Supporting Information

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SI Text

S1. Parameter Ranges for Critical Transitions: Bifurcation Analysis in a Two-Species Mutualistic Model

It is difficult to track analytically the exact mathematical nature of the transitions we observed in our mutualistic communities. Numerical estimation of eigenvalues of the deterministic model at equilibrium shows that indeed all transitions were associated with zero crossing eigenvalues. Such zero-eigenvalue bifurcations, however, can occur continuously (transcritical) or discontinuously (fold). Therefore, we chose to categorize the transitions in our simulations as gradual and abrupt (as defined in *Materials and Methods*) rather than transcritical or fold. Nonetheless, we show that, in a simple two-species mutualistic model, the two environmental stress parameters we studied in the text and *SI Text* (section S2) are associated with crossing a fold bifurcation (1). The two-species mutualistic model reads as follows:

$$\frac{dA}{dt}=\alpha A-bA^2+\frac{\gamma PA}{1+h\gamma P}-lA+u \qquad \textbf{[S1]}$$

$$\frac{dP}{dt}=\alpha P-bP^2+\frac{\gamma AP}{1+h\gamma A}-lP+u, \qquad \textbf{[S2]}$$

where both plant P and animal A biomasses grow with rate α, compete within their respective guilds with interspecific rate β, and enjoy a mutualistic benefit following a saturating function with handling time h and mutualistic strength γ between plants and animals. We also introduced a mortality term l for both plants and animals following ref. 1. We assumed a small immigration rate u for all species.

Just like in our multispecies communities, in the case of obligate mutualism (growth rates $\alpha < 0$), the two species can survive only under strong mutualism ($\sim\gamma > b$) (Fig. S7*A*). Decreasing the mutualistic strength γ makes the two species to suffer until a point where they both crash through a fold bifurcation at which the internal feasible equilibrium (both animal and plant species present) merges with the unstable saddle (Fig. S7*A*). Interestingly, the trivial (extinct) equilibrium is always stable. That means that it is impossible to recover from extinction even when we restore the mutualistic strength back to its starting value. In other words, there is no hysteresis, as the unstable and stable equilibria do not meet (Fig. S7*A*). Nonetheless, they asymptotically approach each other. This implies that the basin of attraction of the extinct equilibrium is very small, and in that sense even a small perturbation could restore plant and animal back from extinction. In effect, a shift back to a feasible plant–animal equilibrium is highly likely through a so-called noise-induced transition, but not through a fold bifurcation. Similarly, in the case of facultative mutualism (growth $\alpha > 0$), increasing the loss rate l causes a sudden shift to extinction through a fold bifurcation (1) (Fig. S7*B*). In this case, restoring conditions (reducing the loss rate back to its starting values), the species shift back from extinction through another fold bifurcation. In this case, contrary to the decreasing mutualistic strength scenario, there is hysteresis.

We can simultaneously mark these two pathways to a critical transition by performing a two-dimensional bifurcation analysis (Fig. S7*C*). For low mutualistic strength ($\sim\gamma < b$), there is no fold bifurcation, but a transcritical (gradual) transition to extinction when increasing background mortality. Similarly, there is no change in stability when decreasing mutualistic strength under a strong background mortality. For in-between conditions, transitions between coexistence and extinction take place through a fold bifurcation. Regardless of the level of mutualistic strength, the recovery to coexistence occurs always when background mortality becomes weaker than the growth rate α (< 1).

S2. Robustness of Critical Slowing-Down Indicators to the Environmental Driver

We tested the robustness of our results in an alternative scenario that also leads to community collapse. In this scenario, we hypothesized that global environmental change may induce a direct stress on the growth rates of animals and plants in our communities following ref. 1. Observations on the decline of bee populations (2, 3), or land use changes that have detrimental effects on plant and animal biomasses (4) are in line with this scenario. We implemented this scenario by introducing a mortality term (l) for both plants and animals as follows:

$$\begin{aligned}\frac{dP_i}{dt}&=P_i\left(\alpha_{Pi}-\sum_j\beta_{Pij}P_j+\frac{\sum_j\gamma_{Pij}A_j}{1+h\sum_j\gamma_{Pij}A_j}\right)-lP_i+u\\ \frac{dA_k}{dt}&=A_k\left(\alpha_{Ak}-\sum_j\beta_{Akj}A_j+\frac{\sum_j\gamma_{Akj}P_j}{1+h\sum_j\gamma_{Akj}P_j}\right)-lA_k+u\\ &\textit{for } i=[1,n],\quad k=[1,m].\end{aligned} \qquad \textbf{[S3]}$$

