, that is

$$\underline{V} = V_c \cap V_2 \tag{8}$$

The CM is defined as

$$CM = \frac{\underline{V}}{\underline{\Delta}} \tag{9}$$

However, under a prescribed and constant value for the offset threshold $\underline{\Delta}$, joint congruence can also be directly assessed by the intersection volume $\underline{V}$. The bigger the $\underline{V}$, the more congruent the considered joint configuration will be.

The analogy between equations (6) and (9) is evident: CM can be considered equivalent to the ratio F/p_0 obtained by the EFCM when applied to a virtual contact, where the indentation is equal to the offset threshold $\underline{\Delta}$. However, with respect to the real indentation, the offset threshold has the advantage to be freely assigned. Its value can thus be chosen in order to make all the significant geometrical quantities easy to measure and kept constant within the analysis of different individuals.

It is worth noting that a desired indentation could be obtained also by moving one of the two bodies with respect to the other. This however would modify the joint configuration at which congruence is evaluated, and it would make the indentation function of six parameters. On the contrary, the offset procedure does not modify the joint configuration and requires the choice of a single parameter, namely, the magnitude of $\underline{\Delta}$.

Evaluation of the reliability of the CM

The main purpose of a measure of joint congruence is to make possible the comparison among two or more articulations, for instance, when characterizing healthy in respect to pathological subjects or when evaluating the progression of a disease such as osteoarthritis in a patient. Thus, to be meaningful, a measure of joint congruence must guarantee the correct sorting of a group of articulations.

CM depends on the value chosen for the offset threshold $\underline{\Delta}$. In order to be reliable, its sorting capability must be proven to be independent by the choice of $\underline{\Delta}$.

For nonconforming surfaces, it is possible to compare the sorting obtainable by the application of equation (9) with the one resulting from the Hertzian formulae, which have been extensively validated and may thus be assumed as gold standard for this kind of contact.

The most general case of nonconforming contact may be reduced to an equivalent contact between a plane and an elliptical paraboloid[20] for which the Hertz theory relates the peak pressure p_0 with the compressive load F by the equation

$$p_0 \cong \left(\frac{6FE^2}{\pi^3 R_e{}^2}\right)^{1/3} \tag{10}$$

where $1/R_e$ is the equivalent relative curvature at the contact and E is the equivalent Young's modulus.[20,40] The F/p_0 ratio grows thus proportionally to R_e.

The evaluation of CM in the same case is depicted in Figure 4. $\underline{V}$ can be computed as the integral along z of the section of the paraboloid on an plane orthogonal to z. This section will be an ellipse whose equation is function of the minimum and maximum relative curvature radii at the point of contact, $R_{\min}$ and $R_{\max}$, respectively, that is

$$\frac{1}{2zR_{\min}}x^2 + \frac{1}{2zR_{\max}}y^2 - 1 = 0 \tag{11}$$

The area of this ellipse will be

$$A(z) = 2\pi z\sqrt{R_{\min}R_{\max}} = 2\pi z R_e \tag{12}$$

where R_e is the equivalent relative curvature radius also employed in the Hertz formulae. The intersection volume can thus be evaluated as

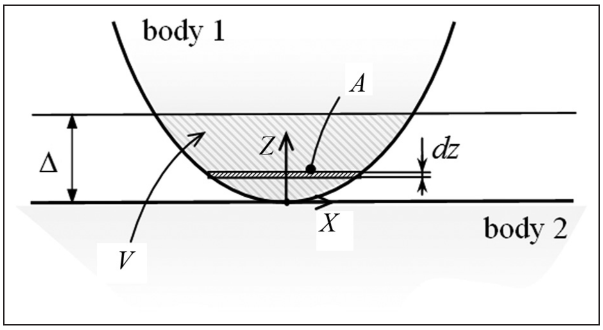

Figure 4. Analytical evaluation of the intersection volume in the case of a nonconforming contact (cross-sectional view).

$$\underline{V} = \int_0^{\underline{\Delta}} A(z)dz = \int_0^{\underline{\Delta}} 2\pi R_e z dz = \pi R_e \underline{\Delta}^2 \tag{13}$$

and CM will finally result as

$$\text{CM} = \frac{\underline{V}}{\underline{\Delta}} = \pi R_e \underline{\Delta} \tag{14}$$

which is also proportional to R_e. For nonconforming surfaces, the sorting provided by CM is thus equivalent to the one provided by Hertz theory.

For what concerns the case of conforming surfaces, it is not possible to provide a mathematical validation of the sorting property of CM since a general analytical description is missing for this kind of contact. It is, however, possible to proceed to a numerical validation.