In our treatment of a simple two-species mutualistic model (section S1), an increase in the loss term l leads to community collapse through a fold bifurcation, and the same has been shown in a model where only pollinators suffered an increase in mortality (1). Contrary to the scenario in the main text, here species do not depend on the presence of a strong mutualism for their survival (facultative mutualism) (5). Growth rates $\alpha_{(P),(A)}$ were sampled uniformly from [0.1, 0.3]. We started simulations in a strong mutualistic regime $\gamma_o = 10\tau$, with no background mortality ($l = 0$), and with random initial species biomasses ([0, 10]). Once the parameterization allowed for all species to be present in each community, we gradually increased the loss term l from 0 to 8 for both plants and animals in 200 steps. At each step, we simulated for 500 time steps until the community reached equilibrium and recorded species biomasses. The identification of abrupt transitions and the quantification of the CSD indicators were performed as described in *Materials and Methods*.

We found similar trends in the pattern of transitions in our communities to the trends in the scenario presented in the main text. In this scenario and for the parameterizations we chose, the onset of collapse was abrupt in 75 out of the 79 communities (Fig. S6*A*). Resilience indicators measured at species and community level also showed that they could be used to signal the proximity of communities to the onset of collapse as we found in the main text (Fig. S6*B*). Similar to the scenario in the main text, not all species equally signaled the proximity to the onset of community collapse. Strong negative correlations were found with species degree (Fig. S6*C*). Interestingly, we confirmed the strong correlations between CSD indicators and the time of extinction of individual species (Fig. S6*D*). Note that the negative correlations we found in Fig. S3*D* are in contrast to the positive correlations of the scenario in the main text due the opposite direction in the environmental stressor (higher background mortality indicated less resilience).

S3. Multiplicative vs. Additive Noise and Variance Indicators

Critical slowing down is reflected as an increase in variance (usually measured as standard deviation, STD). In our simulations, however, changes in species STD generally decreased, contradicting the expected pattern from theory (Fig. S8*A*). This is because we have introduced environmental stochasticity scaled to the actual biomass of each species (multiplicative noise). As a result, when biomass decreased, the magnitude of added noise also decreased. To account for this scaling effect, we measured changes in variability expressed as coefficient of variation (CV). CV is generally an unbiased measure of variability that has been traditionally applied in quantifying species stability (6, 7) and recently used as an early-warning signal for comparing populations at the edge of extinction (8). Thus, although biomasses of all species strongly decreased when approaching the onset of collapse (Fig. S8*A*), this did not imply that the increase in CV we reported was driven by decreases in biomass rather than a rise in STD. To illustrate the effect of multiplicative noise, we simulated all communities without scaling to species biomass (additive noise, Fig. S8*B*). As expected, we found a rising STD, whereas the general decreases in biomass only made increases in CV stronger.

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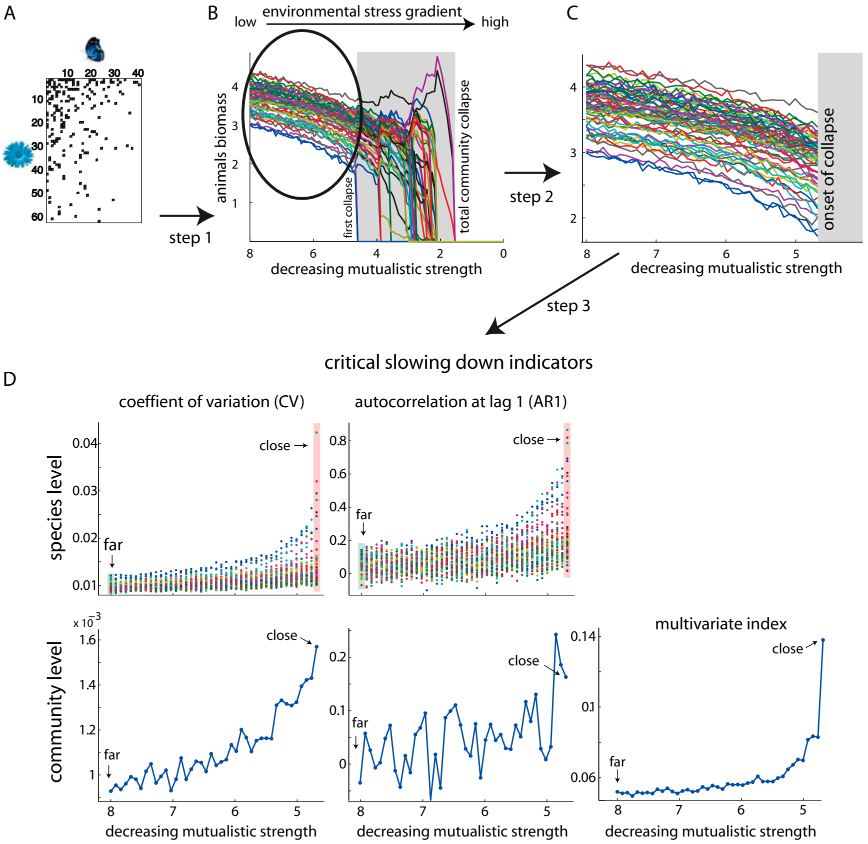

Fig. S1. Flowchart for estimating CSD indicators. Flowchart exemplifying the estimation of CSD indicators. (*A*) Plant–pollinator community from Cordon del Cepo, Chile. Black boxes represent a mutualistic link between plants and animals. (*B*) Decreasing mutualistic strength γ stresses species biomasses until the collapse of the complete community. (*C*) We estimated indicators only in the part of the time series that precedes the first collapsing event (circled in *B*). (*D*) Species-level (CV and AR1) and community-level (CV, AR1, and multivariate index) indicators measured for every level of mutualistic strength up to the first collapsing event. Community-level indicators show an increase, whereas species-level indicators vary in the strength of their trends. We quantified trends for species- and community-level indicators by estimating the natural logarithmic ratio close (one step before the onset of collapse) and far (at the beginning of the experiment) from the transition.

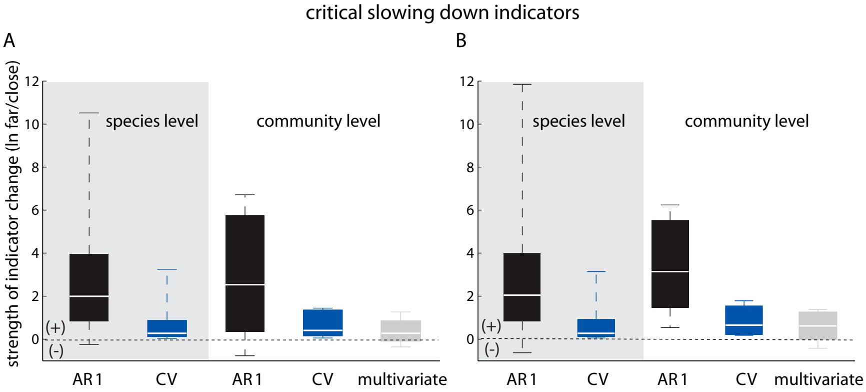

Fig. S2. Performance of resilience indicators at species and community level for Plants and Animals. CSD indicators measured in the 79 mutualistic communities before the onset of their collapse distinguished for (*A*) Plants and (*B*) Animals. Performance of CSD indicators measured as the natural logarithmic ratio of autocorrelation at lag 1 (AR1) and coefficient of variation (CV) close and far from the first transition experienced by the community. Positive values indicate an increase in the indicators. Boxplots include the median, 5th and 95th percentiles, and box whiskers indicate minimum and maximum values.

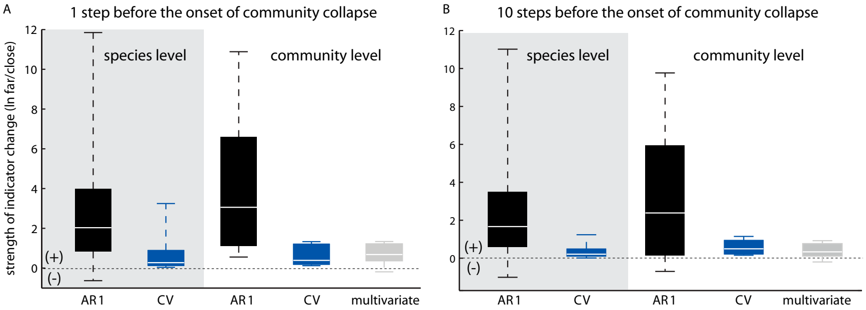

Fig. S3. Performance of resilience indicators at species and community level 1 step and 10 steps away from the onset of collapse. Comparison of changes in CSD indicators when measured at 1 step (*A*) vs. 10 steps (*B*) before the onset of collapse. There are no differences, although the trends are expectedly weaker when CSD indicators are estimated further from the onset of collapse. Negative values indicate decreasing trends. Boxplots include the median, 5th and 95th percentiles, and box whiskers indicate minimum and maximum values.