A significant example of articular contact that does not fit the hypothesis of the Hertz theory is represented by the knee. In this joint, the menisci increase the conformity between the tibial plateau and the femoral condyles, moving the initial contact at the periphery of the articulation and leaving an internal initial gap.[41,42] In Adeeb et al.,[27] it has been shown that this geometry results in a better distribution of the applied load with respect to what a full congruent articulation would do. The application of CM to the same cases confirms this result showing that despite the initial internal gap, the menisci increase the joint congruence in comparison to a full congruent contact (Figure 5).

More in general, CM has been previously used in a kinetic model of the ankle joint, where it was assumed that human articulations move along a maximum congruence path.[18] In that work, the tibiotalar relative motion was considered as a 1-degree-of-freedom motion[43] and parameterized by the flexion angle, defined according to the Grood and Suntay[44] notation. For every value of the flexion within the range of motion of the joint, the five-dimensional space of the remaining parameters of the tibiotalar relative position

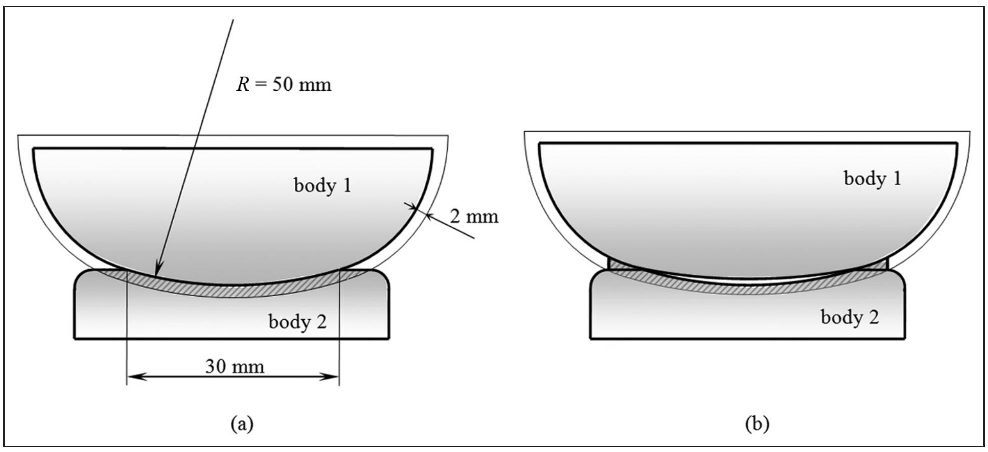

Figure 5. Evaluation of joint congruence for (a) a full conforming contact, CM = 72.34 mm^2, and (b) a contact with peripheral structures, CM = 77.36 mm^2. The internal gap in (b) is 0.25 mm, magnified in the figure for the sake of clarity.

and orientation was spanned through an optimization algorithm, searching for the maximum congruence configuration by means of CM. This operation is not different from sorting among several individuals within a population, where each joint configuration represents a distinct subject.

The algorithm converged on a common trajectory independently from the value of the offset threshold employed for the definition of V_c, thus proving numerically that the capability of CM to sort a group does not dependent by the choice of $\underline{\Delta}$. On the other side, the good matching between this trajectory and experimental data supports the correctness of the sorting.

The measure is thus reliable for the evaluation of congruence both for conforming and nonconforming surfaces, independently form the value chosen for the offset threshold, that may thus be arbitrarily chosen. As an indication, $\underline{\Delta}$ should be greater than the accuracy of the surface registration but not as big as to include bone features not related to the articulation. Clearly, the chosen value must be kept constant within a group to be sorted.

Conclusion

Joint congruence is an estimator of the joint capability to distribute an applied load. Its measure allows the characterization of the healthy and pathological states of an articulation, thus helping the study of the etiology of different pathologies and the definition of new treatments.

In this article, a measure of joint congruence empirically defined in order to be suitable for the clinical application has been recalled. The Winkler EFCM has been used to provide sound theoretical basis to CM by showing the capability of this measure to capture the geometrical feature related to the load distribution at the contact. Also, the reliability of the sorting provided by CM has been proven for both conforming and nonconforming contacts. The soundness of CM is, therefore, analytically supported.

In addition, it has been shown that CM permits the representation of articular surfaces by means of simple triangulated meshes (such as STereoLithography, STL), thus avoiding the need of a differentiable representation. Moreover, CM does not require the identification of contact points. On the contrary, it can be applied even when the articulating bodies do not touch each other, as in the case of fibrocartilaginous structures moving the contact to the periphery of the articulations, thus introducing an internal gap, or where the cartilage layer representation is missing, for instance, when studying articulations from computerized tomography (CT) scans. Future work will aim at validating CM as a possible tool for the identification of the onset and progression of joint osteoarthritis as, for instance, in the first carpo-metacarpal joint.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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