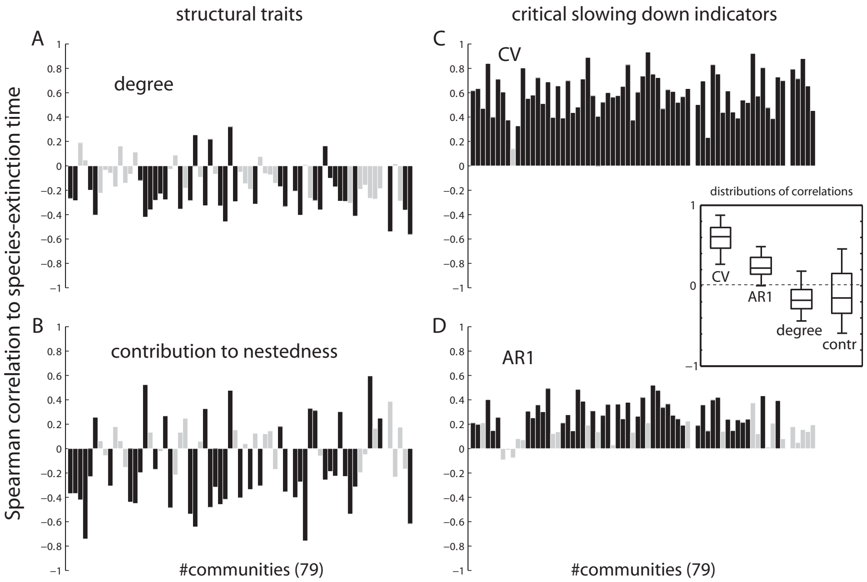

Fig. S4. Correlations between traits and CSD indicators with the timing of species extinctions. (*A* and *B*) Correlations between the timing of species extinction (the level of mutualistic strength at which species went extinct) and species traits: degree, the number of species interactions, and contribution to nestedness, the level of shared interacting partners in the community. (*C* and *D*) Correlations between changes in CSD indicators (coefficient of variation, CV, and autocorrelation at lag 1, AR1) and the timing of species extinction. The *Inset* summarizes the distributions of all correlations. Species that go extinct first tend to show the strongest increase in CV and appear to be specialists. Gray bars indicate nonsignificant correlations ($P > 0.05$). Zero values are assigned to communities where all species went extinct at the same time.

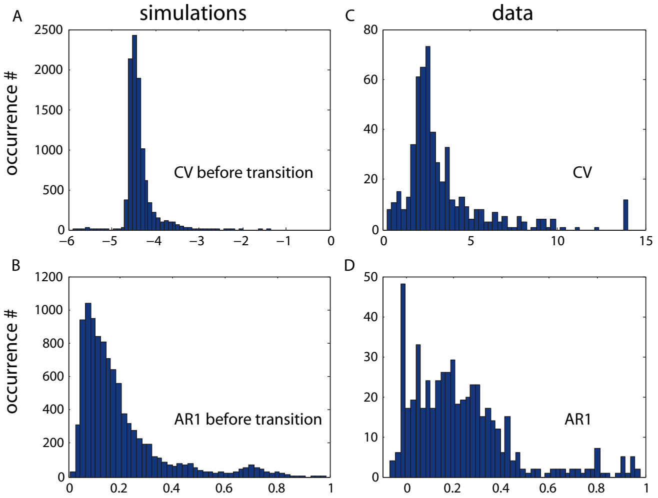

Fig. S5. Comparing variability and autocorrelation in simulated and empirical data. Distributions of variability (measured as CV) and autocorrelation at lag 1 (AR1) estimated in (*A* and *B*) our simulated communities just before the onset of community collapse; (*C* and *D*) in 529 time series derived from the Global Population Dynamics Database (1) with length of >100 points. The ranges for CV are of multiple orders of magnitude different between simulated and empirical data (*A* and *C*; note logarithmic *x* axis for *A*), pointing out that our mutualistic model cannot reproduce realistic population dynamics. On the other hand, the distributions are rather similar. Although species on the right tails of the distributions could be considered most vulnerable to extinction following the critical slowing-down hypothesis in the case of the simulated data, this interpretation does not hold for the empirical data as long as we do not take into account differences among species (like growth rates), environmental stressors, or length and quality of the time series.

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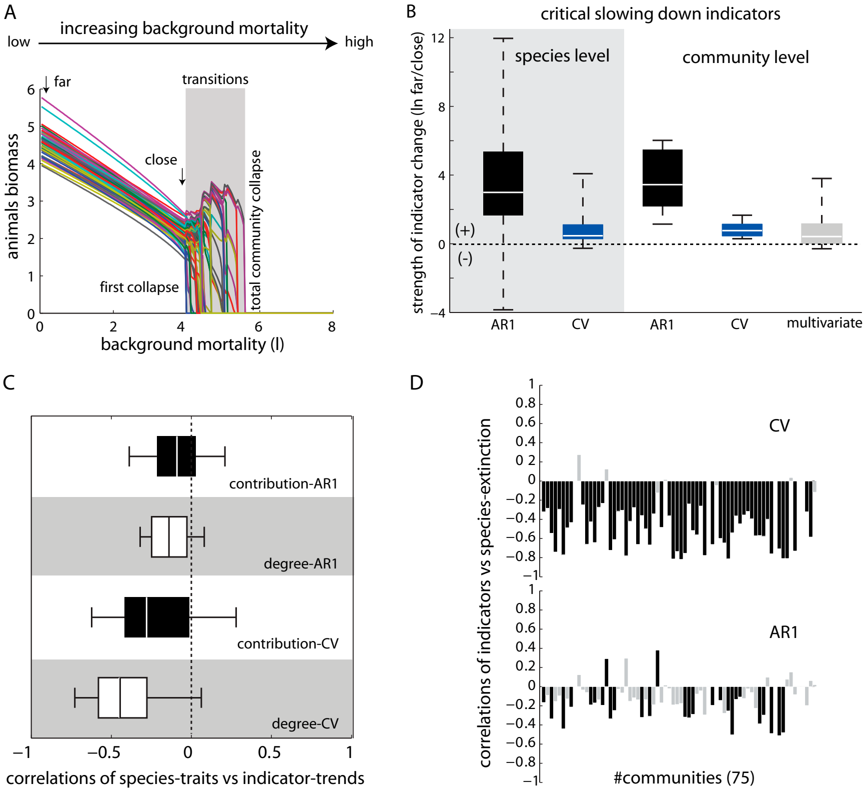

Fig. S6. Community collapse under increasing background mortality. An alternative scenario of gradual environmental change (increasing background mortality). (*A*) Just like in the scenario presented in the main text, there is a point where the environmental stressor induces an abrupt transition marking the onset of a sequence of extinctions until the collapse of the complete community (plant–pollinator community—Cordon del Cepo, Chile, same as in Fig. 1 in the main text). (*B*) CSD indicators measured in 75 mutualistic communities before the onset of their abrupt collapse. Performance of CSD indicators measured as the natural logarithmic ratio of autocorrelation at lag 1 (AR1) and coefficient of variation (CV) close and far from the first transition experienced by the community. Positive values indicate an increase in the indicators. Boxplots include the median, 5th and 95th percentiles, and box whiskers indicate minimum and maximum values. (*C*) Correlations between species traits (degree and contribution to nestedness) and species indicators performance. Correlations of CV with degree were significantly different. Boxplots include the median; box edges, the 25th and 75th percentiles; and box whiskers, the 5th and 95th percentiles. (*D*) Correlations between species extinctions (the level of mutualistic strength that species went extinct) and species resilience indicator (coefficient of variation, CV, and autocorrelation at lag1, AR1) performance. The gray bars indicate nonsignificant correlations ($P > 0.05$). Zero values are assigned to communities where all species went extinct at the same time.

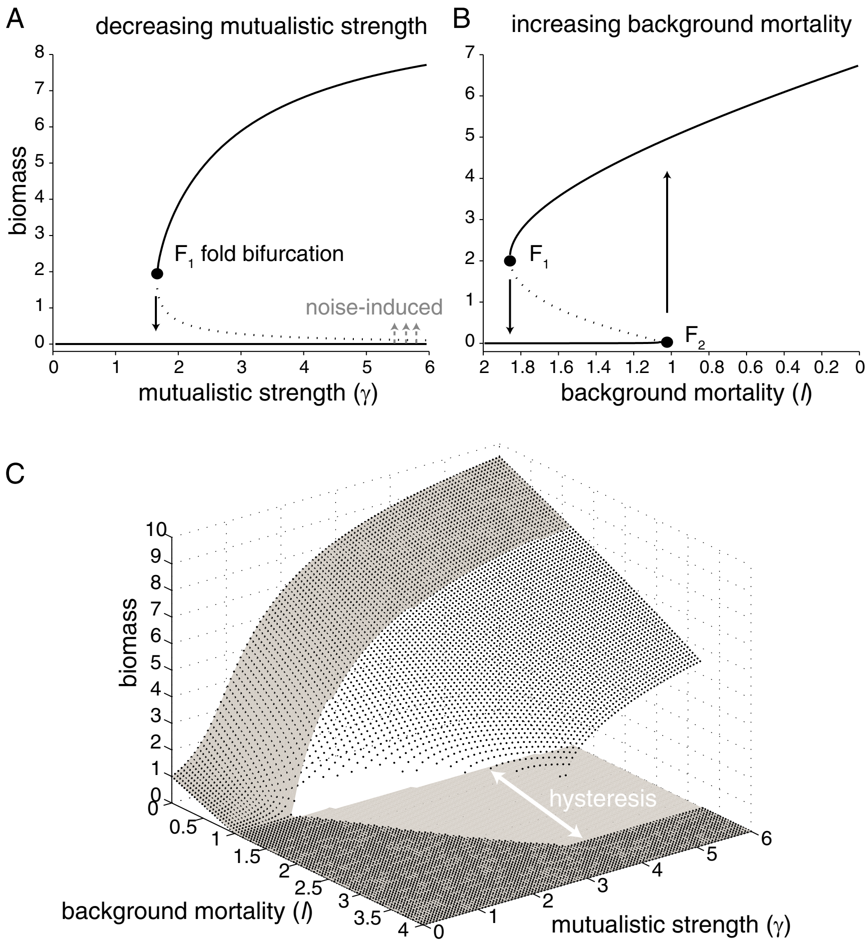

Fig. S7. Bifurcation analysis in a two-species mutualistic model. (*A*) Bifurcation plot as function of mutualistic strength γ. (*B*) Bifurcation plot as function of background mortality *l*. (*C*) A two-dimensional bifurcation plot for mutualistic strength and background mortality.

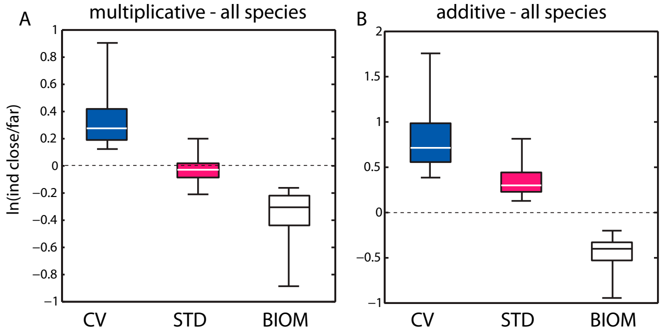

Fig. S8. Trends in species variability and biomass under different implementations of environmental noise. Distribution of changes in coefficient of variation (CV), standard deviation (STD), and biomass (BIOM) in the presence of multiplicative (*A*) and additive noise (*B*). Box edges represent 25th and 75th percentiles, and box whiskers indicate the 5th and 95th percentiles.

Table S1. Summary properties of mutualistic networks

Network property	Median	Max	Min
Richness (*Ntot*)	70	997	22
Animals (*NA*)	44.5	883	5
Plants (*NP*)	25	207	7
No. of links (*L*)	145.5	2,930	24
Connectance	0.142	0.688	0.017
Nestedness	0.415	0.935	0.077
SD of species degree	1.544	2.466	0.560

Median, maximum, and minimum values of the structural properties of the 79 plant–pollinator and plant seed–disperser mutualistic communities we used. Shown are richness (total number of species; *Ntot*), number of animal species (*NA*), number of plant species (*NP*), total number of links (*L*), connectance (the realized density of species links in the community), nestedness (the degree of overlap of species interactions), and species degree (as the standard deviation of species links in each community). We estimated nestedness using the measure described in ref. 1.

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